Forced and Free Variations of Weather and Climate

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(Manuscript received 18 January 1979, in final form 16 April 1979)

ABSTRACT

Variations of weather and climate are termed "forced" or "free" according to whether or not they are produced by variations in external conditions.

In many simple climate models, the poleward transport of sensible heat in the atmosphere has been treated as a diffusive process, and has been assumed to be proportional to the poleward temperature gradient. The validity of this assumption, for various space and time scales, is tested with 10 years of twice-daily upper level weather data. The space scales are defined by a spherical harmonic analysis, while the time scales are defined by a "poor man's spectral analysis." The diffusive assumption is verified for the long-term average and the seasonal variations of the largest space scale, but it fails to hold for most of the remaining scales.

It is shown that diffusive behavior can be expected only for forced scales. It is suggested that most of the scales resolved by the data are free.

1. Introduction

Fluctuations of the state of the atmosphere and the upper layers of the underlying oceans and continents occur on practically all detectable time scales, from fractions of an hour to millions of years. Those variations with periods of decades or longer are generally considered to constitute changes of climate; sometimes variations with periods of years or even months are also called climatic. Atmospheric variations of shorter period are usually looked upon as mere changes in the weather; it is assumed that the climate may remain fixed while the weather is fluctuating.

Numerous explanations for climatic changes have been proposed. Many of these regard climatic variations as being the necessary response to changes in external conditions. Variations of this sort will be called *forced*. The most obviously forced climatic changes would be those resulting from changes in the sun itself or in the earth's orbital parameters.

By contrast, day-to-day weather variations are generally assumed to take place independently of any changes in external conditions. Variations of this sort will be called *free*. The local weather changes which accompany the passage of migratory cyclones and anticyclones are presumably free, since these systems owe their existence, at least in many instances, to the instability of large-scale zonal currents. The latter would be unstable whether the external conditions were varying or steady. Let us consider the matter of forced and free variations in greater detail.

The spatial variations of the external heating which forces the atmospheric circulation are primarily of very large scale. They consist mainly of a contrast between low and high latitudes and, because of the differing thermal capacities and other properties of land and water, a contrast between continents and oceans. If this heating did not vary at all with time, the complete system of equations governing the atmosphere and its surroundings would possess a special steady-state solution, whose features would also be mainly of large scale. The climate represented by this solution would be far different from the one which we experience. At any place it would be raining or snowing all the time, or else it would never rain nor snow, and there would be a preponderance of jungles, glaciers and deserts.

Actually the external heating undergoes regular variations from summer to winter and from day to night. Again, if the forcing did not vary otherwise, the equations would possess a special solution with only annual and diurnal variations. These variations would definitely be forced. Locations with rainy summers and dry winters (or vice versa) would now be possible, but the weather at any place could be accurately forecast by consulting the calendar and the clock.

In reality neither of these special solutions would describe the behavior which the atmosphere would undergo if the assumed regular forcing prevailed, because both solutions would be baroclinically unstable with respect to inevitable disturbances of small amplitude. Prominent among the systems which would consequently appear would be the

0022-4928/79/081367-10$06.50

migratory cyclones and anticyclones of continental or subcontinental size. At individual locations, there would then be fluctuations with periods comparable to the time required for a cyclone to travel a full wavelength, say a few days. These fluctuations would definitely be free.

Returning to the hypothetical case of steady external forcing, we would find that the simplest solution possessing cyclones and anticyclones would be one in which these systems would pass any given location at regular intervals, and where each system would be an exact replica of its predecessor. In the more realistic case with annual and diurnal variations, there should be a somewhat similar solution in which the structure and speed of progression of each cyclone or anticyclone, at a given location, would depend only on the time of year and, to a slight extent, on the time of day. The weather would again be perfectly predictable.

Again, neither solution would describe what would in fact occur, because each would be unstable with respect to still further small-amplitude disturbances. Cyclones and anticyclones would consequently undergo development and decay, and we could expect variations with periods comparable to the life span of a cyclone, say a few weeks. At the same time, the systems would acquire some finer structure, which would exhibit fluctuations of shorter period.

The fluctuations which we have so far been describing are essentially atmospheric; they would not differ greatly in amplitude or scale if, aside from the normal annual and diurnal changes, the state of the underlying ocean and land surfaces did not vary at all. Actually, on account of the ocean's rather large heat capacity, the ocean-surface temperature tends not to vary rapidly, but, on time scales where the variations might be considered climatic, it responds to the atmosphere as well as influencing it. Similar responses characterize the ice-covered portions of the globe. Free climatic variations in which the underlying surface plays an essential role may therefore be physically possible. Of necessity, the processes involved in these variations would be more numerous and more complicated than those involved in simple baroclinic instability. Therefore, it is more difficult to establish by theoretical methods that such variations can actually occur.

