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PROGRESS REPORT ON ATMOSPHERIC PREDICTABILITY

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The accuracy with which weather forecasts can ultimately be produced depends upon the natural amplification rate of errors. By "errors" we mean differences between arbitrary separate states of the atmosphere, or between separate solutions of the governing equations. Recent independent estimates by dynamical and empirical procedures agree that small errors in the synoptic and larger scales will double in about three days, growing more slowly as they become larger.

The greatest uncertainty in estimating the ultimate accuracy of forecasting arises from the possibility that the inevitable errors in the meso- or micro-scales will rapidly induce appreciable errors in the synoptic scale, which will then amplify as if they had been present initially. Preliminary studies support this hypothesis. Further studies using more refined statistical assumptions should be performed. Experimental numerical forecasts with very closely spaced grid points should be undertaken when sufficiently powerful computers become available.

I. Introduction

Weather lore seems to date back to the earliest days of recorded history, but it is hardly more than a hundred years since the various nations began to establish national weather services, charged with the collection and dissemination of weather information. The earliest forecasts were necessarily rather unreliable, based as they were upon meager collections of observations and unsystematic prediction techniques, and they did not always gain public confidence. Indeed, Admiral FitzRoy of the British Navy was severely criticized by some of his scientific colleagues for issuing any forecasts at all to the public, and, following his death a few years later, the publication of the forecasts was discontinued.

During the past century the nations have continually expanded their networks of observing stations, so that a fair portion of the earth's surface is now covered. Perhaps equally important, routine observations now extend throughout the troposphere and well into the stratosphere, whose existence, incidentally, was unknown a century ago. Exchange of information, essential to the success of operational weather forecasting, has generally taken place even among those nations which have been disinclined to cooperate in other matters.

Likewise, new techniques of prediction have continually been introduced and tested. The improvements in weather forecasting which we have experienced over a hundred years can presumably be directly attributed to better observations or better techniques.

It is no secret, however, that many innovations in forecasting have failed to yield the improvements which were anticipated when they were introduced. A familiar example is isentropic analysis, whose developers, incidentally, visualized it as a research tool rather than a forecasting tool. Evidence of widespread belief, however, that isentropic analysis was at least the partial answer to the forecasting problem is the fact that some twenty-five years ago the pressure, specific humidity, stream function, and shear-stability ratio vector at each of three standard isentropic surfaces were mandatory information on every radiosonde message. Yet today how many forecasters even know what a shear-stability ratio vector is?

A notable exception to the rule that new techniques fail to produce appreciable improvement seems to be numerical weather prediction. We do not propose to present any documentation as to how well the technique has worked, but it seems safe to say that many meteorologists do not merely believe that numerical prediction is capable of yielding major improvements; they believe that these improvements have already been attained.

ational technique, it was not at all obvious that real improvements would be forthcoming. Possibly it was the repeated failure of new methods to live up to expectations which from time to time led some meteorologists to ask whether there might be previously unsuspected limitations upon the possible accuracy of weather forecasting. In the past fifteen years or so, the subject of predictability has developed into a recognized field of meteorological research. The basic questions to be answered are, first, "What are the intrinsic limitations upon the extent to which the weather may be predicted?", and, second, "What further practical limitations are imposed by economic and other considerations?"

II. Accumulation and amplification

We begin with the premise that if we knew the current state of the atmosphere and its environment exactly, and if we possessed an exact procedure for forward extrapolation of the state of the atmosphere and its environment, we could predict the future weather at any range without error. This premise is of course not strictly justified, since there is some indeterminacy in the evolution of the weather. In particular, the weather is influenced to some extent by unpredictable human activity. However, in the present discussion the lack of determinacy may be disregarded.

We then conclude that errors in prediction must result from imperfect observations of the state of the atmosphere and its environment, or imperfect techniques of forecasting. In further detail, let us consider a forecast of the behavior of the atmosphere throughout some interval of time. At any moment during this interval, the forecast will contain a specific error. We then find that any subsequent growth of the error must result from one or both of two processes: (1) an accumulation of further error due to a faulty prediction technique, and (2) an amplification of the existing error due to a basic instability. We shall refer to these processes as the accumulation process and the amplification process.

