An Improved Model of the Equatorial Troposphere and Its Coupling with the Stratosphere

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ABSTRACT

The Wind-Induced Sea-Air Heat Exchange (WISHE) Model of the 30–60-day oscillation developed by Emanuel is improved by adding downdrafts to the representation of convection and by coupling the troposphere to a passive stratosphere into which equatorial waves may propagate. The downdrafts are associated with a precipitation efficiency that is less than unity; this means that not all of the adiabatic cooling due to ascent in a wave disturbance can be countered by condensation heating, and the wave therefore "feels" a stable stratification as in the work of Neelin et al. As in the latter's model, growth rates of eastward-propagating Kelvin-like modes asymptote to a constant at large zonal wavenumber.

The presence of the stratosphere is shown to have a profound effect on the unstable tropospheric modes. As the upward group velocity is larger for smaller zonal wavelengths, short waves in the troposphere are strongly damped and the most unstable mode shifts to low wavenumbers.

1. Introduction

The Wind-Induced Surface Heat Exchange (WISHE) Model was originally proposed by Emanuel (1987; hereinafter E87) and independently by Neelin et al. (1987) as an alternative to the wave–CISK mechanism for explaining the 30–60-day oscillation. The acronym WISHE is intended to replace and unify the terms "air–sea interaction" used by E87 and "evaporation–wind feedback" used by Neelin et al. (1987).

Emanuel (1987) has shown that WISHE can explain some of the basic features of the 30–60-day oscillation. However, the model, like most others, fails to predict the observed largest-scale mode selection. Instead it predicts that the smallest scale grows most rapidly, although when an effective stratification is used as in Neelin et al. (1987) the growth rates asymptote to a constant at large wavenumber.

The main purpose of the present paper is to demonstrate that the presence of the stratosphere resolves this deficiency of the WISHE model. In addition to coupling the model troposphere with a passive stratosphere, allowing some of the wave energy to radiate upward, we include a more refined representation of cumulus clouds.

With these improvements, the model exhibits very strong low wavenumber selection.

This paper is organized as follows. Section 2 describes the modification of the representation of convection, while section 3 describes the results of this modification. In section 4, we couple the modified tropospheric model with a stratosphere, and describe the effects of the stratosphere on the Kelvin-like mode. In section 5 we offer a perspective on the present model, which will be considered to be a first step in constructing a comprehensive tropical atmospheric model coupled with both ocean and stratosphere and will explain both ENSO and QBO along with intraseasonal variability as a set of related phenomena.

In order to present these results in a concise manner, we restrict our attention to the eastward-propagating Kelvin-like 30–60 day oscillation mode in this paper. The westward-propagating Rossby–gravity type mode, which requires more complicated analytical treatment, will be left for future studies.

2. Modification of the WISHE model

Emanuel (1987) adopted a very simple thermodynamical/cloud representation to demonstrate in a clear and concise way the relevance of WISHE to the 30–60-day oscillation. The purpose of the present section is to offer an improved cloud representation. The new convective scheme is very similar to that of Emanuel (1989), who applied it to tropical cyclogenesis.

The modification consists of two steps. The first is to introduce a more explicit representation of the cumulus clouds. The second is an improved treatment of interactions between the subcloud layer and the free troposphere. An explicit representation of the moist entropy exchange between the subcloud layer and the midtroposphere will be introduced.

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In E87, it had simply been assumed that fluctuations in the boundary layer moist entropy ($\delta\theta_{eb}$) are linearly proportional to fluctuations in the pressure-weighted mean of moist entropy through the depth of the troposphere ($\delta\overline{\theta_{em}}$), i.e.,

$$\delta\theta_{eb} = \nu\delta\overline{\theta_{em}},$$

where ν is a constant. This assumption will be replaced by a time-dependent exchange process in the present paper. We shall still assume that fluctuations in boundary-layer entropy and midtropospheric temperature preserve the overall convective neutrality of the troposphere, i.e.,

$$\delta\theta_e^* = \delta\theta_{eb}, \tag{1}$$

where $\delta\theta_e^*$ represents fluctuations in the saturation entropy of the troposphere. The condition $\theta_e^* = \theta_{eb}$ approximately represents convective neutrality.

a. Cumulus cloud representation

The subgrid-scale motions are divided into the deep cumulus convective area and the quiescent surrounding environment. By designating the representative vertical velocity of both areas by w_c and w_d, respectively, the vertical ensemble average velocity is given by

$$w = \sigma w_c + (1 - \sigma)w_d, \tag{2}$$

where σ is the fractional areal coverage of cumulus convection.

In addition to the deep convective updrafts, we consider a shallow updraft–downdraft couplet in which the updraft and downdraft mass fluxes are equal (Fig. 1a). These may be considered either as shallow, nonprecipitating clouds or as rainy downdrafts with a return flow from the boundary layer to the free troposphere. The mass flux in the updraft and downdraft will be designated as $\sigma_s w_s$ and $-\sigma_s w_s$, respectively. Figure 1a sketches the configuration of convective drafts.

The precipitation efficiency of the system in the crude vertical structure of this model is simply the ratio of the deep updraft mass flux to the total updraft mass flux:

$$\epsilon_p = \sigma w_c/(\sigma w_c + \sigma_s w_s). \tag{3}$$

Solving for $\sigma_s w_s$ gives

$$\sigma_s w_s = [(1 - \epsilon_p)/\epsilon_p]\sigma w_c. \tag{4}$$

This representation of convection recognizes the importance of downdrafts in maintaining convective neutrality by their cooling and drying effect on the subcloud layer. In deep tropical convective systems, the precipitation efficiency ϵ_p, is probably in the range 0.7–1.0.

b. Subcloud-layer treatment

Figure 1a presents a schematic view of the cloud and subcloud-layer representation of the present model, while Fig. 1b summarizes the moist entropy exchange processes involved.

The subcloud layer of thickness h obtains moist entropy from the ocean by water vapor evaporation E, while it loses moist entropy to the lower troposphere both by cloudy downdrafts, D, and by net subsidence within the surrounding environment, S. Hence, the moist entropy balance within the subcloud layer is given symbolically by

$$hd(\ln\theta_{eb})/dt = E - D - S. \tag{5a}$$

The entropy balance within the free troposphere, with thickness H, is given by

$$Hd(\ln\theta_{em})/dt = -H\dot{R} + D + S', \tag{5b}$$

where the net radiative cooling is $\dot{R}$, D is the import of subcloud-layer entropy by shallow updrafts, and S' is the downward advection of entropy from aloft. We also introduce

$$d/dt = \partial/\partial t + \mathbf{v}_b \cdot \nabla,$$

in which $\mathbf{v}_b$ is the horizontal velocity at the top of subcloud layer.

For the representation of the moist entropy exchange processes, bulk formulas are assumed:

$$E = C_\theta|\mathbf{v}_b|(\ln\theta_{es} - \ln\theta_{eb}), \tag{6a}$$

$$D = \sigma_s w_s(\ln\theta_{eb} - \ln\theta_{em}), \tag{6b}$$

$$S = -(1 - \sigma)w_d(\ln\theta_{eb} - \ln\theta_{em}), \tag{6c}$$

$$S' = -(1 - \sigma)w_d(\ln\theta_{et} - \ln\theta_{em}), \tag{6d}$$

where $\ln\theta_{es}$ is the saturated moist entropy of the ocean surface and $\ln\theta_{et}$ is the moist entropy of the upper troposphere. Consistent with moist neutrality, $\ln\theta_{et} = \ln\theta_{eb}$ so that $S' = S$.