Nevertheless, one may turn to empirical methods and seek to determine, by processing the appropriate data, whether observed weather and climate variations have been forced or free. For variations with periods of centuries or longer the needed data may not exist. However, for periods covered by reasonably complete meteorological observations, a workable procedure is suggested by a consideration of energy-balance climate models.

Numerical models which have been used to investigate climate and its variations range in complexity from simple one-dimensional models, which may consist of as few as a score of ordinary differential equations, to detailed global circulation models, which may contain as many as 10 000 or even 100 000 equations. Among the former are the "energy-balance models", where the only spatial dimension appearing explicitly is latitude, and the only dependent variable is the temperature, averaged over longitude and in the vertical.

The great interest which such models recently have received was doubtless initiated by the spectacular result obtained independently by Budyko (1969) and Sellers (1969). In their models the temperature at each latitude was assumed to determine the albedo, and hence, by controlling the amount of solar heat utilized, to exert a feedback effect on the temperature. Both Budyko and Sellers found that, according to their models, a decrease in the output of solar energy by a small percentage would cause the entire surface of the earth to become permanently ice covered.

It is not surprising that numerous subsequent investigators have sought to verify and extend these findings. Schneider and Gal-Chen (1973), for example, constructed a time-dependent variant of Budyko's model. North (1975) formulated a model where steady-state solutions could be found analytically. These authors note that somewhat different modeling assumptions could lead to less spectacular conclusions. Nevertheless, the possibility that small changes in external conditions may produce large changes in the climate has been forcefully revealed.

It is perhaps a reasonable approximation to assume, following Budyko and Sellers, that the heat received and the heat emitted by the atmosphere at a given latitude can be determined from the temperature at that latitude, together with the known solar output and albedo. In order to render an energy-balance model closed, however, it is necessary to assume that the other principal process affecting the temperature, namely, the transport of sensible heat from one latitude to another, can likewise be expressed in terms of the temperature field. The time-honored procedure for doing this is to treat the heat flux as a thermal diffusion process and to assume, in essence, that the poleward heat transport is proportional to the poleward temperature gradient.

Although large-scale diffusive behavior appears to be the natural generalization of what takes place in many less complicated systems, it cannot easily be justified by physical reasoning. The simplest argument seems to be that at any given latitude, at a given time, the air moving toward colder latitudes constitutes a sample recently selected, more or less randomly, from some warmer latitude. Hence it possesses a mean temperature characteristic of the warmer latitude, while for similar reasons the air moving toward warmer latitudes possesses a mean temperature characteristic of some colder latitude. But atmospheric currents are by no means randomly located with respect to features of the temperature field, since the latter field exerts a controlling influence on these currents.

Because of the shortcomings of the diffusive assumption, other means of parameterizing the heat transport have been proposed. One reasonable assumption is that the currents which transport the sensible heat will somehow conform with baroclinic stability theory. Recently, Stone (1978), noting that the temperature gradient in middle latitudes tends to remain near the critical gradient for baroclinic instability, has proposed that the heat transport adjusts itself to approximately the intensity needed to keep the temperature gradient at its critical value.

The original aim of the present study was to determine, by examining 10 consecutive years of meteorological data, whether a diffusive treatment of the heat transport was supported by observations. For this purpose we resolved the temperature and heat-transport fields into various latitudinal and temporal scales. As we shall see, we found that certain scales failed to agree with the diffusive assumption. In subsequently attempting to account for these results by simple theory, we found that diffusive behavior could be anticipated only for the forced scales. We were thereby afforded a means of estimating the extent to which the various fluctuations of temperature were forced or free.

2. Basic formulas

There are a number of possible procedures for formulating an energy-balance climate model. We choose a framework where the independent variables are time t, longitude λ, latitude ϕ and pressure p; the dependent variables are specific volume α, temperature T, elevation z, horizontal velocity $\mathbf{U}$ with eastward and northward components u and v, and individual pressure change ω; and the relevant physical constants are the earth's radius a, the earth's angular velocity Ω, the earth's acceleration of gravity g, the specific heat c_p of air at constant pressure, and the gas constant R for air. The sensible heat c_pT of a unit mass of atmosphere is then governed by a form of the first law of thermodynamics

$$c_p dT/dt = Q + \alpha\omega, \tag{1}$$

where Q is the rate of diabatic heating per unit mass. Combining (1) with the equation of mass continuity

$$\nabla\cdot\mathbf{U} + \partial\omega/\partial p = 0, \tag{2}$$

we obtain, after some manipulation,

$$c_p\partial T/\partial t = Q + \alpha\omega - c_p\partial(T\omega)/\partial p + C, \tag{3}$$

where

$$C = -c_p\nabla\cdot(T\mathbf{U}) \tag{4}$$

is the convergence of the horizontal flux of sensible heat. Letting a bar denote an average over longitude and pressure, and neglecting variations of surface pressure, we find that

$$c_p\partial\bar{T}/\partial t = \bar{Q} + \overline{\alpha\omega} + \bar{C}, \tag{5}$$

where

$$\bar{C} = -c_p(a\cos\phi)^{-1}\partial(\cos\phi\overline{Tv})/\partial\phi. \tag{6}$$

