# Use of cloud radars and radiometers for tropical cyclone intensity estimation 

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[1] Tropical cyclone intensity is shown to be proportional to the difference between the moist static energy of the eyewall and that of the undisturbed environment, and to the difference between the absolute temperatures of the boundary layer and of the storm top. Measurements of the radial gradient of cloud top altitude and temperature from the eyewall to the outer region of the storm should therefore provide a measure of storm intensity, when coupled with an estimate of the total temperature difference between the sea surface and the cloud top. Here we develop a formalism for making such intensity estimates and apply it to cloud top heights and temperatures produced by an axisymmetric, nonhydrostatic hurricane model. The results are encouraging, and offer the potential for accurate remote detection of tropical cyclone intensity, to supplement existing satellite-based methods. Citation: Wong, V., and K. Emanuel (2007), Use of cloud radars and radiometers for tropical cyclone intensity estimation, Geophys. Res. Lett., 34, L12811, doi:10.1029/2007GL029960.

## 1. Introduction

[2] Tropical cyclones are among the most lethal and destructive natural phenomena, with single events taking more than 100,000 lives in the developing world, and causing more than $\$ 100$ billion in damage in developed nations. Wind damage is caused almost exclusively by storms of Saffir-Simpson Category 3 and higher, and in general, damage increases as at least the cube of the wind speed [Southern, 1979]. Thus forecasts of tropical cyclone intensity are of critical importance.
[3] Central to any forecast are accurate estimates of the current state of the system in question. Today, tropical cyclone intensity and structure are estimated almost exclusively by satellite-based techniques, except in the North Atlantic, where direct measurements by reconnaissance aircraft are still undertaken on a routine basis. These satellite-based techniques were developed by Dvorak [1975, 1984] and are based on algorithms applied to both visible and infrared images [Velden et al., 2006]. In recent years, alternative algorithms have been developed that may supplement or eventually replace the Dvorak-type techniques [Brueske and Velden, 2003; Demuth et al., 2004, 2006]. Although effective, with half of the wind speed estimates having absolute errors of less than $2.5 \mathrm{~ms}^{-1}$, in about $5 \%$ of cases the wind speed error can exceed $12 \mathrm{~ms}^{-1}$ [Brown and Franklin, 2004]. It is therefore of some interest

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to explore alternative means of estimating tropical cyclone intensity, to supplement the aforementioned techniques.
[4] In this paper we introduce a new method based on a balanced vortex model which predicts that cloud-top altitudes should be sensitive indicators of storm intensity, at least when storms are in a quasi-steady state. This offers the hope that satellite-based cloud-top altimetry, augmented by cloud profiling information and radiometric measurements, as available, for example from the A-Train constellation [Stephens et al., 2002], may be used to supplement and refine intensity estimates based on the existing satellitebased methods.
[5] The predictions of the balanced vortex model are reviewed in the following section, and in section 3 we test the new method against output from simulations using a nonhydrostatic, axisymmetric tropical cyclone model. A summary and suggestions for actual application of this technique using existing cloud-top-detecting and cloudprofiling satellites are presented in the closing section.

## 2. Balanced Vortex Model

[6] A hurricane is an approximately axisymmetric vortex, very nearly in a state of hydrostatic and gradient wind balance. Moreover, persistent moist convection insures that the bulk of the vortex is neutral to (slantwise) moist convection [Emanuel, 1986], which means that a quantity called the saturation potential vorticity is zero. (This is not true in the storm's eye, but this will not concern us here.) The author [Emanuel, 1986] showed that under these circumstances, there is a specific relationship between the distributions of angular momentum per unit mass, $M$, and saturation entropy, $s^{*}$, given by

$$
\begin{equation*}
M d M=-r^{2}\left(T_{s}-T_{o}\right) d s^{*}, \tag{1}
\end{equation*}
$$

where $r$ is the radial distance from the storm's rotation axis, $T_{s}$ is the absolute surface temperature, $T_{o}$ is the saturation entropy-weighted absolute temperature of the storm top, and the angular momentum per unit mass and saturation entropy are defined, respectively,

$$
\begin{equation*}
M \equiv r V+\frac{1}{2} f r^{2}, \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
s^{*} \cong c_{p} \ln (T)-R_{d} \ln (p)+\frac{L_{v} q^{*}}{T} . \tag{3}
\end{equation*}
$$