By substituting Eqs. (6) into (5), we obtain the explicit formulas for moist entropy balances:

$$hd(\ln\theta_{eb})/dt = C_\theta|\mathbf{v}_b|(\ln\theta_{es} - \ln\theta_{eb}) - \sigma_s w_s(\ln\theta_{eb} - \ln\theta_{em}) + (1 - \sigma)w_d(\ln\theta_{eb} - \ln\theta_{em}), \tag{7a}$$

$$Hd(\ln\theta_{em})/dt = -H\dot{R} + \sigma_s w_s(\ln\theta_{eb} - \ln\theta_{em}) - (1 - \sigma)w_d(\ln\theta_{eb} - \ln\theta_{em}). \tag{7b}$$

On the other hand, the conservation of dry entropy in the free troposphere is given by

$$gd(\ln\theta)/dt + N^2 w = \dot{Q}_{conv} - g\dot{R},$$

where N^2 is the Brunt–Väisälä frequency and $\dot{Q}_{conv}$ designates the convective heating. Neglecting the local rate of temperature change inside clouds, the latter is given, to a good approximation, by

$$\dot{Q}_{conv} \approx N^2\sigma w_c.$$

Hence, by using the decomposition of vertical velocity (2),

$$gd(\ln\theta)/dt = -N^2(1 - \sigma)w_d - g\dot{R}. \tag{7c}$$

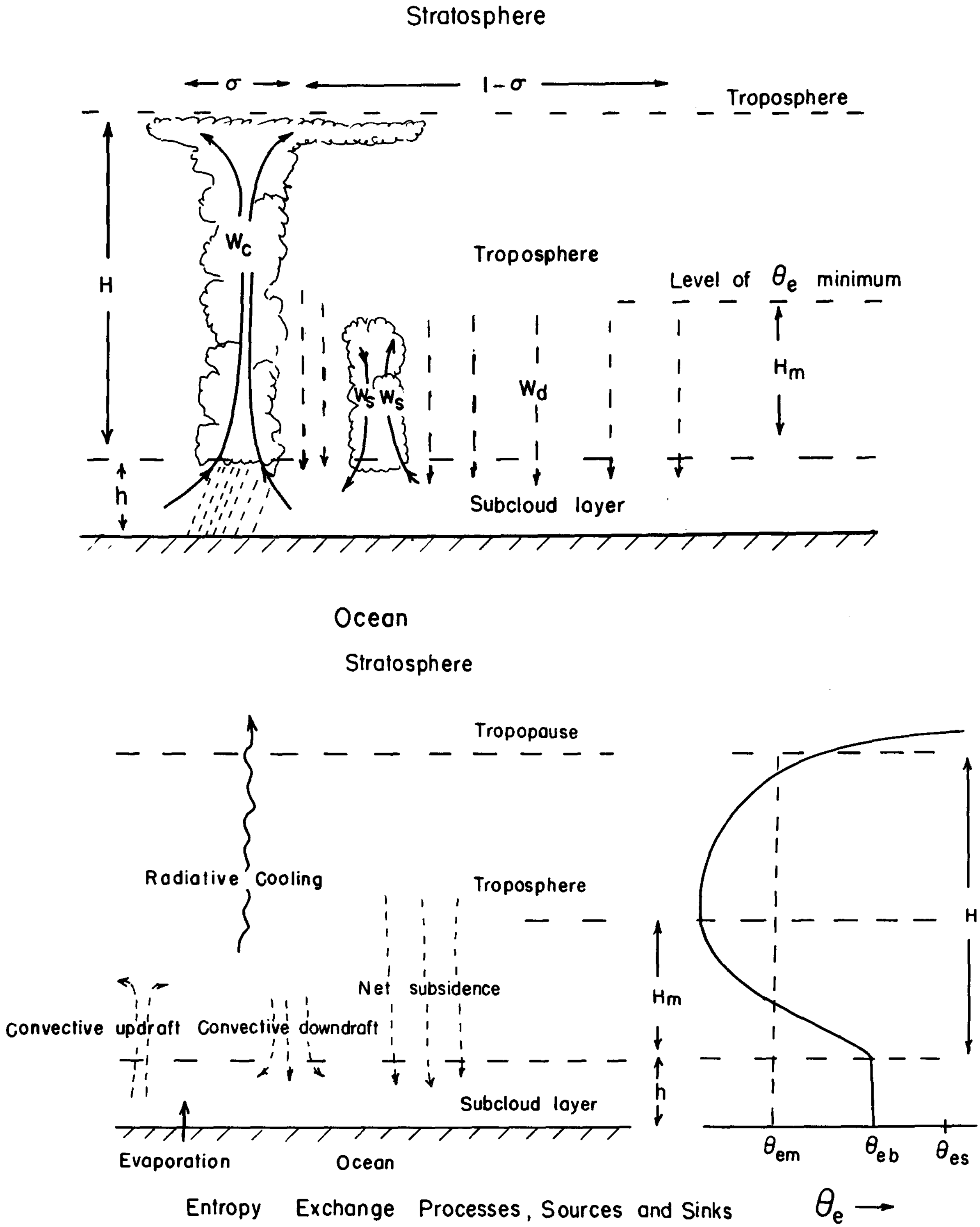

FIG. 1. Schematic representation of the model. (a) Convective representation and model structure. (b) Entropy exchange processes, sources, and sinks; mean entropy distribution.

This equation is considered to be evaluated at the middle level of the model. We adopt a finite difference continuity equation:

$$\nabla \cdot \mathbf{v}_b + H_m^{-1}[(1-\sigma)w_d + \sigma w_c] = 0, \quad (7d)$$

where H_m is taken as the height of the level where the equivalent potential temperature, θ_e, is a minimum (See Fig. 1b).

Finally, the equation of motion on an equatorial β-plane is given by

$$d\mathbf{v}_b/dt = -\nabla\delta\phi_b - \hat{\mathbf{k}} \times \beta y \mathbf{v}_b - C_D h^{-1}|\mathbf{v}_b|\mathbf{v}_b, \quad (7e)$$

where $\delta\phi_b$ is the fluctuating component of geopotential at the top of the subcloud layer, and the third term on the right-hand side is the bulk surface drag. No cumulus momentum transports have been included.

In order to complete the formulation, two supplemental relations are adopted. One is obtained from the condition of neutrality (1) of the troposphere. By taking into account the thermodynamic relation (see E87)

$$\delta \ln\theta = (\Gamma_m/\Gamma_d)\delta \ln\theta_e^*,$$

satisfied for fluctuations at constant pressure, and using (1), we obtain

$$\delta \ln\theta_{eb} = (\Gamma_d/\Gamma_m)\delta \ln\theta, \quad (7f)$$

where Γ_m and Γ_d are the moist and dry adiabatic lapse rates, respectively.

The other supplementary relation is obtained by vertically integrating the hydrostatic relation

$$\partial\delta\phi/\partial p = -\delta\alpha.$$

By taking into account the relation (see E87)

$$(\delta\alpha)_p = (\partial\alpha/\partial \ln\theta_e^*)_p\delta \ln\theta_e^* = (\partial T/\partial p)_{\ln\theta_e^*}C_p\delta \ln\theta_e^*,$$

the vertical integration of the hydrostatic equation gives

$$\delta\phi_b = -C_pT_b\epsilon\delta \ln\theta_{eb} + \delta\phi. \quad (7g)$$

Here we define

$$\epsilon \equiv (T_b - T)/T_b,$$

where $\delta\phi$ is the fluctuating component of geopotential in the free troposphere and the subscript b denotes evaluation at the top of the subcloud layer. Evaluating (7g) at the tropopause yields

$$\delta\phi_b = -C_pT_b\epsilon_t\delta \ln\theta_{eb} + \delta\phi_t, \quad (7h)$$

where $\delta\phi_t$ is the fluctuating component of geopotential at the tropopause and

$$\epsilon_t \equiv (T_b - T_t)/T_b,$$

where T_t is the tropopause temperature. The term $\delta\phi_t$ will be dealt with in section 4, which undertakes to couple the troposphere with the stratosphere.

Applying a vertical average to (7g) yields

$$\delta\phi_b = -C_pT_b\bar{\epsilon}\delta \ln\theta_{eb} + \overline{\delta\phi} \quad (7i)$$

where the overbar represents a mass-weighted tropospheric average, and

$$\bar{\epsilon} \equiv (T_b - \bar{T})/T_b.$$

(We retain the $\bar{\phi}$ term here for later use when the stratosphere is considered with a rigid lid $\overline{\delta\phi} = 0$, as shown by E87.)

Consequently, Eqs. (7a–f) and (7i) constitute a complete set of model equations. We proceed by defining a basic state with constant zonal wind $\bar{U}$, and then consider the linear instability of this basic state.