The term $\bar{Q}$ contains all forms of diabatic heating. The solar energy reaching the atmosphere depends on ϕ and t, so that in a climate model, if the albedo is assumed to be a function of $\bar{T}$ or of ϕ, t and $\bar{T}$, the solar energy absorbed is also a function of ϕ, t and $\bar{T}$. Likewise the outgoing radiation, and also the transfer of sensible heat from the underlying surface to the atmosphere, may be approximated by functions of $\bar{T}$. Other forms of diabatic heating, notably the release of latent heat, are often neglected.

The second term on the right of (5) represents the local adiabatic conversion of kinetic energy into sensible heat. This likewise is frequently neglected.

The final term is the convergence of the poleward transport of sensible heat in the atmosphere. Under a diffusive assumption,

$$\overline{Tv} = -Da^{-1}\partial\bar{T}/\partial\phi, \tag{7}$$

where D is a (positive) diffusion coefficient. It is the appropriateness of this parameterization which we wish to examine.

If D is assumed constant, an equivalent form of (7) is

$$\bar{C} = Dc_p\nabla^2\bar{T}. \tag{8}$$

Eq. (8) is especially convenient for testing with our data set.

The procedure for formulating a climate model may be varied. We could, for example, include kinetic energy with sensible heat on the left side of (5). The conversion term would then drop out, whence no assumption of smallness would be needed, but the transport being parameterized would be that of sensible heat plus kinetic energy, which would be even less certainly related to the temperature gradient. We could also include latent energy with sensible heat and kinetic energy on the left side of (5). The release of latent heat would then not be a part of Q, whence no assumption regarding it would be needed, but the transport of latent heat plus the other forms of energy would now have to be parameterized. Finally, we could include the energy of the upper layers of the ocean and land with that of the atmosphere. The transfer of sensible heat to the atmosphere would then drop out, but the horizontal energy transport to be parameterized would now include the transport by ocean currents.

3. The data

Our data were derived from synoptic analyses of the heights of the 300, 500 and 850 mb surfaces over the Northern Hemisphere, prepared twice daily by the National Meteorological Center. Prior to our study, C. E. Leith had analyzed the height fields at 500 and 850 mb into triangularly truncated series of spherical harmonics, for 10 years and one month beginning 1 December 1962. The series were of the form

$$X(\lambda,\phi,p) = \sum_{n=0}^{N}\sum_{m=0}^{n} [X_{mn}(p)\cos m\lambda + X'_{mn}(p)\sin m\lambda]P_n^m(\sin\phi), \quad (9)$$

where X is an arbitrary scalar variable and P_n^m is the associated Legendre function (or Legendre polynomial, if $m = 0$) of degree n and order m, normalized so that its mean square is unity. To perform the analyses it was necessary to make some assumption regarding the height fields in the Southern Hemisphere, where there were no data, and the fields were treated as being symmetric about the equator. As a result, z_{mn} and z'_{mn} differ from zero only when $m + n$ is even. The series were truncated at $N = 18$. Similar analyses of the 300 mb height fields for nine "winters" 1963–64 to 1971–72, each beginning 15 November and lasting 120 days, had been made by M. L. Blackmon. These winters are contained within the period analyzed by Leith.

We have previously used some of the same data, similarly analyzed into spherical harmonics, in a study of statistical weather forecasting (Lorenz, 1978, hereafter denoted as N). We refer the reader to N for a discussion of the limitations of the data and the spherical harmonic analyses.

To estimate transports of sensible heat using the data, we must evaluate the temperature hydrostatically. We let

$$T = (g/R)\Delta z/\Delta(\ln p), \quad (10)$$

where Δ denotes the magnitude of the difference between values at 300 and 500 mb, or 500 and 850 mb. Likewise we must evaluate the motion geostrophically. As in N, we derive the velocity from a streamfunction ψ which satisfies

$$2\Omega\nabla\cdot(\sin\phi\nabla\psi) = g\nabla^2 z. \quad (11)$$

Eq. (4) then becomes

$$C = -c_p J(\psi,T), \quad (12)$$

where J denotes a Jacobian with respect to λ and $\sin\phi$. The procedure for solving Eq. (11) in spherical harmonics, and a general procedure for evaluating Jacobians in spherical harmonics, are described in N.

