FLOW OF ANGULAR MOMENTUM AS A PREDICTOR FOR THE ZONAL WESTERLIES

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ABSTRACT

An approximate differential equation is presented, relating the change in speed of the zonal westerly winds to the contemporary zonal wind-speed and the meridional flow of absolute angular momentum. This equation is tested statistically by means of values of the momentum flow and the zonal wind-speed, computed with the aid of the geostrophic-wind approximation, from pressure and height data extracted from analyzed northern-hemisphere maps. The momentum flow is found to be positively correlated with the contemporary zonal wind-speed, and also with the contemporary change of the zonal wind-speed, in agreement with the approximate equation. The study suggests that the momentum flow may be a useful quantity for forecasting the zonal wind-speed. It also implies that an important part of the momentum flow is accomplished by means of large-scale horizontal eddies, whose forms are not obscured by the use of subjectively analyzed maps nor by the geostrophic-wind approximation.

1. Introduction

The strength of the prevailing zonal westerly winds in middle latitudes has often been regarded as an index of the state of the general circulation of the atmosphere. Variations in the speed of the zonal westerlies have therefore been the subject of numerous investigations. Another subject which has recently received much attention is the balance of absolute angular momentum in the atmosphere. Since the speed of the westerly wind at a specified point in the atmosphere and the absolute angular momentum per unit mass at that point completely determine each other, the two subjects are closely related.

The functional relation between wind speed and angular momentum contains a latitude factor; however, within a region whose latitudinal extent is small, the average westerly wind-speed and the total angular momentum determine each other fairly closely. Changes in the total angular momentum within a region bounded by two latitudes can result only from a meridional flow of angular momentum across the vertical boundaries, or from a torque exerted by the underlying surface. The possibility of predicting changes in the strength of the zonal westerly winds on the basis of the meridional flow of angular momentum has therefore suggested itself to several investigators [4; 5; 7].

A reasonably conclusive test of the prognostic value of the meridional flow of angular momentum, which for brevity may be called simply the momentum flow, requires the use of data for a long period of time. The earlier studies [5; 7] were handicapped by the absence of more than one or two months of data. The present study has used data for the momentum flow at several latitudes and elevations over a period of four consecutive months. Less complete data for six additional months, collected for the purpose of testing relations which appeared to be significant during the first four months, have also been used. Although the results are of a preliminary nature, they strongly indicate that the momentum flow has some prognostic value. At the same time, they show that it is feasible to compute the momentum flow on a day-to-day basis from analyzed northern-hemisphere maps.

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2. An approximate equation

The absolute angular momentum contained in a unit mass of atmosphere is given by the expression

$$M = \omega r^2 + ru, \tag{1}$$

where ω = earth's angular velocity, r = distance from earth's axis and u = eastward component of wind velocity. From the equation for eastward acceleration and the equation of continuity, it follows that

$$\partial(\rho M)/\partial t + \operatorname{div} \rho M\mathbf{c} + \partial p/\partial\lambda + r\rho D = 0, \tag{2}$$

where t = time, λ = longitude, p = pressure, ρ = density, $\mathbf{c}$ = wind-velocity vector and D = westward acceleration due to friction. If (2) is integrated over the entire volume V lying north of a vertical constant-latitude surface S, the result is

$$\frac{d}{dt}\int_V \rho M\, dV + \int_V \left(r\rho D + \frac{\partial p}{\partial\lambda}\right) dV = \int_S \rho M v\, dS, \tag{3}$$

where v = northward component of wind velocity.

In (3), the first volume integral represents the total angular momentum contained in V, while the second volume integral represents the total torque exerted upon V by the earth's surface, this torque being due partly to skin friction and partly to horizontal pressure-forces exerted by topographic irregularities of the earth's surface (the mountain torque). The surface integral represents the total flow of angular momentum northward across S.