3. Results of the modified WISHE model

After some manipulations, the basic state solution is given by

$$[\ln\theta_{eb}] = [\ln\theta_{es}] - H[\dot{R}]/C_\theta|U|, \quad (8a)$$

$$[\ln\theta_{em}] = [\ln\theta_{eb}] - \epsilon_pN^2H/g, \quad (8b)$$

$$[w_c] = g[\dot{R}]/\sigma N^2, \quad (8c)$$

$$[w_d] = -g[\dot{R}]/(1-\sigma)N^2, \quad (8d)$$

where an overbar designates the basic state value, and we assume that $[w] = 0$. Also note the relation

$$[\dot{R}] = C_\theta H^{-1}|U| \ln([\theta_{es}]/[\theta_{eb}]). \quad (8e)$$

Here U is the mean zonal wind.

The relation (8a) or (8e) simply expresses the balance between entropy loss by radiation and entropy gain from surface fluxes, while (8c) and (8d) show that subsidence warming between clouds balances adiabatic cooling.

The relation (8b) may be understood by first noting that

$$N^2H/g \approx \ln\theta_t - \ln\theta \approx \ln\theta_{eb} - \ln\theta,$$

where $\ln\theta_t \approx \ln\theta_{et} \approx \ln\theta_{eb}$ has been used. Thus, (8b) may be written approximately as

$$\ln\theta_{em} \approx (1-\epsilon_p)\ln\theta_{eb} + \epsilon_p \ln\theta.$$

When all condensed water falls out as rain ($\epsilon_p = 1$), the free troposphere is bone dry and $\ln\theta_{em} = \ln\theta$. At the opposite extreme, if no water falls as rain then the atmosphere is saturated and $\ln\theta_{em} = \ln\theta_{eb}$.

After eliminating σ_sw_s using (4), the equations for perturbations on the basic state defined by (8a–d) are given by

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)u_b = -\frac{\partial}{\partial x}\delta\phi_b + \beta y v_b - \frac{2C_D}{h}|U|u_b \quad (9a)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)v_b = -\frac{\partial}{\partial y}\delta\phi_b - \beta y u_b - \frac{C_D}{h}|U|v_b \quad (9b)$$

$$\frac{\partial u_b}{\partial x} + \frac{\partial v_b}{\partial y} + \frac{1}{H_m}[(1-\sigma)w_d + \sigma w_c] = 0 \quad (9c)$$

$$g\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\delta \ln\theta = -N^2(1-\sigma)w_d - g\frac{\delta \ln\theta}{\tau_{rad}} \quad (9d)$$

$$h\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\delta \ln\theta_{eb} = \frac{H\dot{\bar{R}}}{|U|}\,\mathrm{sgn}(U)u_b - \sigma(1-\epsilon_p)\frac{N^2H}{g}w_c + (1-\sigma)\epsilon_p\frac{N^2H}{g}w_d - \left(C_\phi|U| + \frac{g}{\epsilon_p}\frac{\dot{\bar{R}}}{N^2}\right)\delta\ln\theta_{eb} + \frac{g}{\epsilon_p}\frac{\dot{\bar{R}}}{N^2}\delta\ln\theta_{em} \quad (9e)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\delta \ln\theta_{em} = -\frac{\delta\ln\theta}{\tau_{rad}} + \sigma(1-\epsilon_p)\frac{N^2}{g}w_c + \frac{1}{\epsilon_p}\frac{g\dot{\bar{R}}}{N^2H}\delta(\ln\theta_{eb} - \ln\theta_{em}) - (1-\sigma)w_d\epsilon_p\frac{N^2}{g}, \quad (9f)$$

where every quantity in the equation is a perturbation, if not otherwise designated, and u_b and v_b designate the eastward and northward components of the velocity, respectively. For the perturbation radiative cooling, a Newtonian cooling has been assumed:

$$\delta\dot{R} = \delta\ln\theta/\tau_{rad},$$

where τ_{rad} is a radiative time scale. The order of equations has been altered from (7a–e) for later convenience. In addition to (9a–f), the supplementary relations (7f, i) complete the system of equations.

The equations are then nondimensionalized following E87:

$$\begin{aligned}
x^* &= ax \\
y^* &= a^{1/4}A^{1/4}\beta^{-1/2}y \\
z^* &= H_m z \\
t^* &= a^{1/2}A^{-1/2}t \\
u^* &= a^{1/2}A^{1/2}u \\
v^* &= A^{3/4}a^{-1/4}\beta^{-1/2}v \\
w^* &= H_m A^{1/2}a^{-1/2}w \\
\delta\phi^* &= \bar{\epsilon}C_pT_b\Delta\phi = aA\phi \\
\delta\ln\theta^* &= \Delta T_{eb}/\hat{\Gamma} \\
\delta\ln\theta^*_{eb} &= \Delta T_{eb} \\
\delta\ln\theta^*_{em} &= \Delta T_{em},
\end{aligned}$$

where the asterisks denote the dimensional values. The nondimensionalization of the velocity components u, v, w apply to every level and every subdivision of vertical velocity. Definitions and typical values of the nondimensional constants are

$$A = \bar{\epsilon}C_pT_bC_\theta H^{-1}\ln(\bar{\theta}_{es}/\bar{\theta}_{eb}) \approx 1.26\times10^{-4}\ \mathrm{m\ s^{-2}},$$

$$\Delta = aH^{-1}C_\theta\ln(\bar{\theta}_{es}/\bar{\theta}_{eb}) \approx 4.4\times10^{-2},$$

$$\hat{\Gamma} = \Gamma_d/\Gamma_m \approx 1.7.$$

Also note that

$$A = (\bar{\epsilon}C_pT_b/a)\Delta.$$

We adopt the following physical values:

$a = 6.38\times10^6$ m	Radius of the earth
$\beta = 2.3\times10^{-8}\ \mathrm{s^{-1}\ km^{-1}}$	meridional gradient of Coriolis parameter
$g = 9.8\ \mathrm{m\ s^{-2}}$	gravitational acceleration
$N^2 = 10^{-4}\ \mathrm{s^{-2}}$	Brunt–Väisälä frequency
$H = 8$ km	thickness of the troposphere
$H_m = 5$ km	level of minimum θ_e in the troposphere (Fig. 1A)
$h = 500$ m	thickness of subcloud layer
$C_D = 1\times10^{-3}$	bulk coefficient of momentum exchange rate
$C_\theta = 1.2\times10^{-3}$	bulk coefficient of entropy exchange
$\bar{\epsilon} = 0.1$	thermodynamic efficiency
$\ln(\bar{\theta}_{es}/\bar{\theta}_{eb}) = 0.035$	thermodynamic disequilibrium
$\lvert U\rvert = 2\ \mathrm{m\ s^{-1}}$	magnitude of mean zonal wind
$C_p = 1000\ \mathrm{J\ kg^{-1}\ K^{-1}}$	specific heat at constant pressure
$\tau_{rad} = 50$ d	radiative time scale

Using these values, the characteristic time and velocity scales are

$$a^{1/2}A^{-1/2} = 2.25\times10^5\ \mathrm{s} = 2.6\ \mathrm{d}$$

$$a^{1/2}A^{1/2} = 28.4\ \mathrm{m\ s^{-1}}.$$

From the foregoing, we obtain the nondimensional set of equations

$$(D + 2F)u_b = ik(T_{eb} - \bar{\phi}) + yv_b \quad (10a)$$

$$(D + F)v_b = P[d(T_{eb} - \bar{\phi})/dy - yu_b] \quad (10b)$$

$$iku_b + dv_b/dy + (1-\sigma)w_d + \sigma w_c = 0 \quad (10c)$$

$$(D + \alpha_R)T_{eb} = -\hat{\Gamma}\lambda(1-\sigma)w_d \quad (10d)$$

$$(\delta D + \alpha_e + \alpha_D)T_{eb} = \mathrm{sgn}(U)u_b + \lambda[-(1-\epsilon_p)\sigma w_c + \epsilon_p(1-\sigma)w_d] + \alpha_D T_{em} \quad (10e)$$

$$(D + \alpha_D)T_{em} = \lambda(1-\epsilon_p)\sigma w_c + [\alpha_D - (\alpha_R/\hat{\Gamma})]T_{eb} - \epsilon_p\lambda(1-\sigma)w_d, \quad (10f)$$

where the term $\bar{\phi}$ is retained for the later coupling with the stratosphere. In this section, $\bar{\phi} = 0$ will be taken, corresponding to a rigid lid.