Values of T_{mn}, T'_{mn}, ψ_{mn} and ψ'_{mn} for all values of m are needed to evaluate C_{mn} for $m = 0$. Once the calculations have been performed, however, only the values of T_{mn} and C_{mn} for $m = 0$ are put to further use. These differ from zero only for even values of n. For brevity we let $T_n = \bar{T}_{0n}$ and $C_n = \bar{C}_{0n}$. With the values of T_n and C_n we can construct the fields of $\bar{T}$ and $\bar{C}$, using (9) with $m = 0$.

4. Spatial and temporal resolution

Because of the spatial orthogonality of the Legendre polynomials, the spherical-harmonic analyses provide a resolution into spatial scales, with the larger scales represented by the lower values of n. To test the appropriateness of the parameterization (8) for separate scales, it is therefore unnecessary to reconstruct the fields of $\bar{T}$ and $\bar{C}$. Eq. (8) simply becomes

$$C_n = -n(n + 1)Da^{-2}c_pT_n. \quad (13)$$

The seasonal variations of $\bar{T}$ are strongest in the polar regions. With global data, these variations would be found to be dominated by variations of T_1, which would be generally negative in northern winter and positive in southern winter, since the polynomial P_1 is most strongly negative at the South Pole and positive at the North Pole. With only Northern Hemispheric data, and with equatorial symmetry built into the analyses, the dominant seasonal variations of $\bar{T}$ will instead reveal themselves as variations of T_2, which should be generally negative, and most strongly negative in northern winter, since the polynomial P_2 is most strongly negative at the equator and positive at both poles. Likewise, C_2 should be normally positive, and most strongly so in winter. To this extent there should be qualitative agreement with (13).

Eq. (13) also implies, however, that C_2 should become more strongly positive whenever T_2 becomes more strongly negative, regardless of whether the changes have resulted from the march of the seasons or from some other cause. Similar implications apply to higher values of n. Therefore, we shall examine the extent to which fluctuations of different time scales contribute to a negative mean product of T_n and C_n, whose existence would be implied by (13). For this purpose we shall introduce a special analysis of variance and covariance.

Let the data to be processed consist of 2^K successive values $X_{(0)}, \ldots, X_{(L-1)}$ of a scalar X, where $L = 2^K$, and the corresponding 2^K values of a scalar Y. For $k = 0, \ldots, K$ we define

$$[X,Y]_k = J^{-1}\sum_{j=0}^{J-1} [l^{-1}\sum_{i=0}^{l-1} X_{(lj+i)}][l^{-1}\sum_{i=0}^{l-1} Y_{(lj+i)}], \quad (15)$$

where $l = 2^k$ and $J = L/l$. We then define

$$(X,Y)_k = [X,Y]_k - [X,Y]_{k+1} \tag{16}$$

for $k < K$. We observe that $[X,Y]_0$ is the mean product of X and Y, while $[X,Y]_K$ is the product of the means, so that

$$\sum_{k=0}^{K-1} (X,Y)_k = [X,Y]_0 - [X,Y]_K \tag{17}$$

is the covariance. For convenience we let $(X,Y)_K = [X,Y]_K$.

We have thus expressed the covariance (or the variance, if $Y = X$) as a sum of K terms. When $Y = X$, each term is obviously non-negative. With observations at half-day intervals, $(X,Y)_0$ is the covariance within 1-day periods, $(X,Y)_1$ is the covariance of 1-day averages within 2-day periods, $(X,Y)_2$ is the covariance of 2-day averages within 4-day periods, etc. Our analysis of variance or covariance is therefore a "poor man's spectral analysis"—a scanning of the spectrum or cospectrum with overlapping filters whose corresponding periods center about 1 day, 2 days, 4 days, etc. The analysis contains less information than a conventional spectral analysis, but it involves much less computation. For many purposes the information can be sufficient.

Because we believe that the poor man's spectral analysis should be useful for studying other data sets, particularly when they are very long, we describe an easily programmed algorithm. For analyzing the variance of X, we use $3(K + 1)$ storage locations, denoted by A_{1k}, A_{2k}, and V_k, for $k = 0, \ldots, K$. We set V_k initially to zero. We introduce the observations of X alternately into A_{10} or A_{20}, and add each value of X^2 to V_0. Following each introduction of X into A_{20}, we introduce the sum $A_{10} + A_{20}$ alternately into A_{11} or A_{21}, and add $(A_{10} + A_{20})^2$ to V_1. Following each introduction of $A_{10} + A_{20}$ into A_{21}, we introduce the sum $A_{11} + A_{21}$ alternately into A_{12} or A_{22}, etc. The entire process requires only $3L$ additions and $2L$ multiplications. When this process ends, we convert the values of V_k to $[X,X]_k$ by dividing by 2^{K+k}, and then subtract $[X,X]_{k+1}$ from $[X,X]_k$ to obtain $(X,X)_k$. The procedure is easily modified to analyze the variances and covariances of two or more variables.