Symbolically, we may let $X_1(t)$ denote the exact state of the atmosphere at time t, while $X_2(t)$ denotes the predicted state; the error in prediction is then $X_2 - X_1$. If one were to predict the state at an extremely short range, say a few seconds, he could do little better than choosing the current state as it is believed to exist; thus, the limit of the error in prediction, as the range approaches zero, is simply the error of observation, and it is logical to regard the prediction error at zero range, as being the observational error.

We may also let

$$dX/dt = F(X,t)$$

denote the exact equation governing the atmosphere, while

$$dX/dt = G(X,t)$$

denotes the equation which would govern the atmosphere if the atmosphere really evolved in accordance with the prediction technique being used. It matters not whether the technique itself is numerical or subjective. We then find that the evolution of the error is given by

$$d(X_2 - X_1)/dt = \Big[G(X_2,t) - F(X_2,t) \Big] + \Big[F(X_2,t) - F(X_1,t) \Big] .$$

The two bracketed terms represent the effects of the accumulation process and the amplification process.

Although a given technique gives better results in some weather situations than in others, there is no reason to anticipate any continual increase in the magnitude of the first bracketed term, which depends only upon the extent to which G fails to duplicate F. The term may oscillate about some average value. Thus we may expect that any increase in the total error due solely to accumulation would be quasi-linear. On the other hand, the second bracketed term depends upon the extent to which X_2 fails to equal X_1, i.e., upon the existing error, and may be expected to be large when the error is large. Thus the increase in the total error resulting from amplification may be quasi-exponential. If there is indeed an exponential growth rather than an exponential decay, the phenomenon we are witnessing is instability; two slightly differing states of the atmosphere, governed by the same physical laws, are evolving into considerably different states.

A further complicating factor is that once X_2 no longer resembles X_1 closely, i.e., once the error is moderately large, the growth of the error should no longer be exponential. Mathematically the nonlinear processes will have become dominant. In the limiting case where X_2 and X_1 differ as greatly as randomly chosen states of the atmosphere, no further systematic growth should be expected.

If G does not closely resemble F, i.e., if the forecasting technique is poor, an exponential phase of growth may not be observed at all. If, for example, $G \equiv 0$, so that the forecast is a simple persistence forecast, the growth of the error will certainly not be exponential. If, on the other hand, the forecasting technique is rather good, so that G and F are nearly the same, and if the observations are rather good, so that X_2 and X_1 are nearly the same initially, then, if the atmosphere is unstable, we may expect a range of time during which the amplification process will dominate, and a quasi-exponential growth rate will be observed.

Perhaps because it is hoped that the technique of forecasting will eventually become highly refined, and perhaps merely because of personal preference, many investigators seem to have concentrated their attention upon the process of amplification, i.e., upon the growth which already existing errors would undergo if the forecasting technique were perfect. In the remainder of this discussion we shall consider only the amplification process. We shall find that the topic breaks up into two related but distinct problems.

III. Amplification of large-scale errors

The first problem concerns the amplification of errors which are present in the larger scales of motion. At a range of a day or more, it is only these scales whose details we usually attempt to predict. We may wish to predict that smaller-scale disturbances such as thunderstorms will occur tomorrow, but, except where local geography exerts a controlling influence, we do not try to predict the path of a particular thunderstorm.

In many meteorological undertakings, and in particular in numerical weather prediction, we effectively define the state of the atmosphere in terms of those scales large enough to be resolved by conventional networks of stations, or conventional geographical grids of points, separated by perhaps several hundred kilometers. Smaller scales of motion are acknowledged, but their statistical properties are assumed to be determined by the larger scales of motion on which they are superposed. The effects of the small scales upon the large scales are assumed to be expressible in terms of coefficients of turbulent viscosity and turbulent conductivity.

Most studies of the amplification process have implicitly accepted these assumptions. The problem then becomes fairly well defined. The atmosphere becomes for practical purposes a finite system, with the field of each meteorological variable expressible as a terminating series of standard functions, possibly spherical harmonics. A field of small-amplitude errors becomes governed approximately by a finite system of linear equations, derivable from the equations governing the atmosphere itself. An arbitrary field of errors is resolvable into a set of normal modes, or eigenfunctions (which in turn are linear combinations of the original spherical harmonics), each mode growing or decaying at its characteristic rate. The most rapidly growing mode eventually surpasses all the others in amplitude, so that its growth rate becomes the ultimate growth rate of small errors.