We now assume modal solutions of the form

$$\exp(ikx + \omega t),$$

and also introduce

$$D \equiv \omega + ikU.$$

Note that $\bar{U}$ in the definition of D is nondimensionalized by the factor $A^{1/2}a^{1/2}$.

The nondimensional parameters that appear in Eq. (10) are defined as follows with approximate magnitudes indicated:

$$F \equiv C_D|U^*|a^{1/2}A^{-1/2}h^{-1} \quad \sim 0.9$$

$$\delta \equiv h/H \quad \sim 0.0625$$

$$\lambda \equiv N^2 H_m / g\Delta \quad \sim 1$$

$$\alpha_R \equiv a^{1/2}A^{-1/2}/\tau_{rad} \quad \sim 0.05$$

$$\alpha_e \equiv C_\theta |U^*| a^{1/2}A^{-1/2}/H \quad \sim 0.07$$

$$\alpha_D \equiv \epsilon_p \frac{a^{1/2}A^{-1/2}C_\theta|U^*|}{H^2N^2} g \ln\left(\frac{\bar{\theta}_{es}}{\bar{\theta}_{eb}}\right) \quad \sim \frac{0.03}{\epsilon_p}$$

$$P \equiv \beta A^{-1/2}a^{3/2} \quad \sim 32.7$$

Note that, among these parameters, only the friction coefficient F is poorly estimated because not much is known about cumulus momentum transports out of the subcloud layer. We assume that *all* of the momentum flux convergence occurs in the subcloud layer. It is likely that the actual F is much smaller than the value cited above, which corresponds to a frictional decay time of only about two days. The value $F = 0.1$ will be taken in some of the following cases.

The smallness of δ indicates that the subcloud layer is always nearly exactly in thermodynamic equilibrium with the overlying troposphere so that the time derivative may be ignored in the subcloud-layer thermodynamic Eq. (10e). The parameter λ is proportional to the mean tropospheric stratification; the fact that it is order unity indicates that stratification is as important as surface heat fluxes for long waves [$k \sim O(1)$]. The effect or radiative transfer at 50-day time scales is negligible, judging from the value of α_R, while the relaxation of the boundary layer entropy back to its base state value through the action of surface fluxes, proportional to α_e, is also small.

The term α_D represents the effect of *base state* convective fluxes acting on perturbation entropies. It is small unless the precipitation efficiency, ϵ_p, is very small.

From (10b), the largeness of P shows that the zonal wind perturbation is in geostrophic balance to a very good approximation.

a. Specialization to Kelvin-like modes

The (10) is now adapted to Kelvin-like modes by taking the special case $v_b = 0$ everywhere. Here we consider solutions with a rigid lid at the tropopause, so that $\bar{\phi} = 0$. (In section 4 it was demonstrated that even with a rigid lid, surface friction insures that $\bar{\phi}$ does not vanish, although it will be numerically small. It is neglected here.)

After eliminating $(1 - \sigma)w_d$, σw_c, and T_{em} from the set (10), we obtain

$$(D + 2F)u_b = ikT_{eb} \quad (11a)$$

$$dT_{eb}/dy - yu_b = 0 \quad (11b)$$

$$\Big[(D + \alpha_D)(\delta D + \alpha_e + \alpha_D) - \alpha_D{}^2 + \frac{\alpha_D\alpha_R}{\hat{\Gamma}} + \frac{D}{\hat{\Gamma}}(D + \alpha_R)\Big]T_{eb} = [\operatorname{sgn}(U)(D + \alpha_D) + D(1 - \epsilon_p)\lambda ik]u_b. \quad (11c)$$

Equations (11a) and (11b) lead to

$$T_{eb} = T_{eb0}\exp[iky^2/2(D + 2F)]. \quad (12)$$

Taking $\sigma = \sigma_r + ik(\bar{U} - c)$, we obtain

$$\frac{1}{D + 2F} = \frac{\sigma_r + 2F - ik(U - c)}{(\sigma_r + 2F)^2 + k^2(U - c)^2}.$$

Thus, in order to get a well-behaved solution,

$$U - c < 0 \quad (13)$$

In the following, $\operatorname{sgn}(U) = -1$ is assumed for convenience and in accordance with the actual tropical situation.

By combining (11a) and (11c), a cubic algebraic dispersion relation is obtained:

$$C_1D^3 + C_2D^2 + C_3D + C_4 = 0, \quad (14)$$

where

$$C_1 \equiv \delta + \hat{\Gamma}^{-1},$$

$$C_2 \equiv C_1\alpha_D + \alpha_e + \alpha_D + 2FC_1 + \frac{\alpha_R - \alpha_D}{\hat{\Gamma}},$$

$$C_3 \equiv \alpha_D(\alpha_e + \alpha_R\hat{\Gamma}^{-1}) + 2F(C_2 - 2FC_1) + ik + (1 - \epsilon_p)\lambda k^2,$$

$$C_4 \equiv \alpha_D[2F(\alpha_e + \alpha_R\hat{\Gamma}^{-1}) + ik].$$

The dimensional solutions of (14) are given for various sets of parameter values in Figs. 2–4. Unless otherwise stated, the parameter values are as given earlier. Figure 2 shows the dependence of the growth rate (a), and phase velocity (b) on the precipitation efficiency, ϵ_p. Figures 3 and 4 show the dependence of these on the surface friction F, and on λ, respectively. Remarks on these results will be made after considering some particular limits as follows.

b. Some particular limits

1) $\epsilon_p = 0$

Since $\alpha_D \sim \epsilon_p^{-1} \to +\infty$, hence $T_{em} - T_{eb} \to 0$ in (10e) and (10f). By taking $T_{em} = T_{eb}$, w_c can be eliminated from (10e) and (10f), which leads to

$$[(\delta + \hat{\Gamma}^{-1} + 1)D + \alpha_R \hat{\Gamma}^{-1} + \alpha_e]T_{eb} = \mathrm{sgn}(U)u_b. \quad (15)$$

Equations (10a), (10b), and (15) then constitute a complete system, which is identical to that of E87, except for the factor $1/\hat{\Gamma}$.

From (8b) it is observed that $\epsilon_p = 0$ corresponds to a completely saturated basic state atmosphere. It is for this reason that the system reduces to that of E87.

2) $\delta \sim \alpha_R \sim \alpha_D \sim \alpha_e \to 0$

In this limit, a set of equations identical to that of Neelin et al. (1987) is achieved:

$$(D + 2F)u_b = ikT_{eb} + yv_b, \quad (16a)$$

$$(D + F)v_b = P(dT_{eb}/dy - yu_b), \quad (16b)$$

$$\hat{\Gamma}^{-1}DT_{eb} = \mathrm{sgn}(U)u_b + \lambda(1 - \epsilon_p)(iku_b + dv_b/dy). \quad (16c)$$

By assuming $v_b \equiv 0$, the dispersion relation for the Kelvin-like mode is obtained:

$$D^2 + 2FD + \hat{\Gamma}[ik + k^2\lambda(1 - \epsilon_p)] = 0. \quad (17)$$

In the particular limit of $F \to 0$, we obtain the growth rate and phase velocity:

$$D_r^2 = \frac{1}{2}(1 - \epsilon_p)\hat{\Gamma}\lambda k^2 \times \left\{-1 + \left[1 + \frac{1}{k^2\lambda^2(1 - \epsilon_p)^2}\right]^{1/2}\right\}, \quad (18a)$$

$$D_i^2 = \frac{1}{2}(1 - \epsilon_p)\hat{\Gamma}\lambda k^2\left\{1 + \left[1 + \frac{1}{k^2\lambda^2(1 - \epsilon_p)^2}\right]^{1/2}\right\}. \quad (18b)$$

Hence, in the limit of large wave number ($k \to \infty$), D_r and the phase speed asymptotically approach constant limits:

$$D_r \to [\hat{\Gamma}/4\lambda(1 - \epsilon_p)]^{1/2}$$

$$c = -D_i/k \to [\lambda\hat{\Gamma}(1 - \epsilon_p)/\sqrt{2}]^{1/2}.$$

In this limit the effect of convective downdrafts becomes apparent. When the precipitation efficiency, ϵ_p, is less than unity, some of the heating by updrafts is compensated for by downdraft cooling. Thus, not all of the adiabatic cooling associated with the wave ascent can be compensated for by convective heating, and the atmosphere is *effectively* stably stratified as opposed to neutrally stratified as in E87. Neelin et al. (1987) started off by assuming an effective stable stratification; in the context of the present model cumulus downdrafts provide for this effective stratification.

