Basics of convection and radiative-convective equilibrium

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Program

- Elements of radiative transfer in earth’s atmosphere
- Dry convection and dry radiative-convective equilibrium (RCE)
- Effect of phase changes of water (no big deal)
- Effect of irreversible fall-out of condensed water (big deal)
- Meta-stability
- Wet RCE
- Spontaneous aggregation of wet convection
Elements of Thermal Balance: Solar Radiation

- Luminosity: \(3.9 \times 10^{26} \text{ J s}^{-1} = 6.4 \times 10^{7} \text{ Wm}^{-2}\) at top of photosphere
- Mean distance from earth: \(1.5 \times 10^{11} \text{ m}\)
- Flux density at mean radius of earth

\[
S_0 \equiv \frac{L_0}{4\pi d^2} = 1370 \text{ Wm}^{-2}
\]
Stefan-Boltzmann Equation: \( F = \sigma T^4 \)

\( \sigma = 5.67 \times 10^{-8} \ \text{Wm}^{-2}\text{K}^{-4} \)

Sun: \( \sigma T^4 = 6.4 \times 10^7 \ \text{Wm}^{-2} \)

\( \rightarrow \quad T \approx 6,000 \text{K} \)
Disposition of Solar Radiation:

Total absorbed solar radiation \( = S_0 \left( 1 - a_p \right) \pi r_p^2 \)

\( a_p \equiv \text{planetary albedo} \ (\approx 30\%) \)

Total surface area \( = 4\pi r_p^2 \)

Absorption per unit area \( = \frac{S_0}{4} \left( 1 - a_p \right) \)

Absorption by clouds, atmosphere, and surface
Terrestrial Radiation:

Effective emission temperature:

\[ \sigma T_e^4 \equiv \frac{S}{4} \left(1 - a_p\right) \]

Earth: \( T_e = 255K = -18^\circ C \)

Observed average surface temperature = 288K = 15°C
Highly Reduced Model

- Transparent to solar radiation
- Opaque to infrared radiation
- Blackbody emission from surface and each layer
Radiative Equilibrium:

Top of Atmosphere:

\[
\sigma T_A^4 = \frac{S_0}{4} \left( 1 - a_p \right) = \sigma T_e^4
\]

\[\longrightarrow \quad T_A = T_e\]

Surface:

\[
\sigma T_s^4 = \sigma T_A^4 + \frac{S_0}{4} \left( 1 - a_p \right) = 2\sigma T_e^4
\]

\[\longrightarrow \quad T_s = 2^{1/4} T_e = 303 \text{ K}\]
Surface temperature too large because:

- Real atmosphere is not opaque
- Heat transported by convection as well as by radiation
Elements of the Greenhouse Effect
Principal Atmospheric Absorbers

- **H$_2$O**: Bent triatomic, with permanent dipole moment and pure rotational bands as well as rotation-vibration transitions
- **O$_3$**: Like water, but also involved in photodissociation
- **CO$_2$**: No permanent dipole moment, so no pure rotational transitions, but temporary dipole during vibrational transitions
- **Other gases**: N$_2$O, CH$_4$
Radiation Transmitted by the Atmosphere

- Downgoing Solar Radiation: 70-75% Transmitted
- Upgoing Thermal Radiation: 15-30% Transmitted

Wavelength (μm)

Spectral Intensity

- UV
- Visible
- Infrared

Percent

Total Absorption and Scattering

Major Components:
- Water Vapor
- Carbon Dioxide
- Oxygen and Ozone
- Methane
- Nitrous Oxide
- Rayleigh Scattering
Radiative Equilibrium

• Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
• Radiative heating drives actual state toward state of radiative equilibrium
Extended Layer Models

TOA: \[ \sigma T_2^4 = \sigma T_e^4 \rightarrow T_2 = T_e \]

Middle Layer: \[ 2\sigma T_1^4 = \sigma T_2^4 + \sigma T_s^4 = \sigma T_e^4 + \sigma T_s^4 \]

Surface: \[ \sigma T_s^4 = \sigma T_e^4 + \sigma T_1^4 \]

\[ \rightarrow T_s = 3^{\frac{1}{4}} T_e \quad T_1 = 2^{\frac{1}{4}} T_e \]
Full calculation of radiative equilibrium: (no non-radiative fluxes, fixed relative humidity)
Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)
Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer
When is a fluid unstable to convection?