In practice it is not feasible to find the various normal modes. Most studies of the growth rate of small errors do not attempt to derive systems of equations governing the errors, but proceed to solve numerically one approximate form or another of the equations the general atmospheric circulation. Two time-dependent solutions with slightly differing initial conditions are found. It is then a simple matter to determine how rapidly the difference between the solutions, i.e., the error, has grown.

We cite the most recently documented and probably the most detailed study of this sort, performed by Smagorinsky (1969). In his two initial states the temperature fields possessed a root-mean-square difference of one-half degree. After a brief adjustment period, when much of the initial error in the temperature field seemed to be transferred to the wind field, there was fairly rapid growth, with a doubling time of about three days. The growth rate subsided as the error increased, until, by the time the error acquired more than half of its limiting root-mean-square amplitude of five degrees, the doubling time reached about ten days.

Like all other studies of this sort, Smagorinsky's depends upon an assumed form of the equations governing the atmosphere. It would be desirable to have some results which do not suffer this restriction. We have recently completed such a study (Lorenz 1969a).

This study is based entirely upon observational data, and makes use of the concept of analogues. By analogues we mean two states of the atmosphere occurring at widely separated times but bearing considerable resemblance to one another. If two states qualify as good analogues, either state may be regarded as equivalent to the other state, plus a small error. By observing the behavior of the atmosphere following the occurrences of each state, we may determine how rapidly the error has grown.

In the five years of data which we processed, we were unable to find any truly good analogues. The smallest root-mean-square height-field errors which we encountered, which were more than half as large as differences between randomly chosen states, tended to double in about eight days. There was a strong relation between the size of an error and its growth rate, the smallest errors growing most rapidly. Extrapolation by the most easily justified simple formula indicated that very small errors would double in about 2.5 days.

Smagorinsky's dynamical study and our empirical study therefore show remarkably good if not perfect agreement. They give a rather consistent picture of the expected future progress of an error which is initially small. To translate the results into possible ranges of prediction, we must have some idea of the accuracy and completeness with which the atmosphere will some day be observed. We can present no reliable figures; however, useful forecasts of the positions and intensities of migratory cyclones and anticyclones ten days or even two weeks ahead might well be anticipated within a generation, whereas similar forecasts a month ahead seem entirely unrealistic.

IV. Influence of small-scale errors

The second problem concerns the possibility that errors which are present in the smaller scales of motion may lead at a later time to errors in the larger scales. Strictly speaking the complete state of the atmosphere includes not only the larger scales as seen on standard weather maps, but also the locations and structures of thunderstorms and indeed of the smallest turbulent eddies. Since we do not observe the finer details of thunderstorms and smaller systems, our observed state of the atmosphere must contain errors in the small scales.

These errors should amplify much more rapidly than the larger-scale errors; an error in observing a thunderstorm, for example, should grow at least as rapidly as the thunderstorm itself. At the same time, it may under suitable conditions progress to larger scales. Stated otherwise, two states of the atmosphere which are initially identical in the larger scales, and in the statistics but not in the details of the smaller scales, may evolve differently. Stated again, a slight alteration in the arrangement of the smaller scales of motion may alter the course which the whole atmosphere will follow.

Consider first a sky filled with small fair-weather cumulus clouds. These may be of rather uniform size and shape, and rather uniformly spaced. There are no towering clouds within their midst. Under these conditions it is hard to visualize how a simple reshuffling of the clouds, placing them in different individual locations, could have an appreciable influence upon the ultimate behavior of the air mass in which they are embedded.

On the other hand, consider a large number of small cumulus clouds accompanied by a fair number of towering cumuli and a few giant cumulonimbi; the latter may, in turn, be organized into a squall line. In this event the relocation of a few small cumuli might easily alter the behavior of a nearby larger cumulus; this might in turn affect the growth of a cumulonimbus, which could then alter the course of the squall line and finally the whole air mass. The eventual contamination of a forecast by a progression of the error from small to slightly ·ger and thence to still larger scales therefore looms as a possibility.

In a recent theoretical study (Lorenz 1969b), we have attempted to investigate this possibility quantitatively. We have derived a system of equations whose dependent variables are the mean-square velocity errors in the various scales of motion. For convenience we have allowed each scale to cover an octave of the spectrum. For initial conditions we assumed that systems of diameter greater than 40 meters were perfectly observed, while systems of smaller diameter were unobserved. We found that the errors did indeed propagate to larger scales.