3) $\delta \sim \alpha_R \sim \alpha_e \to 0$, $\epsilon_p \approx 0$, BUT α_D FINITE

The dispersion relation in this limit is given by

$$D^3 + (2F + \alpha_D\hat{\Gamma})D^2 + [2F\alpha_D + k^2\lambda(1 - \epsilon_p) + ik]\hat{\Gamma}D + \hat{\Gamma}\alpha_D ik = 0. \quad (19)$$

Figure 2 demonstrates that the sensitive dependence of small but finite α_D on ϵ_p actually leads to the *strong* scale selection for ϵ_p near zero. The result with ϵ_p near (but not equal to) zero is well approximated by (19).

The reason for this behavior has to do with the dependence of α_D on ϵ_p. Using typical tropical values for the parameters gives

$$\alpha_D \approx 0.03/\epsilon_p.$$

Thus, α_D may remain order unity or less even when ϵ_p is close (but not equal) to zero. Examination of (10e) shows that in this regime ($\alpha_D \sim O(1)$, $\epsilon_p \to 0$) the downward advection of mean state low θ_e air into the subcloud layer becomes negligible, but the advection of the *perturbation* θ_e gradient by the mean downward motion between clouds does not vanish. This term has a strongly damping effect on small-scale motions.

This can be seen more precisely by formally taking the limit of $\epsilon_p \to 0$ but $\alpha_D \sim O(1)$. In this case, Eqs. (10e, f) are given by

$$\mathrm{sgn}(U)u_b - \lambda\sigma w_c + \alpha_D(T_{em} - T_{eb}) = 0 \quad (20a)$$

$$\lambda\sigma w_c - \alpha_D(T_{em} - T_{eb}) = DT_{em} \quad (20b)$$

where the limit of $\delta = \alpha_e = \alpha_R = 0$ has also been taken. By combining (20a) and (20b), we obtain

$$DT_{em} = \mathrm{sgn}(U)u_b,$$

which is interpreted to mean that the exchange of entropy between the subcloud layer and the lower troposphere is so efficient that the latter is immediately affected by perturbation surface fluxes.

Figure 3a shows that, with moderate values of ϵ_p, the growth rate tends to a constant value as wavenumber k increases even without taking the limit of small F. However, the addition of relatively large friction F (say, $F = 0.9$) leads to a strong damping of the modes.

The general effects of ambient stratification on the mode characteristics are shown by Fig. 4, for moderate values of F and ϵ_p. For decreasing thermal stratification, λ, we find that the suppression of the growth rate also decreases, particularly for larger wavenumbers. We also find that the phase velocity decreases (Fig. 4b) as the stratification decreases.

4. Coupling with the stratosphere

We have already incorporated a part of the coupling of the tropospheric model atmosphere with the stratosphere through the tropospheric mean geopotential $\bar{\phi}$ in (10a) and (10b). To complete this part of the coupling, the geopotential ϕ is equated at the bottom of

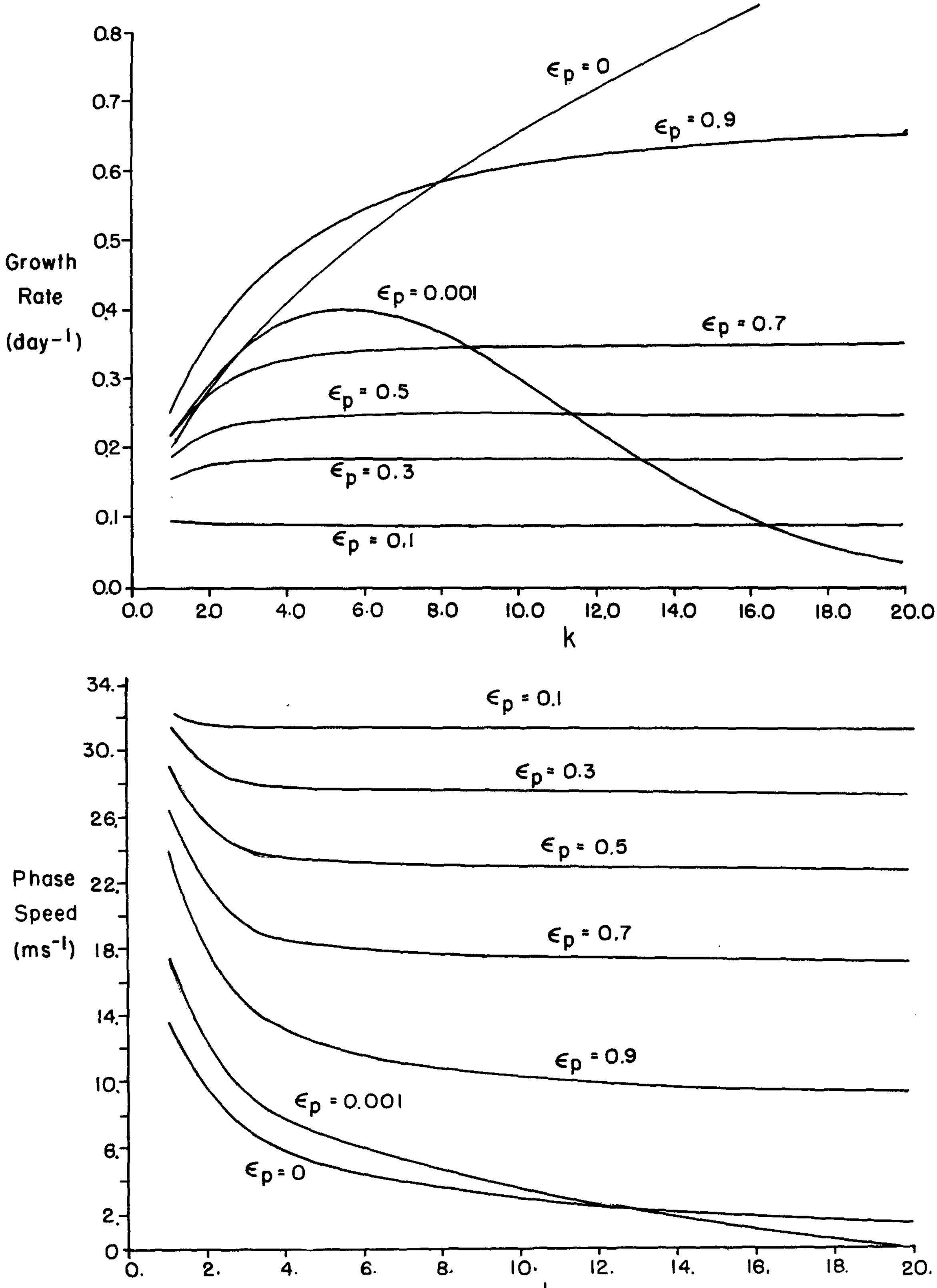

FIG. 2. The dependence of the growth rate (a), and flow-relative phase velocity (b), on longitudinal wavenumber k for various values of the precipitation efficiency, ϵ_p. The other important parameters are $F = 0.1$, $\lambda = 1$.