A very simple approximation to (3) is the equation

$$dU/dt + KU = a\tau, \tag{4}$$

where U is the average low-level westerly wind-speed in V, τ the total flow of angular momentum northward across S, and K and a are positive constants which depend upon the latitude of S. Equation (4) is only an approximation to (3), for a number of reasons. First, the total absolute angular momentum within a region depends not only upon the westerly wind-speed, but also upon the total mass within the region; moreover, to obtain a measure of total angular momentum, the westerly wind-speed should not be simply averaged, but should be weighted more strongly at low latitudes. Second, the low-level westerly wind is not a perfect measure of the westerly wind at higher levels; however, Willett [9] has found that the sea-level and upper-level westerly winds are highly correlated. Third, the torque exerted by skin friction is not exactly proportional to the speed of the low-level westerlies, although Widger [7] has shown that a high correlation exists. Fourth, the mountain torque has been neglected; however White [10] has shown that the mountain torque and the skin friction torque are correlated in latitude; possibly they tend also to be correlated in time, although such a relation has not yet been established.

It is evident that these approximations make (4) less accurate than (3). On the other hand, (4) is so simple that it relates the speed of the zonal westerlies to a single additional quantity, the momentum flow. With sufficient observational data, it is easy to subject (4) to statistical tests. The results of the tests may be regarded as a measure of the justifiability of the approximations used.

Some statistical properties of any quantities U and τ satisfying (4) will now be discussed. If the period of time under study is sufficiently long, the first two terms in (4) must be uncorrelated, since a contemporary correlation between U and dU/dt would imply progressive variations of U^2. The sum of two uncorrelated quantities must be positively correlated with each quantity, but not perfectly correlated with either. Significant positive contemporary correlations of τ with both U and dU/dt should therefore be present in any observational data which claims to satisfy (4), but a nearly perfect correlation between τ and either U or dU/dt is not to be expected and would, in fact, be a denial of (4). It follows as a corollary that correlations with a time lag between τ and U should be significantly positive if the time of U follows that of τ by a suitable interval, but not if the time of U precedes that of τ by a similar interval.

3. Preliminary tests

The first statistical tests were performed with the aid of data which had been previously gathered by the General Circulation Project at the Massachusetts Institute of Technology. The basic data had been extracted from analyzed northern-hemisphere maps, and consisted of individual sea-level pressures, and 700- and 500-mb heights, at each 5 deg of latitude and longitude covered by the analyses, for each day of the four-month period November 1945–February 1946. The sea-level and 500-mb maps were taken from the *Northern hemisphere historical weather maps* [1], while the 700-mb maps were photographic copies of maps analyzed by the Air Weather Service and the U. S. Weather Bureau.

From these basic data, daily values of the momentum flow across various latitudes were obtained according to the procedure developed and described by Widger [8]. Briefly, in the absence of net mass-flow across a given latitude circle at a given elevation, the momentum flow is nearly proportional to the integral, around the latitude circle, of the product of the eastward and northward components of the wind. Pressure or height differences across 10 deg intervals of latitude and longitude were used to measure these components, according to the geostrophic-wind equation. Such differences appear in the formula

$$\tau_\phi{}^5 = \sum_{\lambda/5=1}^{72} [h(\phi - 5, \lambda) - h(\phi + 5, \lambda)] \times [h(\phi, \lambda + 5) - h(\phi, \lambda - 5)], \tag{5}$$

where $h(\phi, \lambda)$ is the 500-mb height at latitude ϕ and longitude λ. Upon multiplication by a suitable factor, $\tau_\phi{}^5$ represents the momentum flow across latitude ϕ, within a layer near 500 mb. Analogous expressions represent the momentum flow within layers near 700 mb and sea level. The sum of the flows in these three layers, here denoted by τ_ϕ, was taken to be the total momentum flow across latitude ϕ.

Daily values of the average sea-level pressure at various latitudes were also obtained from the basic data. The average sea-level pressure P_ϕ at latitude ϕ was assumed to be the simple arithmetic average of the 72 individual pressures at that latitude. The average zonal wind-speeds in various regions were measured geostrophically by linear combinations of average pressures.

Since τ_ϕ was computed from geostrophic-wind observations, it evidently includes none of the momentum flow due to mean meridional cells, *i.e.*, due to a net

TABLE 1. Autocorrelations of daily values of pressure and momentum flow for the season November 1945–February 1946. Values are in hundredths.