Here $V$ is the azimuthal velocity, $f$ is the Coriolis parameter, $c_{p}$ is the heat capacity at constant pressure, $R_{d}$ is the gas
constant for dry air, $p$ the pressure, $L_{v}$ the latent heat of vaporization, and $q^{*}$ is the saturation specific humidity.
[7] Making use of the definition of $M$ from (2), we can write

$$
\begin{equation*}
\frac{M}{r^{2}} d M=\frac{1}{2} f r^{2}+\frac{1}{2} f d(r V)+\frac{1}{2} d V^{2}+\left(f V+\frac{V^{2}}{r}\right) d r \tag{4}
\end{equation*}
$$

If we approximate the tangential flow as being in gradient wind balance, then

$$
\begin{equation*}
\left(f V+\frac{V^{2}}{r}\right) d r=\alpha d p \cong R_{d} T_{s} d \ln (p) \tag{5}
\end{equation*}
$$

where $\alpha$ is the specific volume and, for the last approximation in (5), we have made use of the ideal gas law and have assumed that, along the ocean surface, the air temperature is approximately constant with the value $T_{s}$. Note that in applying (5), we now assume that the derivatives are taken on surfaces of constant altitude. Substituting (5) into the right side of (4) then gives

$$
\begin{equation*}
\frac{M}{r^{2}} d M=\frac{1}{2} f r^{2}+\frac{1}{2} f d(r V)+\frac{1}{2} d V^{2}+R_{d} T_{s} d \ln (p) \tag{6}
\end{equation*}
$$

Finally, substituting (6) into the left side of (1) gives

$$
\begin{equation*}
-\left(T_{s}-T_{o}\right) d s^{*}=\frac{1}{2} f r^{2}+\frac{1}{2} f d(r V)+\frac{1}{2} d V^{2}+R_{d} T_{s} d \ln (p) . \tag{7}
\end{equation*}
$$

Provided that the surface and outflow temperatures are approximately constant with radius, (7) can be integrated exactly, and we do so, starting at the radius of maximum winds, $r_{m}$, and ending at some outer radius $r_{0}$ at which the surface wind is assumed to vanish. In writing down the result, we make the approximations that

$$
f r_{m} \ll V_{m},
$$

and

$$
r_{m}^{2} \ll r_{0}^{2}
$$

where $V_{m}$ is the maximum wind speed. With these, the integral of (7) is

$$
\begin{equation*}
\left(T_{s}-T_{o}\right)\left(s_{m}^{*}-s_{0}^{*}\right) \cong \frac{1}{4} f^{2} r_{0}^{2}+R_{d} T_{s} \ln \left(\frac{p_{0}}{p_{m}}\right)-\frac{1}{2} V_{m}^{2} \tag{8}
\end{equation*}
$$

where $s_{0}^{*}$ and $p_{0}$ are, respectively, the saturation entropy and surface pressure at $r_{0}$, and $p_{m}$ is the surface pressure at the radius of maximum winds.
[8] If the eye of the storm can be characterized as approximately in solid body rotation, then the integral of the gradient wind equation (5) from the storm center to the radius of maximum winds gives

$$
\begin{equation*}
R_{d} T_{s} \ln \left(\frac{p_{m}}{p_{c}}\right)=\frac{1}{2} V_{m}^{2} \tag{9}
\end{equation*}
$$

where $p_{c}$ is the central surface pressure. Eliminating $p_{m}$ between (8) and (9) then gives

$$
\begin{equation*}
\left(T_{s}-T_{o}\right)\left(s_{m}^{*}-s_{0}^{*}\right) \cong \frac{1}{4} f^{2} r_{0}^{2}+R_{d} T_{s} \ln \left(\frac{p_{0}}{p_{c}}\right)-V_{m}^{2} \tag{10}
\end{equation*}
$$

This shows that the increase of saturation entropy from the ambient environment to the eyewall is related to the maximum wind speed and the pressure drop between the storm center and its environment.
[9] It is well known that there exists a strong empirical relationship between the pressure drop across tropical cyclones and their maximum winds speeds, as reviewed recently by Knaff and Zehr [2007]; this is related to the fact that the wind field of these storms is close to being in gradient balance, as given by (5). Knaff and Zehr [2007] develop a semi-empirical relationship that includes the effects of storm diameter and latitude on the pressure-wind relation, but for simplicity, we here use a simple form (their equation 3) widely used in the past:

$$
\begin{equation*}
V_{m}=6.0 \sqrt{\left(p_{0}-p_{c}\right)}, \tag{11}
\end{equation*}
$$