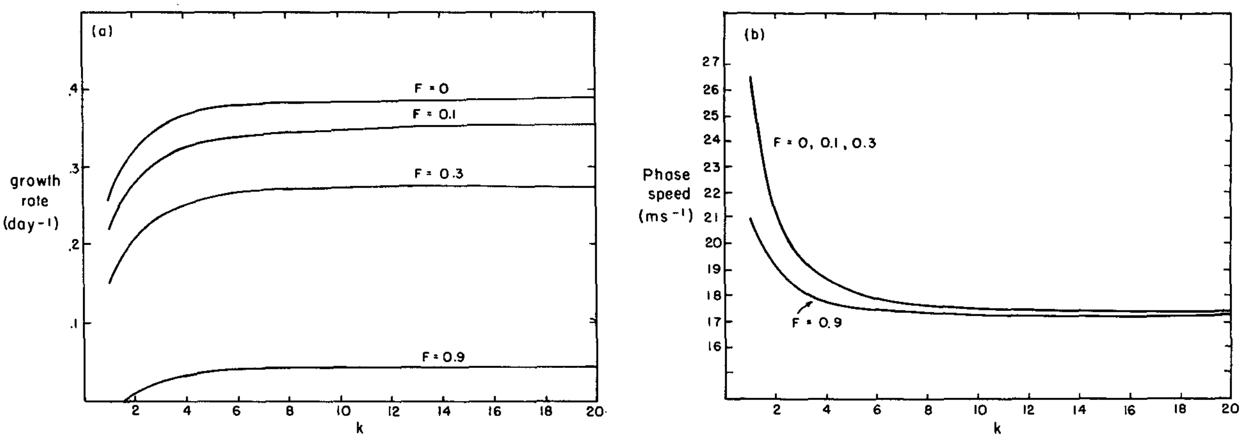

FIG. 3. Same as Fig. 2, but with various values of the surface friction F. Here, $\epsilon_p = 0.7$ and $\lambda = 1$.

the stratosphere to ϕ_t as expressed by (7h):

$$\phi(z = H) = \phi_t, \quad (21a)$$

where H is the nondimensional level of the tropopause scaled by H_m. Another coupling condition required at the tropopause is the continuity of vertical velocity

$$w(z = H) = w_t, \quad (21b)$$

where w_t is the tropospheric vertical velocity at $z = H$. Because the troposphere model (10a)–(10f) does not explicitly contain the tropopause-level vertical velocity w_t we must relate w_t to known quantities of the model. This is accomplished in the following way.

The vertical integration of the continuity equation

$$\nabla \cdot \mathbf{v} + \rho^{-1}\, \partial \rho w / \partial z = 0$$

gives

$$w(z = H) = -\nabla \cdot \int_0^H \rho \mathbf{v} dz, \quad (22)$$

which expresses the tropospheric vertical velocity in terms of vertically integrated horizontal divergence. Here, the nondimensional density ρ assumes the form

$$\rho = \exp[(H - z)/H_s],$$

and H_s is the scale height.

In order to obtain an explicit expression for vertically integrated horizontal divergence, we consider the vertically integrated linear momentum equation

$$D \int_0^H \rho u dz = y \int_0^H \rho v dz - ik\bar{\phi} - 2F'u_b, \quad (23a)$$

$$D \int_0^H \rho v dz = -y \int_0^H \rho u dz - \frac{d}{dy}\bar{\phi} - F'v_b, \quad (23b)$$

where

$$\bar{\phi} = [\phi_t - (\nu - 1)T_{eb}]H_e, \quad (24a)$$

$$\nu \equiv (T_b - T_t)/(T_b - \bar{T}), \quad (24b)$$

$$H_e \equiv (H_s/H_m)(e^{H/H_s} - 1), \quad (24c)$$

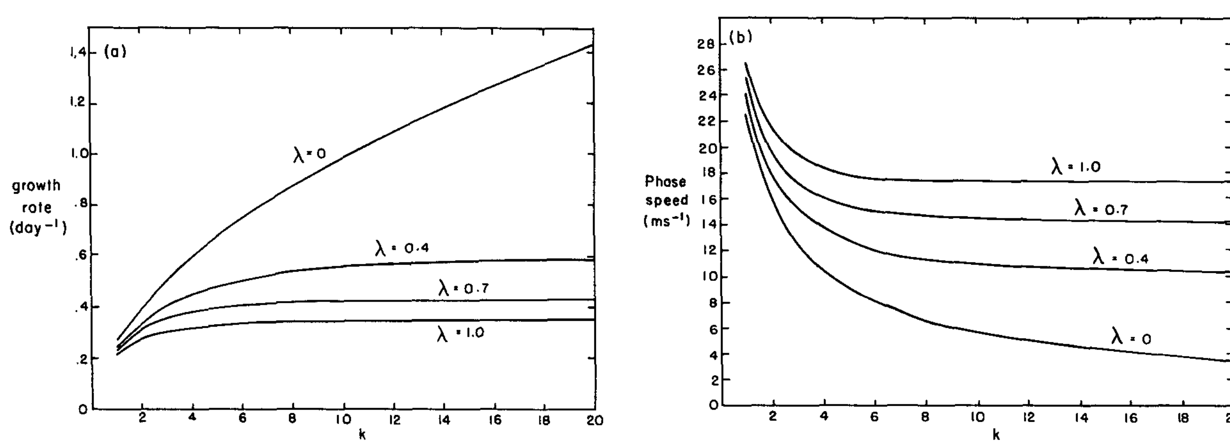

FIG. 4. Same as Fig. 2, but for various values of the tropospheric stratification, λ. Here, $\epsilon_p = 0.7$ and $F = 0.1$.

and

$$F' \equiv F H_e h H_m^{-1}. \quad (24d)$$

The quantities $\bar{\phi}$ and $\bar{T}$ have been defined previously.

After some manipulation of (23a,b), we obtain expressions for the vertically, integrated horizontal components of the velocity in terms of $\bar{\phi}$, u_b, and v_b:

$$(D^2 + y^2) \int_0^H \rho u dz = -\left(ikD + y\frac{d}{dy}\right)\bar{\phi} - F'(yv_b + 2Du_b), \quad (25a)$$

$$(D^2 + y^2) \int_0^H \rho v dz = \left(iky - D\frac{d}{dy}\right)\bar{\phi} + F'(2yu_b - Dv_b). \quad (25b)$$

By substituting (25a,b) into (22), we finally obtain

$$(D^2 + y^2)w(z = H) = \left\{\left[\left(-k^2 + \frac{d^2}{dy^2}\right)D - ik\right]\bar{\phi} + F'\left[2\left(-\frac{d}{dy}y + ikD\right)u_b + \left(D\frac{d}{dy} + iky\right)v_b\right]\right\} + \frac{2y}{(D^2 + y^2)}\left[\left(iky - D\frac{d}{dy}\right)\bar{\phi} + F'(2yu_b - Dv_b)\right], \quad (26)$$

which constitutes the second coupling condition.

As a stratosphere model, we adopt the system expressed in terms of log$p$-coordinates (cf., Andrews et al. 1987, Ch. 4), which is given in nondimensional form by

$$Du - yv = -ik\phi$$

$$Dv + Pyv = -Pd\phi/dy$$

$$iku + (\partial v/\partial y) + \rho^{-1}\partial \rho w/\partial z = 0$$

$$D(\partial\phi/\partial z) + Sw = -\alpha_R \partial\phi/\partial z,$$

where the stratospheric stratification S is defined by

$$S = H_m^2 N_s^2 / aA.$$

Here, N_s is the stratospheric Brunt–Väisälä frequency and H_m is a scale height. By assuming $H_s = 8$ km and $N_s = 4 \times 10^{-4}$ s^{-2}, in addition to the physical values already defined, we obtain nondimensional values of S and H_e of

$$S = 13.2,$$

$$H_e = 2.8.$$

The system is further simplified by assuming solutions of the form

$$u = \rho^{-1/2}\tilde{u} \exp[im(z - H)],$$

$$v = \rho^{-1/2}\tilde{v} \exp[im(z - H)],$$

$$w = \rho^{-1/2}\tilde{w} \exp[im(z - H)],$$

$$\phi = \rho^{-1/2}\tilde{\phi} \exp[im(z - H)].$$

These lead to

$$D\tilde{u} - y\tilde{v} + ik\tilde{\phi} = 0, \quad (27a)$$

$$D\tilde{v} + P(y\tilde{u} + d\tilde{\phi}/dy) = 0, \quad (27b)$$

$$ik\tilde{u} + (d\tilde{v}/dy) + m^2 D\tilde{\phi}/S = 0, \quad (27c)$$

with

$$\tilde{w} = -imD\tilde{\phi}/S, \quad (27d)$$

where the approximations

$$D + \alpha_R \approx D,$$

$$m^2 + (1/4H_s^2) \approx m^2$$

have been made.