When the sequences are very long, say with 2^{20} terms, it may be inconvenient or impossible to store all the values of X before performing the analysis, or even to allow space for storage, and it is in fact unnecessary. Any routine which produces the proper value of X just before it is introduced may be used. This routine, for example, could consist of reading a block of data from a tape, or integrating a system of equations for one or more time steps.

5. Results

We have applied our analysis procedure to the variances of T_n and C_n and the covariance of T_n with C_n, for n(even) = 2, . . . , 18. We originally defined $\bar{T}$ and $\bar{C}$, and hence T_n and C_n, as averages over the depth of the atmosphere. With our available data it is more convenient to redefine them as averages from 850 to 500 mb and, subsequently and separately, as averages from 500 to 300 mb.

For the 850–500 mb layer each series consists of 7366 successive observations, and we could, for example, have analyzed the first 2^{12} observations in each series. However, this would have partially obscured the strong annual cycle. We have therefore constructed the series to be analyzed by choosing the first 2^9 observations in each year (beginning 1 December) for the first four and the last four years. The resulting discontinuities would have caused

TABLE 1. Components $(T_n,T_n)_k$ occurring in analysis of variance of zonally averaged temperature $\bar{T}$ for 850–500 mb layer. Units are 10^{-4} K^{-2}.

	n									
k	2	4	6	8	10	12	14	16	18	Total
0	42	50	41	47	42	41	32	25	19	340
1	44	66	82	94	77	77	60	39	24	562
2	90	137	181	175	146	138	95	46	28	1037
3	200	287	336	291	198	145	86	35	18	1597
4	424	392	363	346	197	109	60	23	14	1929
5	1106	395	336	410	177	97	34	19	9	2582
6	2848	762	260	364	83	35	20	10	6	4389
7	15282	472	101	295	151	22	15	5	4	16346
8	42451	4869	205	1081	327	12	23	1	13	49011
9	230	52	63	12	23	11	5	1	1	398
10	250	100	83	5	19	1	1	1	0	461
11	8	152	5	7	0	0	0	0	0	172
12	954578	1383	4926	4187	1	3	0	13	49	965140
Total	1017553	9117	6982	7314	1471	691	431	218	185	1043964

TABLE 2. Components $(C_n, C_n)_k$ occurring in analysis of variance of convergence of sensible-heat transport $\bar{C}$ for 850–500 mb layer. Units are 10^{-4} K^2 deg^{-2}.

	n									
k	2	4	6	8	10	12	14	16	18	Total
0	35	101	238	389	542	709	667	599	475	3756
1	68	233	528	776	1079	1122	953	820	657	6236
2	121	344	746	1006	1175	1177	988	659	513	6729
3	104	324	557	690	711	670	512	462	422	4452
4	103	272	333	367	312	330	310	237	204	2469
5	50	68	180	196	211	163	136	116	91	1212
6	113	83	133	120	156	101	59	84	63	912
7	130	81	125	61	104	112	68	46	65	793
8	1305	795	898	485	154	97	74	26	60	3895
9	1	3	20	2	26	9	2	4	33	99
10	0	0	7	21	2	18	5	21	0	75
11	0	4	9	2	19	0	5	0	0	40
12	4641	5441	1250	752	854	50	60	12	54	13120
Total	6671	7749	5030	4867	5345	4558	3839	3086	2637	43788

havoc in a conventional spectral analysis, but in our procedure they have little effect, since there is no averaging or differencing across the discontinuities except in whole-year blocks.

Table 1 presents the space-time analyses of variance (and mean square) of $\bar{T}$, i.e., the components $(T_n, T_n)_k$ for $n = 2, \ldots, 18$ and $k = 0, \ldots, 12$. Also shown are column and row totals, which form spatial and temporal analyses, respectively, and the grand total, which is simply the mean square which is being analyzed. By far the greatest contribution comes from $(T_2, T_2)_{12}$, the square of the temporal mean of T_2. Of the remaining components, the greatest contributors are $(T_2, T_2)_8$ and $(T_2, T_2)_7$. The former includes the contrast between "winter" (1 December–7 April) and "summer" (8 April–13 August) averages. The latter, which is somewhat smaller, includes the contrast between the first and last halves of winter, and also between those of summer. For large n and small k (small-scale short-period variations), the values fit a rather smooth pattern; one could almost draw isopleths on the table.