Specifically, the errors in one scale induced errors mainly in the next largest scale; these in turn induced errors in the next scale, etc. By the end of an hour, the initial errors in the 20-40 meter scale had propagated to the cumulus scales (1-10 kilometers). After a day they had invaded the synoptic scales (1250-5000 kilometers). By 17 days there was little predictability left in any scale.

Actually, the error growth <u>after</u> the first day, when only the larger scales retained appreciable predictability, was very much as it would have been if no small-scale motion had been present at all. The principal difference between the results of this study and those of earlier studies is that the presence of smaller scales assured us that within one day there would be errors <u>of moderate size</u> in the larger scales. Without the small-scale motions, the errors in the larger scales at the end of one day would have been only slightly larger than the errors of observation.

These conclusions have such an important bearing upon the future of forecasting that we must quickly emphasize that, like most theoretical conclusions, they are based upon a number of assumptions, and, in this case, assumptions which may not be justified. Indeed, the conclusions are considerably more pessimistic than those which other assumptions would have yielded. Of primary importance is the assumption that the atmosphere possesses all scales of motion in abundance. Specifically, the energy per unit wave number is assumed to fall off from a peak in the larger synoptic scales according to a -5/3 power law.

It is evident that if certain intermediate scales are actually more or less absent in the atmosphere, i.e., if there is a decided "spectral gap", the inevitable errors in the smaller scales can only induce errors in the larger scales by jumping the gap. This they can do only with difficulty, since the strongest influence of errors in one scale is upon errors in scales only slightly larger. In any event, jumping the gap requires considerable time, so that a gap would increase the range of predictability. Moreover, even without a gap, the progression of errors to larger scales is slower if the energy falls off more rapidly with increasing wave number, and it appears to be extremely slow if the energy follows a -3 power law.

A further shortcoming of our study is the assumption that quadratic functions of the field of errors are independent of quadratic functions of the field of motion upon which the errors are superposed. This assumption could not remain valid over a period of time if, for example, errors grow most rapidly when superposed upon the most intense fields of motion.

V. The future outlook

It is apparent, then, that we cannot yet say how far into the future we may ultimately predict the weather, but it is equally apparent that we know where to proceed in order to advance our knowledge. For one thing, we need a mathematical model with more realistic statistical assumptions than those so far used. Steps in this direction have already been taken by Kraichnan (1969). Qualitatively, at least, his results seem to confirm those based upon the simpler assumptions.

Perhaps our greatest immediate need is for a more definitive estimate of the spectrum of atmospheric energy. The often-quoted spectrum of Van der Hoven (1957) shows a pronounced spectral gap, but it is a time-spectrum rather than a space-spectrum, and it is based only upon observations in the lowest hundred meters. Observations which would yield the high-frequency end of the spectrum in the free atmosphere do not yet seem to be sufficiently abundant.

A number of recent studies (e.g., Wiin-Nielsen 1967) suggest that from a peak at a wave length of perhaps 5000 kilometers, the energy spectrum falls off at a rate close to the -3 power law, at least down to 2000 kilometers or less. Such a drop-off cannot continue indefinitely, since there would then be virtually no cumulus activity; however, assignment of a reasonable amount of energy to the cumulus scales produces a spectral gap. Thus the contamination of the forecast by errors initially confined to the smaller scales may well require considerably longer than the time indicated in our theoretical study. Pending further confirmation, we may still visualize the day when the positions of migratory cyclones and anticyclones may be predicted ten days or so in advance, rather than the four or five days which our study has suggested.

It has often been stated that one could in principle study the effects of the smaller scales of motion numerically, simply by using such an enormous network of points that the smaller scales would be resolved. It is generally added that such a procedure would require a prohibitive amount of computation, and would, furthermore, be ridiculously wasteful, because only the statistical properties of the smaller scales are of true interest. Be that as it may, I should like to see a numerical experiment performed, at least in two dimensions, with an enormous network of points. Perhaps we shall never see a grid of a million by a million points, but a thousand by a thousand should be easily handled within a few years. The procedure could be identical to the one which Lilly (1968) has already carried out with a 64 by 64 grid. A few runs with slightly differing initial conditions would suffice. I believe that such an experiment would not only be helpful in establishing the validity, or invalidity, of some of the statistical assumptions appearing in current studies, but that it might well reveal some unexpected new properties of the growth of errors.

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