Lag (days)	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}	τ_{55}	τ_{45}	τ_{35}
1	87	80	72	89	87	44	48	30
2	74	57	50	76	76	11	23	08
3	64	43	36	70	69	08	18	01
4	54	37	21	69	62	14	13	−06

flow of mass at individual elevations. Instead, it includes only the flow due to horizontal eddies, *i.e.*, due to correlations in the horizontal direction between the eastward and northward wind-components. Furthermore, since the wind components used were averages over 10-deg intervals, only the flow due to large-scale horizontal eddies is included.

It was anticipated from the beginning that the most significant results, statistically, might not be those predicted by (4). It was therefore decided to make a rather exhaustive study involving the momentum flow τ_ϕ across each of the four latitudes 35, 45, 55 and 65°N, and the average sea-level pressure P_ϕ at each of the six latitudes 25, 35, 45, 55, 65 and 75°N. Daily values of each of these ten quantities were correlated with values of each other quantity, simultaneously and also with time lags of from one to ten days in each direction, for the "season" November 1945–February 1946. This procedure yielded a set of 1045 distinct correlation coefficients, including 375 of pressure with pressure, 504 of pressure with momentum flow, and 166 of momentum flow with momentum flow. Because such a procedure necessarily involved much repetitious labor, it seemed feasible to perform much of the computing by means of punched-card machines. The punched-card computations were performed by the Statistical Services Section at the Massachusetts Institute of Technology.

The most pertinent of these 1045 correlations appear in tables 1, 2 and 3. The outstanding feature of table 1 is the contrast between the high day-to-day persistence of the pressures and the low day-to-day persistence of the momentum flows. This feature will appear to be of considerable significance from the forecasting point of view.

Table 2 reveals distinct negative correlations between pressures near 70°N and pressures near 40°N. The correlation pattern is similar to patterns obtained

TABLE 2. Contemporary correlations of daily values of pressure (heading columns) with pressure (heading rows), and standard deviations of pressure, for the season November 1945–February 1946. Correlation values are in hundredths, and standard deviations are in mb.

	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}
P_{75}	100	70	−20	−54	−36
P_{65}	70	100	06	−57	−50
P_{55}	−20	06	100	41	−01
P_{45}	−54	−57	41	100	65
P_{35}	−36	−50	−01	65	100
Standard deviation	6.6	4.4	2.9	3.5	2.3

by the writer [2] for correlations between five-day mean sea-level pressures during seven winter seasons. The pattern is interpreted as showing that the principal sea-level pressure variations during the season resulted from shifts of mass between two zones, one centered near 70°N and one near 40°N. The double maximum of standard deviation seems to support this idea.

If the strength of the zonal westerlies is measured by the difference between a pressure in the southern zone and a pressure in the northern zone, it is evident that strong zonal winds may be identified with a concentration of mass in the southern zone. One can, indeed, compute correlations involving zonal wind-speeds from the standard deviations and correlations in table 2. For example, if $U_{55} = P_{45} - P_{65}$, the contemporary correlations between U_{55} and P_{75}, P_{65}, P_{55}, P_{45} and P_{35} are found to be −0.71, −0.91, 0.17, 0.86 and 0.64, respectively. The previously mentioned study by the writer [2] suggested that the zonal wind-speed U_{55} was a good index for the major fluctuations of the general circulation.

Correlations of momentum flow with sea-level pressure and wind speed appear in table 3. It is perhaps a matter of opinion which of these correlations are significant, but the following features stand out. With no time lag, and also with pressure following momentum flow, the momentum flow at each latitude is correlated positively with P_{35} and P_{45}, insignificantly for the most part with P_{55}, and negatively with P_{65}

TABLE 3. Correlations of daily values of sea-level pressure and zonal wind-speed (heading columns) with momentum flow (heading rows), for the season November 1945–February 1946. Values are in hundredths. Positive lag indicates that pressure or wind speed follows momentum flow.