Table 2 is similar, for $\bar{C}$. The components $(C_2, C_2)_{12}$ and $(C_2, C_2)_8$ are important, but are not so overwhelming as $(T_2, T_2)_{12}$ and $(T_2, T_2)_8$, while $(C_4, C_4)_{12}$, the square of the temporal mean of C_4, surpasses $(C_2, C_2)_{12}$. Again smooth variations are evident for large n and small k.

Table 3 presents the analysis of covariance (and mean product) of $\bar{T}$ and $\bar{C}$. There are large negative values where $n = 2$ and $k = 12$, 8 or 7, in agreement with our earlier conjecture, and with diffusion theory. Nevertheless, there is a conspicuous scarcity of negative values in the remainder of the table, particularly in the smoothly varying part where n is large and k is small. Our initial impression is that

TABLE 3. Components $(T_n, C_n)_k$ occurring in analysis of covariance of $\bar{T}$ and $\bar{C}$ for 850–500 mb layer. Unit is 10^{-4} K^2 deg^{-1}.

	n									
k	2	4	6	8	10	12	14	16	18	Total
0	0	1	7	8	19	31	36	28	27	157
1	3	10	29	38	70	87	83	55	47	421
2	−1	21	92	52	113	120	133	64	53	648
3	8	96	114	108	73	115	87	50	42	692
4	16	101	91	121	84	34	43	19	26	537
5	−46	39	111	70	43	44	17	21	6	306
6	−374	153	46	−13	42	15	17	5	0	−109
7	−1258	126	33	52	−30	12	5	10	4	−1047
8	−7414	1951	−77	698	213	−11	33	4	−21	−4622
9	8	10	33	3	14	6	1	1	−1	76
10	1	−3	−19	8	6	−4	0	0	0	−12
11	2	26	6	4	−2	0	0	0	0	37
12	−66561	−2743	−2487	1774	−26	−13	3	12	−51	−70093
Total	−75616	−212	−2021	2923	619	436	458	269	132	−73009

as it stands, Eq. (13) does not afford a satisfactory parameterization of $\bar{C}$. Qualitatively similar results recently have been obtained by van Loon (1979), who applied conventional statistical procedures to 29 years of 700 mb winter data.

For a further interpretation of our findings, we note that, according to (5),

$$\tfrac{1}{2}c_p\partial(\bar{T}^2)/\partial t = \bar{T}\bar{C} + \ldots, \tag{18}$$

the omitted terms being those not involving the transport. The separate components in Table 3 may therefore be regarded as separate contributors to the growth of the spatial variance of $\bar{T}$. Under a diffusive assumption all components would contribute negatively; our computations show that, even though the grand total is decidedly negative, most components contribute positively.

It thus appears that the negative relation between $\bar{T}$ and $\bar{C}$ may be due only to the contribution of the long-term mean values and the normal seasonal cycle. To check this possibility, we removed the means and the bulk of the seasonal variations from the data. This we did by subtracting from each series T_n or C_n the series of the form

$$\hat{X}_{(j)} = X_0 + X_1 \cos\omega j + X_1' \sin\omega j + X_2 \cos 2\omega j + X_2' \sin 2\omega j, \tag{19}$$

where $\omega = 2\pi/730$, which best approximated T_n or C_n, in the least-squares sense, over the 10-year time span. We then repeated the computations which led to Tables 1–3.

Table 4 takes the place of Table 3. The large negative values which had appeared when $k = 12$, 8 or 7 have now disappeared, and there is only a scattering of minus signs. It is particularly significant that every column total is positive, as is every row total except one. The failure of the covariances for $k = 12$ to vanish identically results from using only a portion of the data in the computations, while the means were removed from the entire set.

Our results thus far have been based only on the layer from 850 to 500 mb. For the remainder of the atmosphere our data restrict us to the layer from 500 to 300 mb, for winter only. More extensive coverage would have been highly desirable.

For this layer each time series contains 240 observation per year. For our analysis we could use the first 2^7 observations in each of eight years, but this would eliminate more than half of the data. Instead we have increased the number of "observations" to 2^8 per year by retaining the 240 real observations and following them with 16 zeros. We have done this only for series from which the means and seasonal variations have been removed, so that additional zeros are "normal" values. The effect of the zeros should be only to reduce the absolute values of the variances and covariances; no negative covariances will be changed into positive ones.

Table 5 is the equivalent of Table 4, for the new layer. The long-term mean which has been removed is of necessity the winter mean. Because there are fewer observations per year, variations within a year now correspond to $k = 0, \ldots, 7$ instead of $k = 0, \ldots, 8$. The tables are rather similar. Again all the column totals are positive, as are all the row totals which correspond to periods less than a year. Therefore, it seems safe to conclude that our earlier results, at least for the shorter periods, were not the consequence of accidentally choosing a special layer of the atmosphere.

