Climatic Change as a Mathematical Problem

EDWARD N. LORENZ
Dept. of Meteorology, Massachusetts Institute of Technology, Cambridge

(Manuscript received 16 January 1970)

ABSTRACT

Formulating reasonable hypotheses regarding climatic change requires physical insight and ingenuity, but subsequently testing these hypotheses demands quantitative computation. Many features of today's climate have been reproduced by mathematical models (equations arranged for numerical solution by digital computers), similar to those used in weather prediction. Models currently in use generally predict only the atmosphere, and pre-specify the state of its environment (oceans, land surfaces, sun, etc.). Newer models, where certain environmental conditions enter as additonal dependent variables, should be suitable for testing climatic-change hypotheses. Aspects of the atmosphere which play no role in these hypotheses may be highly simplified. A super-model where virtually all not-strictly-constant features of the atmosphere and its environment enter as variables may ultimately lead to an acceptable theory of climatic change.

1. Introduction

The problem of climatic change occupies but one corner of the field of climatology. Yet, perhaps because it requires its followers to visualize an age when things did not all look as they do today, it has succeeded in attracting the imagination and effort of many scholars who might have looked upon general climatology as something rather prosaic. Probably for the same reason, it is highly conducive to speculation, and hypotheses easily outnumber established results. When, some years from now, someone will see fit to assemble the body of knowledge which may properly be called the theory of climatic change, the greater part of this knowledge will likely consist of facts and results which are not known today.

The complete problem of climatic change entails several distinct sub-problems. First, there is the observational task of establishing that changes of climate actually have occurred—by no means a trivial undertaking—and of determining the nature and extent of these changes. At the other extreme, there is the theoretical task of determining just what changes in climate would take place as a result of specified hypothetical causes. An intermediate problem is that of identifying the principal cause or causes of those changes in climate which have actually happened.

The first of these tasks is fairly well in hand, although it is by no means completed. During the past century or two, routine meteorological measurements have revealed certain progressive changes, such as a general warming trend during the first half of the twentieth century. Earlier historical times have seen changes in vegetation of the sort which evidently demand changes in rainfall or temperature regimes. However, the most spectacular changes are presumably those which accompanied the advance and retreat of the prehistoric continental glaciers. We feel confident that only a climate different from today's could have produced and maintained the great ice sheets, while, conversely, the presence of the ice must have produced and maintained a climate different from today's. When, however, we ask how greatly the ancient temperature and precipitation patterns differed from the current ones, we find no general agreement.

In the matter of determining the response of the climate to specified influences, we are still in the speculative era. Moreover, it seems unlikely that we shall obtain new results in which we can place very much confidence until we have perfected a more quantitative approach. We shall presently consider this matter in greater detail.

The intermediate problem, that of properly identifying the causes of well-established changes of climate, is the one whose solution would seem to advance our general knowledge the most. It is mainly this problem which will concern us in this discussion.

At first glance it might not appear that mathematics would play an important role in attacking the problem. What would seem to be called for is physical insight and ingenuity. We prefer the point of view that physical insight is indeed required, especially in the formulation of hypotheses, but that the ultimate choice among the numerous hypotheses which have been proposed, and the many more which presumably will be appearing, must be based upon mathematical considerations.

In the following we shall first describe a mathematical procedure which has been and is being successfully applied to the more general problem of climate. We shall then indicate the types of modification needed to make the procedure applicable to the problem of climatic change, and the resulting degree of success to be anticipated.

2. Mathematical models of climate

In a mathematical treatment, we may define the climate as the collection of all long-term statistical properties of the state of the atmosphere. We may represent the instantaneous state of the atmosphere by the three-dimensional fields of temperature, pressure, density and wind velocity, and water in its gaseous, liquid and solid phases. The variations of these fields as time progresses are governed by familiar physical laws; we may express these as mathematical equations which specify the time derivative of each atmospheric variable as a function of the state of the atmosphere and its environment.

If we are to solve these equations, we must know how the environment will behave. Frequently, we simplify the problem by assuming that the state of the environment is known. If we do not wish to do this, we may introduce additional variables describing the state of the ocean surface and land surface and other environmental features, and formulate appropriate additional equations governing their behavior.

If long-term statistics are understood to mean statistics taken over an infinite span of time, there is sometimes just one set of statistics compatible with a particular system of equations. In this event the system is said to be *transitive,* and the set of statistics constitutes the climate. It is also possible that two or more sets of statistics are compatible with a given system of equations. The system is then said to be *intransitive,* and the various sets of statistics constitute alternative, physically possible climates. The selection of a particular climate by a real physical system is then perhaps fortuitous. We do not know whether the atmosphere, or, more appropriately, the atmosphere-ocean-earth system, is transitive or intransitive, although there are some reasons for believing that it is transitive.