	Lag (days)	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}	U_{55}
	−3	−12	00	09	12	21	06
	−2	−12	−02	11	15	21	09
	−1	−19	−08	15	27	27	19
	0	−33	−28	34	36	32	36
τ_{55}	1	−43	−36	14	37	36	41
	2	−41	−37	−07	29	36	38
	3	−32	−31	−07	26	34	32
	4	−31	−31	−10	33	39	36
	−3	−02	12	13	03	11	−06
	−2	−10	04	20	11	12	03
	−1	−15	−06	15	24	29	16
	0	−27	−31	13	45	41	42
τ_{45}	1	−41	−38	12	46	49	47
	2	−39	−37	04	38	47	42
	3	−29	−20	03	28	44	27
	4	−21	−18	04	32	45	27
	−3	02	07	03	00	10	−04
	−2	−03	01	07	09	10	04
	−1	−06	−10	10	13	20	13
	0	−10	−16	04	22	33	21
τ_{35}	1	−24	−23	03	27	42	28
	2	−29	−21	−04	26	38	26
	3	−30	−28	−04	22	37	29
	4	−23	−22	−02	24	32	26

and P_{75}. With pressure preceding momentum flow, the correlations are mostly insignificant. In the light of the interpretation of table 2, it appears that above-normal values of momentum flow in middle latitudes tend to be accompanied and also followed, but not preceded, by a concentration of mass in the southern zone, whence they must also tend to be accompanied by increasing mass within the southern zone. Equivalently, above-normal values of the momentum flow tend to be accompanied by strong zonal westerly winds, and also by increasing zonal westerly winds, while below-normal values of the momentum flow tend to be accompanied by weak and decreasing zonal westerly winds.

The correlations between momentum transport and wind speed in table 3 therefore have the signs predicted by (4). It must be admitted, of course, that the average wind speed U_{55} between 45 and 65°N is not the same as the average wind speed north of a given latitude, to which (4) refers. Nevertheless, the theoretical usefulness of (4) seems to be confirmed.

TABLE 4. Correlations of four-day mean values of momentum flow (heading rows) with values of sea-level pressure and zonal wind-speed on the fifth day (heading columns), for the season November 1945–February 1946. Values are in hundredths.

	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}	U_{55}
τ_{55}	−55	−50	−04	46	53	55
τ_{45}	−45	−40	08	50	65	50
τ_{35}	−43	−38	−03	40	60	44

The magnitudes of the correlations seem perhaps disappointingly small, and appear at first to cast some doubt upon the practical value of (4). A closer inspection, however, shows that tables 1 and 3 together imply the existence of considerably higher correlations. For example, table 3 shows that U_{55} has a moderately high correlation with the value of τ_{55} at the same time, and also with the value of τ_{55} two days earlier. Table 1 shows that values of τ_{55} separated by two days are almost independent quantities statistically, in the sense that the correlation is near zero. When a quantity, here U_{55}, is correlated with each of two independent quantities, its correlation with some linear combination of these quantities is considerably higher. The correlation can further be increased by introducing a third independent quantity, the value of τ_{55} four days earlier. It would be possible to compute the linear combinations which give the highest possible correlations, but, to illustrate the point, it seems sufficient to exhibit the correlations between values of momentum flow averaged for four successive days and pressure and wind speed on the fifth day. The increased magnitude of these correlations, which appear in table 4, over those in table 3 is apparent. The correlations suggest that (4) may lead, after all, to some rules having forecasting value.

4. Further tests

It is evident, from the preceding section, that zonal wind-speed and momentum flow were related during the season studied. Rather than find the best relations for that season, it seems more desirable to find relations which hold during each of several seasons.

The choice of additional seasons was determined by the readily available data. The basic data in this case were made available through the kindness of the U. S. Weather Bureau—Massachusetts Institute of Technology Extended Forecasting Project. The data consisted of individual sea-level pressures and 500-mb heights, which had been extracted from analyzed northern-hemisphere maps, at each 5 deg of latitude and 10 deg of longitude covered by the analyses, for each day of the year 1949. The maps were taken from the latest northern-hemisphere weather-map series [6]. From this year, two "seasons" of three months each were chosen, one consisting of January, February and March, and the other consisting of October, November and December.