6. Further theoretical considerations

Although the positive covariances found between the temperature and the convergence of sensible-heat transport in the smaller scales and shorter periods may have been unanticipated, we can, hav-

TABLE 4. As in Table 3 except with seasonal trend removed from the data.

	n									
k	2	4	6	8	10	12	14	16	18	Total
0	1	1	7	8	19	31	36	28	27	158
1	3	10	29	38	70	87	83	55	47	422
2	0	20	92	51	113	120	133	64	53	648
3	13	97	115	109	74	114	87	50	42	702
4	38	99	91	117	84	35	43	20	26	553
5	35	46	111	76	38	43	16	21	7	393
6	19	30	70	−15	25	18	13	4	0	164
7	−11	−7	43	23	3	15	7	8	6	85
8	7	27	72	0	20	4	1	2	2	135
9	8	10	33	3	14	6	1	1	−1	76
10	1	−3	−19	8	6	−4	0	0	0	−12
11	2	26	6	4	−2	0	0	0	0	37
12	0	0	4	3	−1	0	2	0	0	7
Total	115	356	654	424	463	469	423	253	210	3367

TABLE 5. As in Table 4 except for 500–300 mb layer.

k	n = 2	4	6	8	10	12	14	16	18	Total
0	11	−1	3	2	15	11	28	10	13	90
1	4	9	13	12	43	30	35	29	24	200
2	7	24	59	42	60	69	52	29	18	360
3	14	−22	50	56	53	71	29	22	19	291
4	4	50	111	55	42	5	19	4	18	309
5	−29	27	25	40	7	24	14	12	12	131
6	−2	15	47	15	11	1	10	11	4	112
7	2	−5	5	21	3	13	9	1	8	57
8	37	6	13	0	31	−12	2	2	−2	77
9	−36	−2	−24	−4	0	0	1	−2	0	−68
10	10	−11	2	6	2	1	−2	−1	0	7
11	−1	−6	1	0	0	0	1	0	0	−6
Total	21	83	303	245	265	213	198	118	115	1560

ing noted them, offer rather simple arguments to account for them. For this purpose, we first recall the reason why the small-scale and short-period variations exist at all. As we noted, the external forcing is primarily of larger scale and longer period. The forced variations, however, are unstable. The smaller scales and shorter periods owe their existence to this instability; hence they are free.

We now approximate Eq. (6) by

$$c_p\partial\bar{T}/\partial t = \bar{F} - hc_p\bar{T} + \bar{C}, \qquad (20)$$

where $\bar{F}$, the "forcing", is the portion of $\bar{Q}$, notably the solar radiation reaching the earth, which is not physically influenced by $\bar{T}$; in a climate model it may be treated as a prespecified function of t and ϕ. Likewise, $-hc_p\bar{T}$, where h is a positive constant, is the portion of $\bar{Q}$ which depends physically on $\bar{T}$; we treat is here as a linear damping effect. Thereby we omit the portion of $\bar{Q}$ which, while physically dependent on $\bar{T}$, bears no linear relation to $\bar{T}$. Likewise we have omitted the conversion from kinetic energy.

If we combine (20) with a parameterization of the form

$$\bar{C} = G(t,\phi,\bar{T},\partial\bar{T}/\partial\phi,\partial^2\bar{T}/\partial\phi^2, \ldots) \qquad (21)$$

or

$$\partial\bar{C}/\partial t = H(t,\phi,\bar{T},\bar{C},\partial\bar{T}/\partial\phi,\partial\bar{C}/\partial\phi, \ldots), \qquad (22)$$

where G or H is a prespecified function, we obtain a closed system of equations. Starting from arbitrary initial conditions, we may solve the system numerically and then process the solution as if it were observational data, to obtain the climate or, if the system admits more than one climate, to obtain one of the climates. A particular example of (21) is, of course, the parameterization (8).

An equivalent form of (20) is

$$c_pdT_n/dt = F_n - hc_pT_n + C_n. \qquad (23)$$

It follows that for any value of k,

$$c_p(T_n,dT_n/dt)_k = (T_n,F_n)_k - hc_p(T_n,T_n)_k + (T_n,C_n)_k. \qquad (24)$$

It is evident from the definitions (15) and (16) that as long as T_n is not progressively increasing or decreasing

$$\langle(T_n,dT_n/dt)_k\rangle = 0, \qquad (25)$$

where the angle brackets denote an expected value. It follows that

$$\langle(T_n,C_n)_k\rangle = hc_p\langle(T_n,T_n)_k\rangle - \langle(T_n,F_n)_k\rangle. \qquad (26)$$

We consider now a scale and a period for which the direct forcing $\bar{F}$ is essentially absent, e.g., the case of large n and small k. Here the final term in (26) drops out. It then follows that the expected covariance component $\langle(T_n,C_n)_k\rangle$ is *positive*. This is precisely what shows up in Tables 4 and 5 for the presumably unforced components. For scales and periods which are forced no such conclusion is possible, and Table 3 in fact reveals negative covariances.