Analytical procedures for determining the climate or climates from a system of governing equations include the derivation and solution of new equations whose dependent variables are statistics, and the evaluation of statistics from analytic solutions of the original equations. However, the extreme nonlinearity of the atmospheric equations renders these procedures unfeasible. There remains the possiblility of solving the equations numerically, and compiling statistics; it is this procedure which is currently in use.

In the numerical method, the continuous fields of atmospheric variables are replaced by their values at a pre-chosen three-dimensional grid of points. Partial derivatives in the governing equations are replaced by finite differences, and integrals are replaced by sums. Initial conditions are chosen, often arbitrarily, and the equations are solved in a stepwise manner with the aid of a digital computer.

Our concept of climate now requires a slight modification. Numerical solutions necessarily extend over finite spans of time, and infinite statistics cannot be compiled. However, this apparent shortcoming proves to be an advantage when we come to the problem of climatic change. By their very nature, statistics taken over an infinite time span do not vary as time progresses, and changes of climate defined by such statistics are non-existent. Statistics taken over long but finite spans of time are more in keeping with the concept of climate which we wish to pursue.

It has become common practice to refer to a particular system of governing equations, together with a specific procedure for solving it, as a mathematical *model* of the atmosphere. Likewise, the process of obtaining a particular solution for a special purpose is often called a numerical *experiment.*

It must not be supposed that such an approach to the problem of climate was or could have been developed overnight, once computers had become available. For one thing, the development of mathematical models has accompanied the development of computers rather than following it, and has always been limited by the size and speed of the computers available. Of greater importance, certain technical questions had to be answered before the models would work properly. What finite difference operator must replace a partial derivative, for example, if spurious sources or sinks of energy are to be excluded? If a grid contains only a few thousand points, how are systems like thunderstorms to be taken into account, when an individual thunderstorm may occupy no more than one ten-millionth of the atmosphere? What aspects of the atmosphere and its environment may be considered irrelevant, and completely disregarded, in order to reduce the problem to manageable size, and what properties must be retained?

These and similar questions have by now been at least partially answered. Yet the very size of the task has placed it almost beyond the reach of the individual worker who happens to have a computer at his disposal. We find instead that much of the progress has come from the efforts of several groups addressing themselves to the specific problem. We mention one such group, the Geophysical Fluid Dynamics Laboratory of ESSA, which has been headed by Dr. Joseph Smagorinsky since its founding during the 1950's. Its staff presently includes some two dozen scientists, some of whom are specialists in certain physical or mathematical aspects of the subject. If it has not become big science, it has certainly left the realm of little science.

During its lifetime this group has constructed and tested a succession of models. The earlier ones (Smagorinsky, 1963) used a few thousand grid points, and disregarded the presence of water in its various phases. One of the more recent models (Miyakoda *et al.,* 1969) uses about 50,000 grid points, and contains a complete hydrological cycle. Many of the principal climatological features are fairly realistically reproduced.

Nevertheless, some important aspects have yet to be introduced. For example, the model does not produce its own clouds; liquid water is assumed to fall out immediately as rain. The amounts of absorption, emission and reflection of radiation by clouds, which exert a profound influence upon climate, are taken to be the amounts which would accompany a climatological normal distribution of clouds. Likewise, the influence of the atmosphere upon the oceans is omitted, and climatological normal sea surface temperatures are assumed. In short, the behavior of the environment is assumed to be known in advance. The omission of these and other aspects does not stem from a lack of regard for their importance, nor from an inability to incorporate them; it has simply not been possible to do everything at once while retaining confidence that one is doing it correctly.

3. Models and climatic change

We now come to the central question. How can one use mathematical models to study climatic change, and, in particular, to identify and establish the principal causes of climatic change? It is not certain that we can presently use them at all, at least insofar as formulating hypotheses is concerned. For example, in a number of current hypotheses (e.g., Donn and Ewing, 1968), increases and decreases in the extent of sea ice, and the subsequent influence of the ice upon atmospheric conditions, play an essential role. In current mathematical models the presence of sea ice, when it is recognized at all, is represented by constants rather than dependent variables. If an investigator has chosen to regard sea ice as a constant feature, no amount of mathematical finesse will reveal to him the positive results which he might have obtained by treating it as a variable feature instead. Likewise, in one hypothesis (Weyl, 1968), variations of oceanic salinity are assumed to exert their control upon the amount of sea ice, which in turn influences the atmosphere. An investigator using a model where salinity is assumed constant, or, more likely, where it is disregarded altogether, even though variations of sea ice are included, could never have arrived at such a hypothesis.