where $V_{m}$ is given in meters per second, and $p$ in hPa. Since the fractional pressure drop is small, even in very intense storms, we can approximate the second term on the right of (10) as $R_{d} T_{s}\left(\frac{p_{0}-p_{c}}{p_{0}}\right)$, whereupon, after eliminating $p_{0}-p_{c}$ using (11), (10) becomes

$$
\begin{equation*}
V_{m}^{2} \cong \frac{\left(T_{s}-T_{o}\right)\left(s_{m}^{*}-s_{0}^{*}\right)-\frac{1}{4} f^{2} r_{0}^{2}}{\frac{R_{d} T_{s}}{36 p_{0}}-1} \tag{12}
\end{equation*}
$$

[10] Along the top of the storm's outflow, where we approximate the absolute temperature as $T_{o}$, entropy is related to static energy by

$$
\begin{equation*}
T_{o} d s^{*}=d h^{*} \tag{13}
\end{equation*}
$$

where $h^{*}$ is the saturation moist static energy, defined

$$
\begin{equation*}
h^{*} \equiv c_{p} T+L_{v} q^{*}+g z \tag{14}
\end{equation*}
$$

where $g$ is the acceleration of gravity and $z$ is the altitude. Combining (13) and (12) gives

$$
\begin{equation*}
V_{m}^{2} \cong \frac{\left(\frac{T_{s}-T_{o}}{T_{o}}\right) \Delta h^{*}-\frac{1}{4} f^{2} r_{0}^{2}}{\frac{R_{d} T_{s}}{36 p_{0}}-1} \tag{15}
\end{equation*}
$$

where $\Delta h^{*}$ is the total change of saturation moist static energy from the eyewall to the environment. At the storm top, the absolute temperature is usually less than $-50^{\circ} \mathrm{C}$, so the contribution of the saturation specific humidity to the saturation moist static energy given by (14) is negligible.
[11] Thus if sufficiently accurate estimates of cloud top height across the eyewall can be made, and if also there are sufficiently accurate estimates of the sea surface temperature and temperature at the outflow level, one can attempt to use (15) to estimate storm intensity. Differentiating (15), we


Figure 1. Evolution with time of the maximum surface wind in simulations using the nonhydrostatic, axisymmetric tropical cyclone model of Rotunno and Emanuel [1987]. The different colors represent two different imposed stratospheric temperatures. The solid lines represent observed winds while the dashed lines have been predicted from cloud top altitude and temperature using (15).
estimate that to achieve an accuracy of $2 \mathrm{~ms}^{-1}$ in the maximum wind speed, it would be necessary to estimate $T_{s}$ to within $8 \mathrm{C}, T_{o}$ to better than 5 C , and $\Delta h^{*}$ to better than $400 \mathrm{~m}^{2} \mathrm{~s}^{-2}$.
[12] In the following section, we estimate the variables in (15) from numerical simulations of hurricanes, and compare the estimates with actual wind speeds in the simulations.

## 3. Numerical Simulations

[13] As a preliminary test of this technique, we perform numerical simulations using the nonhydrostatic, axisymmetric model of Rotunno and Emanuel [1987], as modified by Bister and Emanuel [1998]. This model does not have a cumulus parameterization, and explicitly, albeit crudely, simulates convective clouds. The model is run with horizontal and vertical grid spacings of 3.75 km and 312.5 m , respectively. We begin, as did Rotunno and Emanuel [1987], with a convectively neutral sounding and insert a small amplitude, warm-core vortex, which decays for some tens of hours before amplifying into a tropical cyclone. The model achieves a statistically nearly steady state after about a week.
[14] As the integration proceeds, we record the cloud top height and temperature as a function of radius and time. Here we define the cloud top as the first level, in a downward-directed search, where the condensed water content exceeds $0.2 \mathrm{~g} / \mathrm{Kg}$. Naturally, different thresholds would be associated with different determinations of cloudtop height, but here we are interested in the radial gradient of the quantity, which would be expected to be less sensitive to the particular threshold, as long as the threshold lies within the strong negative vertical gradient of cloud water near the tops of the model's clouds.
[15] For all the integrations, we use the same initial and environmental sounding, except that we impose an isothermal stratosphere whose temperature is varied from one
integration to the next. The height of the tropopause is determined as the altitude where the initial tropopause sounding first becomes as cold as the imposed temperature of the stratosphere. By increasing this imposed stratospheric temperature, we can vary the effective outflow temperature and thus the intensity of the mature storm, so that the method can be tested for various intensities. In the experiments described here, the imposed stratospheric temperature ranges from 203 K to 253 K ; for clarity of presentation, we show results only for stratospheric temperatures of 203 and 243 K. Since we are using the same tropospheric sounding, the initial and environmental moist static energy is identical in each case, and so we can approximate $\Delta h^{*}$ in (15) as

