Comments on "The Circulation Associated with a Cold Front. Part I: Dry Case"

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In their paper on circulations associated with cold fronts, Orlanski and Ross (1977) purport to simulate the development of a more or less steady nonfrontogenetical circulation in association with a cold front which is present in the initial conditions. Several of their conclusions regarding the dynamics of the circulation are misleading, and I must question whether the circulation they simulate is truly frontal in character.

The statement contained in the abstract that "In the moving cold front, the vertical shear of the synoptic wind which advects the front produces an ageostrophic residue as a result of the differential advection of the vertical shear of the frontal jet and the horizontal temperature gradient across the front" is not a correct view of the dynamics of the circulation which they simulate. In fact, this circulation results from the differential advection of the *horizontal* shear (vorticity) of the frontal jet by the flow across the front, and the horizontal shear of the temperature advection *along* the front. As the vertical shear of the frontal jet and the temperature gradient across the front are nearly in thermal balance, the two quantities are advected at the same rate. To see that this is so, one need only examine their Eq. (4.11), which may be written in terms of the perturbation streamfunction which they use in their figures:

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$$N^2\frac{\partial^2\psi'}{\partial x^2}+f\left(f+\frac{\partial v}{\partial x}\right)\frac{\partial^2\psi'}{\partial z^2}-2f\frac{\partial v}{\partial z}\frac{\partial^2\psi'}{\partial x\partial z}=-2f\frac{\partial v}{\partial x}\frac{\partial U_g}{\partial z}.$$

That this balance equation duplicates the essential features of their steady numerical solution is adequately illustrated in their Fig. 10. The above, in fact, is identical to the quasi-geostrophic ω equation except that the second term on the left is a product of f and the absolute vorticity rather than f^2, and the third term on the left is not contained in the quasi-geostrophic formulation. The forcing on the right is contributed to equally, in the quasi-geostrophic sense, by advection of the frontal jet's vorticity by the thermal wind across the frontal and the zonal gradient of the temperature advection *along* the front. These two terms are identical and additive in the quasi-geostrophic formulation, and also in the model since the zonal shear is always in thermal balance with the temperature gradient along the front. The vertical shear of the frontal jet appears only in the non-geostrophic term, and this shear does not necessarily enhance the circulation. In fact, for a particular case, one can show that the shear inhibits the circulation:

Suppose we take a simple flow in which N^2, $\partial v/\partial z$ and $\partial U_g/\partial z$ are constant, and $\partial v/\partial x$ may be neglected compared to f. Then the preceding balance equation becomes

$$N^2\frac{\partial^2\psi'}{\partial x^2}+f^2\frac{\partial^2\psi'}{\partial z^2}-2f\frac{\partial v}{\partial z}\frac{\partial^2\psi'}{\partial x\partial z}=-2f\frac{\partial v}{\partial x}\frac{\partial U_g}{\partial z}. \quad (1)$$

Upon this simple constant meridional shear flow we will superpose a vertically oriented vorticity wave

$$\frac{\partial v}{\partial x}=\bar{V}_xe^{ilx},$$

where V_x is the amplitude of the vorticity of the frontal jet, and l is the horizontal wavenumber of the vorticity wave. Hence (1) will reduce to a simple linear equation which will be elliptic, parabolic or hyperbolic according to whether Ri is greater than, equal to, or less than unity. The solution to the elliptic equation with the boundary conditions $\psi=0$ at $z=0, H$ is

$$\psi=A\Bigg[\cos lx-e^{-bz}\cos(lx+az)-\frac{\sinh bz}{\sinh bH}\{\cos[lx+a(z-H)]-e^{-bH}\cos(lx+az)\}\Bigg], \quad (2)$$

where

$$A\equiv\frac{2f\bar{V}_x\partial U_g/\partial z}{l^2N^2},\quad a\equiv lf^{-1}\partial v/\partial z$$
$$b\equiv lf^{-1}\partial v/\partial z(\mathrm{Ri}-1)^{1/2}.$$