For these two seasons, only the 500-mb data were used in computing momentum flow. Daily values of the momentum flow were computed from the formula

$$\tau_\phi' = \tfrac{1}{2}\sum_{\lambda/10=1}^{36} h(\phi - 2\tfrac{1}{2}, \lambda)[h(\phi + 2\tfrac{1}{2}, \lambda + 10) - h(\phi + 2\tfrac{1}{2}, \lambda - 10)]. \quad (6)$$

In (6), the height difference in brackets is a geostrophic measure of the northward component of the wind, so that τ_ϕ' appears to be a sum of products of a height with a wind component, rather than a sum of products of wind components, like $\tau_\phi{}^5$ in (5). However, it can be seen that τ_ϕ' actually is a sum of products of wind components, when it is observed that (6) may be expanded to become

$$\tau_\phi' = \tfrac{1}{2}\sum_{\lambda/10=1}^{36} [h(\phi - 2\tfrac{1}{2}, \lambda) - h(\phi + 2\tfrac{1}{2}, \lambda)] \times [h(\phi - 2\tfrac{1}{2}, \lambda + 10) - h(\phi - 2\tfrac{1}{2}, \lambda - 10) + h(\phi + 2\tfrac{1}{2}, \lambda + 10) - h(\phi + 2\tfrac{1}{2}, \lambda - 10)]. \quad (7)$$

In (7), the eastward component of the wind at latitude ϕ is multiplied by the average of the northward components at latitudes $\phi - 2\frac{1}{2}$ and $\phi + 2\frac{1}{2}$.

Formula (6) is perhaps the simplest possible formula for computing geostrophic-momentum flow directly from tabulated data. It may easily be altered slightly, to be applicable to data tabulated for each 5 deg of longitude rather than each 10 deg. In such a form, it has been discussed in detail by the writer [3].

From the daily values of momentum flow, a set of four-day mean values,[2] overlapping every two days, was constructed. That is, if D is the first day of the

[2] Four-day means were chosen for convenience in computation. Nearly the same results should be expected from the use of five-day means, such as those used by the U. S. Weather Bureau in extended forecasting.

season for which momentum flow is measured, and D' is the last day, the first four-day period consists of D, $D + 1$, $D + 2$ and $D + 3$, the second consists of $D + 2$, $D + 3$, $D + 4$ and $D + 5$, *etc.*, and the last consists of $D' - 3$, $D' - 2$, $D' - 1$ and D'.

Daily values of the average sea-level pressure at various latitudes were also obtained for these two seasons. Four-day mean values were then determined. Zonal wind-speeds were again represented by linear combinations of pressures. In order that contemporary and lag correlations during one season might all be based upon the same number of pairs of values, average sea-level pressures were determined for four additional days at the beginning and also at the end of each season. Thus, the first four-day period for pressures and wind speeds consists of $D - 4$, $D - 3$, $D - 2$ and $D - 1$, while the last consists of $D' + 1$, $D' + 2$, $D' + 3$ and $D' + 4$.

Similar four-day averages were also determined for the season November 1945–February 1946. As with the other two seasons, only the momentum flow computed from the 500-mb maps was used.[3]

Four-day mean values of momentum flow were then correlated with values of pressure and wind speed, simultaneously and also with time lags of two and four days in each direction, for each of the three seasons. The highest correlations discovered involved the momentum flow $\tau_{52.5}$ across latitude 52.5°N and the zonal wind-speed $U_{55} = P_{45} - P_{65}$. The results are summarized in table 5. In this table, τ denotes the four-day mean value of $\tau_{52.5}$, while U denotes the four-day mean value of U_{55}. The symbols U_{--}, U_{-}, U_{+} and U_{++} also denote four-day mean values of U_{55}, occurring, respectively, four days earlier, two days earlier, two days later and four days later than the values of U. Thus, the correlation of τ with U is contemporary, while those of τ with U_{--}, U_{-}, U_{+} and U_{++} are lag correlations. The difference $U_{++} - U_{--}$ evidently represents the eight-day change of the four-day mean value of U_{55}. Thus, the correlation between τ and $U_{++} - U_{--}$ may be regarded as a contemporary correlation between τ and the rate of change of U.

[3] Actually, to make use of earlier computations, the quantity $\tau'_{52.5}$ was replaced by a nearly equal quantity, the average of τ^5_{50} and τ^5_{55} as computed by (5). It is not believed that any of the results are noticeably affected by this substitution.