As a further simplication, attention will be restricted to the Kelvin mode with $v \equiv 0$ in the remainder of this section.

It is straightforward to obtain the dispersion relation for the Kelvin mode from (27a–c), which is

$$D = \pm i S^{1/2} k/m.$$

Because the mode is physically forced from the troposphere and propagates upward in the stratosphere, a growing wave should decay with height. To satisfy this condition, the imaginary part of vertical wavenumber should be positive:

$$m_i > 0,$$

which means the positive sign in the dispersion relation given above should be taken because the unstable mode is the focus of interest. Hence,

$$D = iS^{1/2}k/m. \quad (28)$$

We also require that the wave energy propagate upward, hence, the vertical group velocity should be positive. Since the group velocity is given by

$$c_g \equiv -\frac{\partial}{\partial m_r}\mathrm{Im}[D] = \frac{S^{1/2}k}{|m|^4}(m_r^2 - m_i^2),$$

it is required that

$$m_r^2 > m_i^2.$$

The group velocity c_g may also be written in terms of $D = (D_r, D_i)$ as

$$c_g = (D_i^2 - D_r^2)/S^{1/2}k.$$

Hence, positive group velocity requires

$$D_i^2 > D_r^2.$$

In the limit of small F and thermal damping with finite λ, the dispersion relation given by (18a,b) leads to

$$c_g = (1 - \epsilon_p)\hat{\Gamma}\lambda k/S^{1/2},$$

assuming for the moment that the mode itself is not affected by the stratosphere. The result means that the smaller-scale modes more effectively propagate into the stratosphere. Consequently, we expect a stronger suppression of the growth rate for the smaller-scale modes due to the energy propagation into the stratosphere. On the other hand, in the E87 model, the dispersion relation is given by

$$D_r = -D_i = (k/2)^{1/2},$$

which gives no energy propagation ($c_g = 0$) into the stratosphere. Hence, we also expect that, in the absence of tropospheric stratification, there will be no strong scale selection due to stratosphere wave propagation.

Finally, since the latitudinal structure is given by

$$\tilde{u} \sim \tilde{\phi} \sim \exp(iky^2/2D), \qquad (29)$$

well-behaved solutions require

$$-D_i \equiv k(c - U) > 0,$$

which again means eastward propagation.

Next, we solve the tropospheric model. Even by restricting attention to the Kelvin-like mode, the problem is still complicated. Hence, the problem is further restricted to the limit

$$\delta \sim \alpha_R \sim \alpha_e \sim \alpha_D \sim F \to 0.$$

It has been shown that physically realistic values of α_R, α_e, and α_D are very small, and that growing solutions are only obtained in the limit of small F. We shall not separately consider the formal limit of $\epsilon_p \to 1$, which recovers the E87 model because the result for this limit is simply provided by taking $\lambda = 0$ and $\hat{\Gamma} = 1$ in the following. Note that, from the previous section, taking small but finite ϵ_p does not recover the result for the E87 model.

The tropospheric model equations reduce to

$$Du_b = ik(T_{eb} - \bar{\phi}), \qquad (30a)$$

$$d(T_{eb} - \bar{\phi})/dy - yu_b = 0, \qquad (30b)$$

$$DT_{eb} = \hat{\Gamma}[-1 + ik\lambda(1 - \epsilon_p)]u_b, \qquad (30c)$$

for the Kelvin-like mode in this limit. Equations (30a,c) lead to

$$u_b = \frac{-ikD}{D^2 + ik\hat{\Gamma}(1 - ik\lambda(1 - \epsilon_p))}\bar{\phi}, \qquad (31a)$$

$$T_{eb} = \frac{ik\hat{\Gamma}[1 - ik\lambda(1 - \epsilon_p)]}{D^2 + ik\hat{\Gamma}[1 - ik\lambda(1 - \epsilon_p)]}\bar{\phi}. \qquad (31b)$$

By substituting (31a,b) into (30b), we obtain

$$\bar{\phi} = \bar{\phi}(y = 0)\exp(iky^2/2D), \qquad (31c)$$

which is consistent with the stratosphere solution (29) connected by condition (21a).

The final procedure is to complete the solution by satisfying the other condition (26), which reduces to

$$w(z = H) = \frac{1}{D^2 + y^2}\left[\left(-k^2 + \frac{d^2}{dy^2}\right)D - ik\right]\bar{\phi} + \frac{2y}{(D^2 + y^2)^2}\left(iky - D\frac{d}{dy}\right)\bar{\phi}$$

in the present limit. By substituting (27d) and (31b) into the above, using (7h) and (7i) to relate $\bar{\phi}$ to ϕ_t, we obtain

$$D(D^2 + iK_e\nu) + kS^{1/2}H_e(D^2 + iK_e) = 0, \qquad (32)$$

where

$$K_e \equiv \hat{\Gamma}k[1 - ik\lambda(1 - \epsilon_p)].$$

Note that by taking $S \to \infty$, the tropospheric-trapped result of the previous section is recovered: compare with (17). It is equivalent to taking a rigid lid at the tropopause because the boundary condition (27d) reduces to

$$w_t(z = H) = 0$$

in this limit.

Before presenting numerical solutions of the cubic equation (32), we note the existence of some asymptotic solutions. First, we write (32) in the form

$$D^2 + iK_e + D(D^2 + iK_e\nu)/kS^{1/2}H_e = 0. \qquad (33)$$

Note that $S^{-1/2}H_e^{-1} \approx 0.1$. Regarding this quantity as a small parameter, the solution D may be expanded in a power series involving the small parameter:

$$D = D_0 + S^{-1/2}H_e^{-1}D_1 + S^{-1}H_e^{-2}D_2 + \cdots.$$

Substituting this into (33) and equating like orders of $S^{-1/2}H_e^{-1}$ gives, to first order,

$$\omega_r = k\left\langle\frac{1}{2}\lambda(1 - \epsilon_p)\hat{\Gamma}\left\{-1 + \left[1 + \frac{1}{k^2\lambda^2(1 - \epsilon_p)^2}\right]^{1/2}\right\}\right\rangle^{1/2} - \frac{\lambda\hat{\Gamma}(1 - \epsilon_p)(\nu - 1)}{2S^{1/2}H_e}k$$

$$c = U + \left\langle\frac{1}{2}\lambda(1 - \epsilon_p)\hat{\Gamma}\left\{1 + \left[1 + \frac{1}{k^2\lambda^2(1 - \epsilon_p)^2}\right]^{1/2}\right\}\right\rangle^{1/2} + \frac{\hat{\Gamma}(\nu - 1)}{2S^{1/2}H_e}k^{-1}. \qquad (34)$$

Clearly, the $O(S^{-1/2}H_e^{-1})$ corrections to the complex growth rate diminish the real growth rate at a rate proportional to k, and increase the phase speed at a rate that falls off as k^{-1}. This shows that what are inferred from examining the vertical group velocity of stratospheric Kelvin waves does indeed turn out to be the case: short waves are preferentially damped.

Exact solutions of (32) are shown in Figs. 5 and 6 as a function of k, ϵ_p, and λ with $\nu = 3$. All other parameter values are as stated earlier, bearing in mind that all damping coefficients have been set to zero. The asymptotic solution (34) is also shown for the case $\epsilon_p = 0.9$ for comparison.