In view of the manner in which mathematical models have evolved, and in view of our failure to have yet incorporated every feature which we *know* to be relevant, it is inconceivable that in the near future we shall construct a model possessing every feature which could possibly be relevant, i.e., which treats every not-strictly-constant feature of the atmosphere and its environment as a dependent variable. We therefore ought not to look upon a mathematical model as a means of by-passing the physical imagination needed to formulate hypotheses. We should, however, regard a model as a valuable tool for *testing* hypotheses. For this purpose, we can and must incorporate into our model each individual feature, such as variable sea ice or salinity, suspected of being important.

Such testing seems essential if the hypotheses are not simply to remain hypotheses forever. For example, one might argue convincingly that a decrease in evaporation from the ocean would bring about a decrease in surface salinity, which would inhibit vertical overturning and thereby favor the formation of sea ice, which would in turn bring about increased reflection of solar radiation, and thereby lower the atmospheric temperature. Such reasoning could be completely sound, and yet not be particularly relevant to the problem of climatic change, if the decrease in temperature arising from a given decrease in evaporation should prove to be negligibly small, or if the decreased evaporation should simultaneously initiate a second chain of events which would favor a rise in temperature. Yet all the essential features of this reasoning *can* be incorporated into a mathematical model, and the step-by-step numerical integration of the equations will then constitute a system of bookkeeping for the ensuing temperature changes. We hasten to add that although the chain of events appearing in our example is modeled after certain currently proposed hypotheses, it is not intended to be an accurate presentation of the content of any particular hypothesis. We are not aiming to criticize specific pieces of work for not being numerical; after all, the formulation must come first. Also, we are not accusing all current hypotheses of being non-quantitative; we simply maintain that further exploitation of quantitative procedures is essential.

The manner in which we may put a model to work depends upon the nature of the hypothesis being tested. Some hypotheses regard changes in climate as the direct result of changes in the external environment, i.e., the portion of the environment which is not in turn appreciably influenced by the atmosphere. Changes in the intensity or spectral distribution of solar energy reaching the earth would fit this category. So also, most likely, would changes in the geographical locations of continents, although in the absence of numerical computations one might argue that the continual wind stress over the centuries plays some part in continental migration. Here the simplest procedure would be to compare numerical experiments already performed, using environmental conditions typical of today's with additional experiments to be performed with an altered environment. In essence, one would be assuming that today's climate is the one which would continue to prevail if the external environmental status quo could be preserved.

Other hypotheses, however, involve only the immediate environment, i.e., the part of the environment whose variations result at least partly from atmospheric effects. A typical feature of the immediate environment would be sea ice. In investigating such hypotheses, the physical system, i.e., the thing which is described by the dependent variables, should include all portions of the immediate environment as well as the atmosphere itself. The envisioned climatic changes would then become completely internal.

One might simply perform two or more numerical experiments of limited duration, with different arbitrarily chosen initial conditions, to see whether more than one climate could ensue. However, the possibility of more than one climate is not quite the same as a change of climate, especially when intransitivity looms as a possibility; and in any event it gives no indication of the time required for a change to be realized. A more satisfactory test would be a single experiment of sufficiently long duration to capture the climatic changes.

This ideal procedure has obvious practical drawbacks. Experiments so far performed with the more realistic models have extended over less than a year of simulated time. Climatic-change experiments may require hundreds or even tens of thousands of years. Even with the anticipated continual improvement in computer speed, the envisioned experiments could be prohibitively lengthy.

One is therefore tempted to settle for the performance of a few shorter experiments with differing initial conditions. However, alternative possibilities should not be overlooked. First of all, in many qualitative hypotheses the meteorological portions of the arguments are rather naive. There is no suggestion that the atmosphere is a system requiring several hundred thousand numbers for its proper description. One cannot escape the feeling that 50,000 grid points, although not irrelevant, are somehow redundant. To investigate the plausibility of a hypothesis as it has been formulated, the numerical description of the atmosphere should not have to be more sophisticated than the verbal description entering the hypothesis. A model with a few hundred grid points rather than many thousand is therefore suggested, even though it may be unrealistic in its treatment of aspects of the atmosphere which do not enter the hypothesis. If the hypothesis is sound, the model should reproduce the envisioned climatic changes. The work required to obtain solutions extending over centuries should be no more than that needed to extend more detailed experiments over years.