TABLE 5. Correlations involving four-day mean values of the momentum flow across latitude 52.5°N and the sea-level zonal wind-speed between 45° and 65°N. Values are in hundredths.

Item	Quantities correlated		Nov. 45–Feb. 46	Jan.–Mar. 49	Oct.–Dec 49
1	τ	U_{--}	14	09	01
2	τ	U_{-}	35	27	30
3	τ	U	60	50	59
4	τ	U_{+}	68	62	60
5	τ	U_{++}	60	59	40
6	τ	$U_{++} - U_{--}$	41	53	36
7	τ	$U + \frac{1}{2}(U_{++} - U_{--})$	72	67	66
8	U	U_{++}	64	79	52
9	$U + \frac{1}{2}\tau$	U_{++}	70	82	53

The correlation coefficients in table 5 are in general agreement with (4). The first five items show that, as anticipated, the momentum flow is not highly correlated with the speed of the zonal westerlies if the westerlies precede the flow, but that there are fairly high correlations if the westerlies accompany or follow the flow. Thus, as shown by the sixth item, the flow is positively correlated with the change in the speed of the zonal westerlies.

In the seventh item, $U + \frac{1}{2}(U_{++} - U_{--})$ is merely a typical linear combination which has a high correlation with τ for each season. It is not necessarily the combination having the highest correlation for any particular season. Since $U_{++} - U_{--}$ represents an eight-day change, the coefficient $\frac{1}{2}$ would be in agreement with (4) if $1/K = 4$ days. These correlation coefficients may be regarded as a measure of how closely the computed values of the two sides of (4) actually balance.

The eighth item is simply an autocorrelation, which measures the persistence of the zonal wind-speed after four days. In the ninth item, $U + \frac{1}{2}\tau$ is a typical combination of U and τ which is well correlated with U_{++} for each season.[4] Evidently each correlation in item 9 is higher than the corresponding one in item 8, so that $U + \frac{1}{2}\tau$ is a quantity which in each season leads to a better-than-persistence forecast for the speed of the zonal westerlies four days later.

It frequently occurs that two meteorological quantities are highly correlated during a period of several months simply because they have similar normal seasonal trends. It might thus appear that some of the high correlations in table 5 are merely the result of seasonal variations, rather than shorter-period irregular fluctuations. A standard method for testing such an idea consists of removing the seasonal trend from the data.

The values of τ so far computed are insufficient for determination of the seasonal trend of τ. On the other hand, the seasonal trend of U can be determined with moderate precision. "Normal" values of U have been estimated by the writer [2] from monthly normal values, based upon forty years of northern-hemisphere maps. These normal values were subtracted from the observed values of U for the three seasons under study. The computations summarized in table 5 were then repeated.

Although some changes were observed for individual seasons, the average correlations for the three seasons of τ with U, and with $U_{++} - U_{--}$, were virtually unaltered. It therefore does not appear that the correlations in table 5 are the result of a relationship between the seasonal trends of τ and U.

It can hardly be claimed that the results presented in table 5 constitute a complete verification of (4).

[4] Since U and τ are dimensionally different, the coefficient $\frac{1}{2}$ in $U + \frac{1}{2}\tau$ must be dimensional. It is applicable, in this study, when $U_{55} = P_{45} - P_{65}$ is expressed in tenths of mb, and $\tau_{52.5}$ is computed from (6), with $h(\phi, \lambda)$ expressed in hundreds of ft.

For one thing, it is hard to identify the measured quantities U and τ in table 5 with the quantities U and τ in (4), since the average wind-speed between 45 and 65°N is not the same as the average wind-speed north of 52.5°N. It must be admitted that the quantities $\tau_{52.5}$ and U_{55} were chosen for presentation in table 5 because of the resulting high correlations.

Aside from this consideration, the correlations in item 7 of table 5 are far from perfect. Nevertheless, they appear to the writer to be gratifyingly high. It would be difficult to say exactly how large a correlation must be under these conditions to be "significant," but the stability of the correlations from one season to another, together with their general agreement with the theory, makes it seem highly improbable that they are merely the result of chance.