For Ri <1, a solution of the hyperbolic equation is

$$\psi=A\Bigg[\cos lx-\cos[lx+(a-b'z)]+\frac{\sin b'z}{\sin b'H}\{\cos(lx+az-b'H)-\cos[lx+a(z-H)]\}\Bigg], \quad (3)$$

where

$$b'\equiv lf^{-1}(\partial v/\partial z)(1-\mathrm{Ri})^{1/2}.$$

The amplitude coefficient is proportional to the forcing and inversely proportional to the static stability and wavenumber. Note that as the vertical shear of the frontal jet becomes very large (Ri $\to 0$), a and b become equal and the structure function in (3) vanishes. [Asymptotic analysis of (1) for large $\partial v/\partial z$ yields a solution for ψ (or ω) whose amplitude coefficient vanishes as $\partial v/\partial z\to\infty$.] Inspection of (2) shows that when $\partial v/\partial z=0$, the structure function becomes separable. Evidently, the effect of the vertical shear of the frontal jet in this case is to tilt the streamfunctions and to inhibit the circulation. For this simple case in which $\partial v/\partial z$ and $\partial U_g/\partial z$ are constant and thus proportional to each other, changing one or the other is equivalent to rotating the potential isotherms with respect to the vorticity isopleths which, since they depend only on $\partial v/\partial x$, are not dependent on the existence of a front. The conclusion is intuitive: the strongest vertical motions occur when the thermal wind is orthogonal to the vorticity isopleths.

In the model, of course, $\partial v/\partial z$ and $\partial U_g/\partial z$ are not constant (nor is N^2) and the role of the front itself, as reflected in the former quantity, is not clear. The dependence of the circulation strength on the frontal intensity is related to the vorticity of the frontal jet, not its vertical shear. Although the existence of a front is a sufficient condition for horizontal shear, which provides the forcing in this model if there exists a temperature gradient along the front, it is not a necessary condition; thus the dynamics of the circulation are not uniquely frontal in character. It appears that Orlanski and Ross have simulated a special case of a two-dimensional baroclinic wave in which the circulation, in the steady-state, is maintained against dissipation by the conversion of the unlimited amount of available potential energy associated with the temperature gradient along the front. This circulation is described quite well by the quasi-geostrophic omega equation, except perhaps where Ri is small. Since this is essentially a quasi-geostrophic baroclinic system, one would expect it to develop in the real atmosphere at very different scales than are constrained by the initial conditions in this model.

These problems bear on the results of their moist simulations as well (Ross and Orlanski, 1978). If indeed steady frontal circulations do exist, the hypothesis (e.g., Williams, 1974) that frontogenetical processes are balanced by dissipation and diffusion seems more attractive. In this case, the potential energy is supplied by the temperature gradient across the front.

Finally, it should be pointed out that the numerical simulation of the symmetric instabilities does not suffer the drawbacks cited above, and may be the more interesting result of the analysis. In reviewing their results, it should be kept in mind that areas of low Richardson number are not uniquely associated with fronts, and that higher amplitude circulations might occur if the dimensions of the low Richardson number region were greater. Also, the exact criterion for inertial instability in a nearly inviscid fluid with constant shear, as shown by McIntyre (1970), is

$$\mathrm{Ri} < \frac{f}{f + \partial v/\partial x}\left[\frac{(1+\sigma)^2}{4\sigma}\right],$$

where σ is the Prandtl number. In fact, the smallness of the absolute vorticity $(f + \partial v/\partial x)$ does guarantee instability in symmetric flow with infinite Richardson number [see footnote, Orlanski and Ross (1977, p. 1631)].

In conclusion, it is my belief that the results of this simulation are not representative of circulations in frontal zones in view of both the lack of observational evidence of strong vorticity advection by the component of thermal wind across fronts, which advection forces the circulation in this model, and the artificiality of the initial conditions in light of the processes which accompany frontogenesis (i.e., deformation). It seems more likely that steady-state frontal circulations, if they exist, represent a balance between frontogenetical and diffusive processes.

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