A further simplification could, if realizable, lead to even greater savings. Current experiments use the equations of short-range weather forecasting, even though they are not short-range experiments, and in the course of generating their climates they recreate the life history of each transient weather system, such as the familiar migratory cyclones and anticyclones. Perhaps there is some way to filter out explicit reference to these systems, while still retaining their overall effects. The numerical solution of the equations could then proceed in time increments of days or even weeks, instead of hours or minutes, and experiments extending over millenia would become feasible.

Meanwhile, it is of interest to ask what would happen if we took the mathematical models which are currently being used to simulate climate, without any modifications to accommodate existing climatic change hypotheses, and performed experiments lasting centuries or more. Would climatic changes be revealed? If we include as one hypothesis of climatic change the proposition that no processes other than those commonly considered in short-range weather forecasting are needed to bring about changes in climate, we would be testing this hypothesis.

The proposition is by no means preposterous. There are extremely simple and also very complicated systems of equations possessing solutions which behave in one manner for an extended period of time, and then change more or less abrubtly to another mode of behavior for an equally long time. Such systems have been described as *almost intransitive* (Lorenz, 1968).

There are certain indications against almost-intransitivity as a major cause of climatic change, if the system hypothesized to be almost intransitive is taken to be the atmosphere alone. Enough numerical experiments with different initial conditions have already been performed by different groups to see whether widely differing climates are likely to appear. Invariably, the climate is found to look more like today's climate than an ancient one. It is, of course, possible that some investigators have obtained climates which do not look like today's and have simply assumed that something must have gone wrong, and that their results are not worth publishing. It does seem likely, however, that something favoring the older climates is missing from the experiments.

The situation is quite different if the system includes some portion of the environment, even if this portion is nothing more than the sea-surface temperature field. Almost-intransitivity becomes still more plausible if ocean currents are included. Models which generate their own oceanic properties as well as their own atmospheric properties are only beginning to be explored. When the system under consideration includes not only sea ice but also such features of the continents as snow cover and storage of water in the ground, almost-intransitivity becomes an attractive hypothesis. Indeed, it may be said that any hypothesis which does not invoke changes of the external environment is effectively attributing climatic changes to almost-intransitivity.

There appears to be in the mathematical model of the atmosphere a new and powerful tool for studying the phenomenon of climatic change. Availability of this tool to those who logically should be using it poses a problem; perhaps there should be a center for climatic-change hypothesis testing. It has been said that new hypotheses are being introduced much more rapidly than older ones are being rejected; the next ten years could see a reversal of this trend if the models are properly exploited.

If, instead, we look into the 21st century, and make an optimistic forecast concerning the type of computer which will be available, we find that yet another approach to climatic change may become feasible. We may construct a super-model, including as variables every feature of the atmosphere and its environment which can conceivably have varied over the ages. Included will be such features as the detailed composition of the atmosphere and the oceans, the extent of continental glaciation, and the distribution of vegetation. We can probably omit human activity on the grounds that human tampering was not responsible for *past* climatic changes. When we integrate the equations, if they are correct, we shall necessarily obtain changes in climate, including the great ice ages.

Such a solution may give us little insight as to why the changes took place. However, we can now eliminate various features, singly or in combination, and see whether climatic changes are still produced. In this manner we can eventually say what features or combinations of features *could have* produced the changes. In essence, we shall have reached the day when mathematical procedures will be instrumental in formulating hypotheses as well as testing them. This is a brute-force approach, and undoubtedly involves much computing which a little careful planning could eliminate, but this appears to be the way of modern computations. As to what features *did* produce climatic changes, we shall still have the privilege of arguing.

REFERENCES

Donn, W. L., and M. Ewing, 1968: The theory of an ice-free Arctic Ocean. *Meteor. Monogr.*, 8, No. 30, 100–105.

Lorenz, E. N., 1968: Climatic determinism. *Meteor. Monogr.*, 8, No. 30, 1–3.

Miyakoda, K., J. Smagorinsky, R. F. Strickler and G. D. Hembree, 1969: Experimental extended prediction with a nine-level hemispheric model. *Mon. Wea. Rev.*, **97**, 1–76.

Smagorinsky, J., 1963: General circulation experiments with the primitive equations. I. The basic experiment. *Mon. Wea. Rev.*, **91**, 99–164.

Weyl, P., 1968: The role of the oceans in climatic change: A theory of the ice ages. *Meteor. Monogr.*, **8**, No. 30, 37–62.