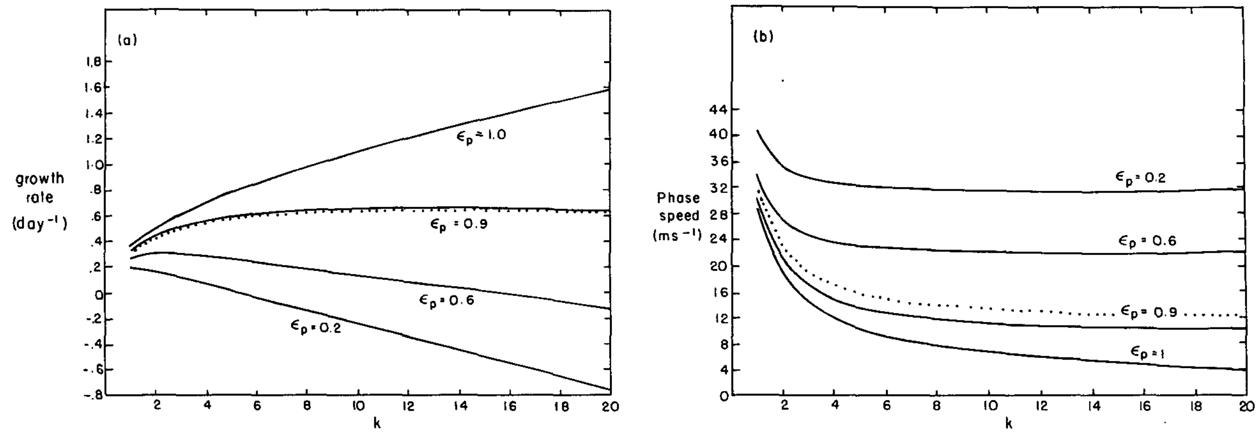

FIG. 5. The growth rate (a) and flow-relative phase speed (b) of WISHE modes with a coupled stratosphere for various values of the precipitation efficiency, ϵ_p. The other parameter values are $\lambda = 1$, $\nu = 3$, and $S^{1/2}H_e = 10.17$. Asymptotic solutions (34) for $\epsilon_p = 0.9$ are shown by dotted lines.

The presence of the stratosphere clearly leads to strong damping of all but the longest waves, as inferred from the asymptotic solutions. The effect on the phase speed is far less dramatic and falls off with increasing wavenumber. As the upward propagation of wave energy into the stratosphere is clearly a real feature of the equatorial atmosphere, this effect is to be expected in nature. Only low wavenumber disturbances can amplify, and for realistic values of λ and ϵ_p wavenumbers 1 and 2 have the highest growth rate.

5. Discussion

Two first-order effects have been added to the model of intraseasonal oscillations proposed by E87. The first has been the addition of convective downdrafts, which explicitly recognizes their important role in keeping the atmosphere convectively neutral by drying and cooling the subcloud layer. The ratio of the deep convective updraft flux to the total flux, which includes nonprecipitating updrafts, is the precipitation efficiency (ϵ_p). When this is less than unity, not all of the adiabatic cooling associated with large-scale ascent can be compensated for by heating, and the free atmosphere cools while the subcloud-layer entropy decreases so as to maintain neutrality. Thus, with $\epsilon_p < 1$ the large scale "feels" an effective stratification roughly proportional to $N^2(1 - \epsilon_p)$. With the addition of downdrafts in a convectively neutral large-scale atmosphere, the E87 model becomes mathematically very similar to the model of Neelin et al. (1987), predicting growth rates that increase with zonal wavenumber but asymptote to a constant at high wavenumber.

The same effect of downdrafts was cited by Emanuel (1989) as the reason for the failure of weak vortices over the tropical oceans to amplify into full-blown tropical cyclones: the initial ascent associated with the vortex leads to cooling of the free troposphere and concomitant drying of the subcloud layer, so that the warm core diminishes and downdrafts reduce the subcloud-layer entropy near the vortex center.

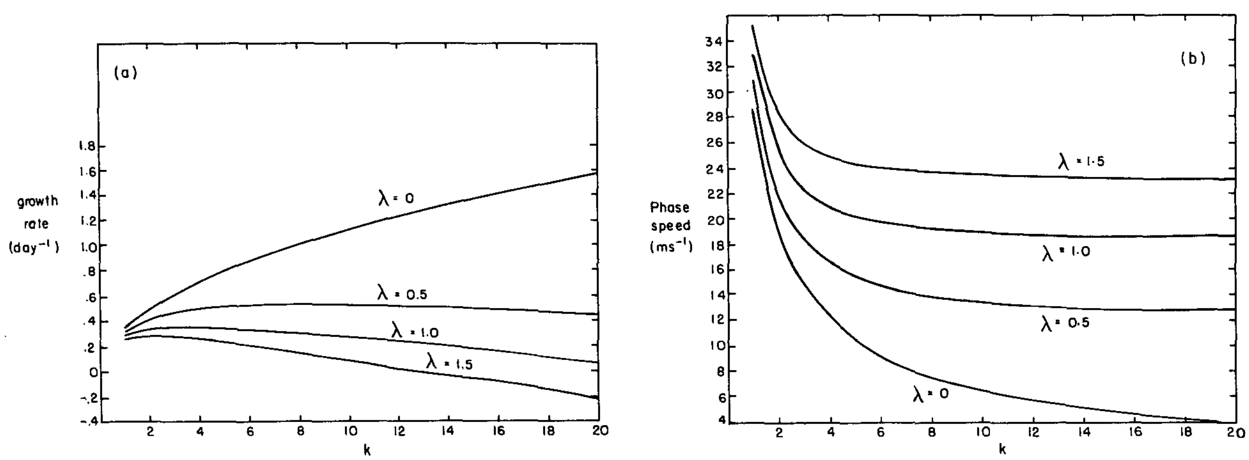

FIG. 6. Same as Fig. 5, but for various values of λ, taking $\epsilon_p = 0.8$.

The second main improvement to the E87 model is the coupling of the troposphere, formerly taken to be bounded by a rigid tropopause, to a passive stratosphere into which waves may radiate from below. As the upward group velocity increases with zonal wavenumber, short waves lose proportionally more energy to the stratosphere than long waves. Quantitatively, the short-wave instability is eliminated and only relatively long waves may amplify due to Wind-Induced Surface Heat Exchange (WISHE). For typical values of the parameters, the wavenumbers 1 and 2 WISHE modes amplify most rapidly. Examination of Figs. 2b and 5 shows that the phase speed of WISHE modes is close to observed values when $\epsilon_p \approx 0.8$. With the stratosphere coupled to the troposphere, the wavenumber 1 WISHE mode has an *e*-folding time of approximately 20 days, about half a wave period.

It is worth noting that with or without a stratosphere, the phase speeds of WISHE modes are nearly constant, except at low wavenumbers (see Figs. 2 and 5), as also found by Neelin et al. (1987). The growth rates are also fairly insensitive to zonal wavenumber except when the latter is small. This suggests that the actual phenomenon may not be a single mode at all, but rather a superposition of quasi-linear modes that gives an isolated, eastward-propagating disturbance whose zonal wind component is mainly wavenumber 1, but whose vertical velocity is restricted to a small region. Nonlinear effects might produce a similar structure.[1]

[1] This idea was suggested to the authors by I. Held, based partly on his experience with nonlinear CISK models.

We have not explored the nongeostrophic modes described by E87. Coupling these modes to the stratosphere proves to be very complex because each mode must be described by a series of orthogonal functions rather than by single functions, as in the case of the Kelvin-like modes. The nongeostrophic modes are of interest because they include westward-propagating structures similar to that of Yanai waves and with periods of 5–6 d. Coupling these to the stratosphere will be left for future work. It is the hope that the complete spectrum of WISHE modes coupled with the stratosphere may yield a comprehensive description of equatorial waves, including those that drive the quasi-biennial oscillation.

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REFERENCES

Andrews, D. G., J. R. Holton and C. B. Leovy, 1987: *Middle Atmospheric Dynamics.* Academic Press, 489 pp.

Emanuel, K. A., 1987: An air–sea interaction model of intraseasonal oscillations in the tropics. *J. Atmos. Sci.,* **44,** 2324–2340.

——, 1989: The finite-amplitude nature of tropical cyclogenesis. *J. Atmos. Sci.,* **46,** 3431–3456.

Nakazawa, T., 1988: Tropical super clusters within intraseasonal variations over the western Pacific. *J. Meteorol. Soc. Japan,* **66,** 823–839.

Neelin, J. D., I. M. Held and K. H. Cook, 1987: Evaporation–wind feedback and low-frequency variability in the tropical atmosphere. *J. Atmos. Sci.,* **44,** 2341–2348.