- Buoyancy
- Stability
Buoyancy

\[ B \equiv g \frac{\alpha'}{\bar{\alpha}} , \]

\( \alpha' = \) fluctuation of specific volume on isobaric surface

\( \bar{\alpha} = \) mean specific volume on isobaric surface
Buoyancy and Entropy

Specific Volume: \( \alpha = \frac{1}{\rho} \)

Specific Entropy: \( s \)

\[ \alpha = \alpha(p,s) \]

\[ (\delta\alpha)_p = \left( \frac{\partial\alpha}{\partial s} \right)_p \delta s = \left( \frac{\partial T}{\partial p} \right)_s \delta s \]

\[ B = g \left( \frac{\delta\alpha}{\alpha} \right)_p = \frac{g}{\alpha} \left( \frac{\partial T}{\partial p} \right)_s \delta s = -\left( \frac{\partial T}{\partial z} \right)_s \delta s \equiv \Gamma \delta s \]

Buoyancy is produced by fluctuations of entropy on isobaric surfaces
\[ \Gamma = \frac{g}{c_p} \]

Earth's atmosphere:

\[ \Gamma = \frac{1K}{100m} \]
Prototypical Convective Problem: The Prandtl Problem


\[ T = \text{constant} \]
\[ F \equiv \overline{w' B'} \]

\[ q \sim (zF)^{\frac{1}{3}} \]

\[ g\beta |T'| \sim F^{\frac{2}{3}} z^{-\frac{1}{3}} \]

\[ q_0 \sim \left( z_0^T F \right)^{\frac{1}{3}} \]

\[ g\beta |T_0'| \sim F^{\frac{2}{3}} \left( z_0^T \right)^{-\frac{1}{3}} \]

\[ g\beta \frac{d\overline{T}}{dz} = -c_1 F^{\frac{2}{3}} z^{-\frac{4}{3}} \]

\[ \rightarrow \overline{T} = \overline{T}_0 - \frac{3c_1}{g\beta} F^{\frac{2}{3}} \left[ \left( z_0^T \right)^{-\frac{1}{3}} - z^{-\frac{1}{3}} \right] \]

\[ \overline{T}_\infty = \overline{T}_0 - \frac{3c_1}{g\beta} F^{\frac{2}{3}} \left( z_0^T \right)^{-\frac{1}{3}} \]
$T_0 = 320K \quad T_A = 300K \quad z_T = 0.001 m$
Model Aircraft Measurements
(Renno and Williams, 1995)
Full calculation of radiative-dry convective equilibrium
(Surface dry turbulent enthalpy flux, fixed relative humidity)
Pure radiative equilibrium

Radiative-dry convective equilibrium
Moist Prandtl Problem

Air is everywhere saturated and filled with cloud

$T_0 = \text{constant}$
Phase change is fast!
Problem isomorphic to classical Prandtl problem with some transformations:

\[ \text{Entropy: } s = \left( c_{pd} + r_t c_l \right) \ln\left( \frac{T}{T_0} \right) - R_d \ln\left( \frac{p_d}{p_0} \right) + \frac{L_v r^*}{T}, \]

where \( c_{pd} \) is the heat capacity at constant pressure of dry air, \( c_l \) is the heat capacity of liquid water, \( r \) is the mixing ratio (mass of water vapor per unit mass of dry air), \( r_t \) is the total water mixing ratio (mass per unit mass of dry air), \( R_d \) is the gas constant for dry air, \( p_d \) is the partial pressure of dry air, \( L_v \) is the latent heat of vaporization

\[ B = \frac{\Gamma_m}{1 + r_t} s', \quad \Gamma_m = - \left( \frac{\partial T}{\partial z} \right)_{s,r_t} = \text{moist adiabatic lapse rate} \]
TKE Budget: \[
\frac{\partial}{\partial z} \left[ \overline{w'^3} + \alpha_0 \overline{p'w'} \right] = \frac{\Gamma_m F}{(1 + r_t) T_0}.
\]

New vertical coordinate: \[
\mu \equiv \frac{T_0 - \overline{T}}{\Gamma_d}
\]

TKE: \[
\frac{\partial}{\partial \mu} \left[ \overline{w'^3} + \alpha_0 \overline{p'w'} \right] = \frac{\Gamma_d F}{(1 + r_t) T_0} \equiv F_B.
\]

Back to one-parameter problem; all dry scalings are valid, e.g. :

\[
q \sim \left( \mu F_B \right)^{1/3}
\]
Irreversible fall-out of precipitation

“Pseudo-adiabatic” limit: A demon removes all condensed water as soon as it forms
Pseudo-adiabatic process on a tephigram
Precipitation Formation:

- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes
Moist RCE with precipitation

Narrow updrafts “entrain” environmental air!
Ascent not adiabatic.
Properties of Moist RCE

- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
  \[ MT \frac{\partial s_d}{\partial z} = -\dot{Q} \]
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive
Unsolved Problems in Moist RCE

- What determines fractional area covered by updrafts? (Observed to be ~1%)
- What determines characteristic updraft velocity? (Really same as first problem)
- What sets the average vertical temperature profile?
Rawinsondes
Sounding at Majuro, August 1 2016 0 GMT
Sounding at Majuro, August 1 2016 0 GMT
Plot soundings from anywhere in the world, from any time back to 1973 and lift parcels from any level, adiabatically or with entrainment

ftp://texmex.mit.edu/pub/emanuel/CLASS/Tropical/Plotsoundpak.zip

Requires MATLAB
Coincidence of saturated and unsaturated air allows for metastable states: 

*Conditional Instability*

- Relatively unusual
- Mostly over some mid-latitude continents in spring
- Important ingredient in severe convective storms (including large hail, strong winds, tornadoes)
Stability Assessment using Tephigrams:

Convective Available Potential Energy (CAPE):

\[ \text{CAPE}_i = \int_{p_n}^{p_i} \left( \alpha_p - \alpha_e \right) dp \]

\[ = \int_{p}^{p_i} R_d \left( T_{\rho_p} - T_{\rho_e} \right) d \ln(p) \]
Sounding at Norman, Oklahoma, May 5th 2007
Sounding at Norman, Oklahoma, May 5th 2007
Back to Moist RCE
Radiative-Moist Convective Equilibrium
Response of Moist RCE State to Perturbations
Response to Large-Scale Vertical Motion

- Free tropospheric temperature tied to boundary layer moist entropy
- Large-scale ascent requires more convection
- More convection causes more downdrafts
- Boundary layer entropy decreases
- Free tropospheric temperature decreases
- STABLE
Little evidence that perturbations to moist RCE state are unstable in the absence of external diabatic processes

RCE state can be destabilized by
- Interactions between perturbation surface winds and surface enthalpy fluxes (WISHE)
- Interactions between radiation and clouds and/or water vapor
If moistening of the free troposphere results in positive perturbation radiative heating, the moist RCE state can be unstable and lead to self-aggregation of moist convection.
Cloud-permitting models of radiative-convective equilibrium run in moderately large domains


- Horizontal Resolution: 2km
- Vertical Resolution: 64 levels
- Periodic lateral boundaries
- Initial sounding from domain average of smaller domain run in RCE
- No solar fluxes; specified radiative cooling rate.

Courtesy Tim Cronin, MIT
Explicit numerical simulations of radiative-moist convective equilibrium
Cloud Top Temperature and Precipitation, Day 80
Local Solar Time : 13 hours
Physics of Self-Aggregation
Aggregation Depends on Surface Temperature (in some models and boundary conditions)

Larger domain needed for high SSTs to aggregate

Allison Wing, PhD thesis, 2014
Perturbation net radiative heating rates in response to an instantaneous reduction of specific humidity of 20% from the RCE states for SSTs ranging from 25 to 45°C.
Self-Aggregation on an f-plane
Ongoing work with Marat Khairoutdinov

Set-up

- Model: System for Atmospheric Modeling (SAM; Khairoutdinov and Randall 2003)
- Domain: 40,000 km x 10,000 km; top at 30 km
- Zonally periodic; solid walls at 46°N/S; equator at the center
- Grid spacing 20 km
- Aqua-planet with uniform SST=300K
- Constant insolation: 650 W/m² with constant zenith angle of 52°
- Realistic Coriolis parameter

Precipitable Water

Snapshot of precipitable water in CONTROL simulation
Summary

- Atmospheric convection is mostly a quasi-equilibrium phenomenon, though there are important exceptions.

- Phase change of water, because it occurs near thermodynamic equilibrium, does not fundamentally change the character of convection.

- Irreversible fall-out of precipitation dramatically changes the character of convection.
Moist RCE itself presents ongoing challenges, such as fundamental scale of velocities in clouds.

Moist RCE appears stable to perturbations that do not excite external enthalpy fluxes.

RCE state can be destabilized by interactions with surface enthalpy fluxes and radiation, giving rise to a rich variety of large-scale phenomena.