5. Conclusion

The flow of angular momentum across certain latitudes is found to be positively correlated with both contemporary and subsequent values of the zonal wind-speed at certain latitudes. The relations appear to be in general agreement with theory. Considerable further research, involving several years of additional data, will be required to determine what relations will most likely prove consistently good. When these relations are found, an additional basis for forecasting the speed of the zonal westerly winds will have been established.

Although the results so far obtained have not yet greatly improved the statistical prediction of the zonal wind-speed, they do possess far-reaching implications concerning the nature of the flow of angular momentum It should be remembered that the computations involved are based on geostrophic winds, determined from subjectively analyzed maps, by means of contour heights at points separated by several hundred kilometers, at the single level of 500 mb, once a day. Since fairly high correlations were found, it would seem that an important part of the flow of angular momentum is obtained from the computation procedure Therefore, there appears to be strong evidence in favor of the following claims:

1. An important part of the flow of angular momentum is accomplished through the medium of horizontal eddies.
2. An important part of the flow of angular momentum due to horizontal eddies is due to large-scale horizontal eddies, whose forms can be described by specifying the wind vectors at points separated by several hundred kilometers.
3. The forms of the large-scale horizontal eddies are so definite that they are not obscured by the necessarily subjective analysis of northern hemisphere maps, nor by the use of the geostrophic-wind approximation.
4. An important part of the flow of angular momentum can be deduced from measurements at one level.
5. An important part of the flow of angular momentum can be deduced from observations taken once a day.

It should be noticed that each of the above claims refers to an important part of the flow of angular momentum, and not to the entire flow Thus, the very same observations which lead to these claims also suggest methods for increasing the numerical values of the correlation coefficients obtained. Because of the sparse distribution of observing stations, particularly at certain longitudes, it hardly seems feasible at present to measure the desired flow of angular momentum on a daily basis through observed winds, rather than geostrophic winds. Neither does it seem feasible to measure the flow due to small-scale eddies, although the use of data at every 5 deg rather than every 10 deg of longitude might be desirable. On the other hand, it is altogether feasible to measure the large-scale geostrophic flow at several levels instead of one. Such a procedure may yield numerically higher correlation coefficients. Finally, the low day-to-day persistence of the flow of angular momentum suggests that, even though an important part of the flow can be deduced from obsevations taken once a day, a significant part may also be lost. Computations of the flow at 12-hr rather than 24-hr, intervals may well give numerically higher correlations than those so far obtained.

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REFERENCES

1. Air Weather Service, 1945–46: *Northern hemisphere historical weather maps, sea level and 500 millibar*, Washington, D. C.
2. Lorenz, E. N., 1951: Seasonal and irregular variations of the northern hemisphere sea-level pressure profile. *J. Meteor.*, 8, 52–59.
3. ——, 1951: *Computations of the balance of angular momentum and the poleward transport of heat.* Report no. 6, Contract no. AF 19(122)–153 between Geophys. Res. Direct. and Mass. Inst. Tech.
4. Mintz, Y., 1949: *The maintenance of the mean zonal motion of the atmosphere.* Ph.D. thesis, Univ. of Calif. at Los Angeles (unpublished).
5. Panofsky, H., *et al.*, 1950: *Progress report on a study of day-to-day changes of angular momentum of atmospheric zonal rings.* Report no. 5, Contract no. AF 19(122)–48 between Geophys. Res. Direct. and Univ. of Calif. at Los Angeles.
6. U. S. Weather Bureau, 1949: *Daily series, synoptic weather maps, northern hemisphere sea level and 500 millibar charts with synoptic data tabulations*, Washington, D. C.
7. Widger, W. K., 1949: *A study of the generation and transport of angular momentum in the atmosphere.* Sc.D. thesis, Mass. Inst. Tech. (unpublished).
8. ——, 1949: A study of the flow of angular momentum in the atmosphere. *J. Meteor.*, 6, 291–299.
9. Willett, H. C., 1947: *Final report of the Weather Bureau—M.I.T. Extended Forecasting Project for the fiscal year July 1, 1946–July 1, 1947.* Cambridge, Mass. Inst. Tech.
10. White, R. M., 1949: The role of mountains in the angular-momentum balance of the atmosphere. *J. Meteor.*, 6, 353–355.