$$
\begin{equation*}
\Delta h^{*} \cong\left(h_{\text {eyewall }}^{*}-h_{0}^{*}\right) \tag{16}
\end{equation*}
$$

where $h_{\text {eyewall }}^{*}$ is the estimated saturation moist static energy in the eyewall region, and $h_{0}^{*}$ is the constant value of the environmental saturation moist static energy, which for the sounding we use is $3.35 \times 10^{5} \mathrm{~m}^{2} \mathrm{~s}^{-2}$. Once again, in estimating these moist static energies at cloud top altitudes, we neglect the very small contribution of the saturation specific humidities, although, as they are only a function of temperature and pressure, they could be estimated from the cloud top altitude and temperature. The sea surface temperature in (15) is the constant value of 300 K used in all the integrations, while the outflow temperature used for the first factor in (15) as well as in the calculation of $h_{\text {eyewall }}^{*}$ is taken to be its average value between 75 km and 125 km from the axis. Likewise, the cloud top altitude is also averaged over the same range of radii. In the numerical model, the value of $f$ is $5 \times 10^{-5} \mathrm{~s}^{-1}$, and the outer radius of the circulation does not exceed about 900 km , so the last term in the numerator of (15) is about $500 \mathrm{~m}^{2} \mathrm{~s}^{-2}$, which is typically an order of magnitude smaller than the first term in the numerator. Here we approximate it by a constant value of $500 \mathrm{~m}^{2} \mathrm{~s}^{-2}$.
[16] Results of the simulations are summarized in Figure 1. The two colors represent two different imposed stratospheric temperatures. The solid line represents the evolution with time of the maximum tangential wind speed in the simulation (not necessarily at the surface), while the dashed line represents the estimate based on (15) as described in the preceding paragraph.
[17] The technique performs very well for the deeper, more intense simulation, even when the simulated tropical cyclone is intensifying rapidly. The estimated intensity is zero, however, during the gestation phase, when the imposed, initial warm core vortex decays into a cold core cyclone before intensification. For the simulation with the lower and warmer tropopause, the technique based on cloud-top altitude and temperature overestimates intensity during rapid intensification, but underestimates it just after the intensification phase, although the steady-state is well estimated. Overall, the estimated intensity is close enough to the actual intensity to warrant further exploration of the utility of this method.

## 4. Summary

[18] We have presented a new technique for estimating tropical cyclone intensity using estimates of cloud top
altitude and temperature in the storm core. The technique is based on the idealization of a tropical cyclone as a balanced, warm-core vortex that is neutral to slantwise moist convection and thus has zero saturation moist potential vorticity. This technique predicts that the intensity of a mature tropical cyclone should depend on surface temperature, saturation entropy-weighted outflow temperature, and the difference between the saturation moist static energy in the eyewall and that of the undisturbed environment, with a small contribution from a term proportional to the overall storm diameter. The saturation moist static energy of the eyewall is very nearly a function only of its temperature and altitude, so measurements of these quantities from infrared imagery and cloud radar, respectively, may provide the necessary quantities for estimating storm intensity, given independent estimates of sea surface temperature.
[19] Tests of the new technique using cloud top altitude and temperatures derived from numerical simulations of tropical cyclones using a nonhydrostatic, axisymmetric model show good agreement between estimated and observed maximum wind speeds.
[20] While this technique clearly shows promise based on these axisymmetric simulations, it remains to evaluate its performance in more realistic, three-dimensional settings, where the cloud top altitude and temperature can vary greatly with azimuth as well as radius, and where vertical wind shear and other environmental influences affect storm intensity. The ultimate goal is to apply this technique to estimating the intensity of real storms using satellite-borne cloud radar to detect cloud-top altitudes and infrared radiometers to estimate cloud-top temperatures. This will allow for estimates of the outflow temperature and eyewall saturation static energy in (15); the saturation static energy of the environment could be estimated from gridded analysis data or perhaps from measurements of cloud top
altitude and temperature well outside the storm core, where cumulus towers more reflect the moist static energy of the storm environment.

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