An essential point is that our conclusion has been drawn entirely from the "thermodynamic" equation (20). No assumption whatever has been made concerning the nature of the parameterization (21) or (22). It is of interest then to ask the consequence of insisting that (21) should reduce to the diffusive parameterization (8), with its implied negative covariance. What would happen is that, if we solved the system numerically with arbitrary initial conditions, the unforced scales and periods would simply damp out. Assuming that only the regular variations were included in $\bar{F}$, each year would eventually be a repetition of the previous year. Some time-dependent climate models have indeed exhibited precisely this behavior.

7. Concluding remarks

We have developed a straightforward procedure for partially determining whether the variations of a particular scale in the field of zonally and vertically averaged temperature are forced or free. We simply evaluate the contribution of that scale to the covariance of the temperature and the convergence of sensible-heat transport. For free variations the covariance must be positive; hence a negative covariance implies that the variations are forced. A positive covariance of the temperature and the convergence of the transport is equivalent to a negative covariance of the temperature gradient and the transport itself.

We regard the determination as partial because we cannot establish the converse of our result. That is, it is possible to have a positive covariance even when some forcing is present.

The physical reasoning involved is quite simple. If a certain scale in some field is not directly forced, the only process which can maintain the energy represented by its variations against dissipative effects is the nonlinear transfer of energy from some other scale. If instead the scale is forced, but only weakly, the nonlinear transfer might combine with the forcing and oppose the dissipation; but if the forcing is strong, it is more likely that the transfer will combine with the dissipation to oppose the forcing. When the field in question is that of zonally and vertically averaged temperature, the transfer takes the form of a convergence or divergence of sensible-heat transport.

In examining 10 years of meteorological data, we have found, as expected, that the annual temperature variations and, of course, the spatial variations of the long-term mean are forced. We have encountered little evidence that any of the remaining variations resolved by the data are forced, although, as noted, we cannot prove that they are free. Variations with periods exceeding two years ($k = 10$ or 11 for 850–500 mb, $k = 9$ or 10 for 500–300 mb) show as many negative as positive covariances, but these are based upon too few degrees of freedom for the signs to be significant. In any event there is a strong suggestion that, for many fluctuations of extended period, forcing may not be an important factor.

There is some question as to how much the results have told us about forced and free variations of *climate*. First of all, it is not universally agreed that fluctuations within a decade should be called climatic. Furthermore, if they are, there is still much more to climate than the zonally averaged temperature which appears in the energy-balance models. There are large regional systems, such as the Asiatic monsoon, which would be obscured by a zonal averaging process. There are other atmospheric quantities, such as rainfall, which may in theory be related to the temperature field, but where, with present knowledge, any relationship would be hard to describe. Finally, there are additional quantities, such as ice cover, which describe the state of the underlying oceans and continents.

It would appear that the variations of some of these quantities might be investigated by a study similar to the present one. Any feature which is subject to dissipation and which is not directly forced must draw its energy from some other feature. For any particular feature, we may attempt to formulate the appropriate expression for the transfer of energy from other features, and, if we succeed, we may determine whether the data needed to evaluate the appropriate covariances are available.

Our original hope in performing this study was that we might discover an improved means of parameterizing the sensible-heat transport in simple climate models. We have not succeeded in doing so. Certainly, the diffusive parameterization is unsatisfactory for many purposes. Although, at most extratropical latitudes, the heat transport is usually downgradient, it is, at a fixed time of year, less strongly downgradient when the gradient itself is stronger. Indeed, a comparison of the parameterization (13) with the simple model equation (23) shows that, scale by scale, the parameterization does little more than increase the effective dissipation coefficient, replacing h, whose value is not too well known in any case, by $h + n(n + 1)Da^{-2}$.

Use of a negative D, which would be suggested by the upper right portions of Tables 3–5, would do no better; it would merely decrease the dissipation. In fact, if $h + n(n + 1)Da^{-2}$ should become negative, the model would blow up.

The diffusive parameterization might be suitable in studies having the primary purpose of determining the expected response of the climate to some change in the forcing, such as a change in solar output. Here we might prefer to suppress the free variations, so that we could obtain significant statistics from shorter numerical runs. This is precisely what the parameterization would do.

A modified parameterization which permits both forced and free variations is

$$\bar{C} = Dc_p\nabla^2\bar{T} + \bar{r} \tag{27}$$

or

$$C_n = -n(n + 1)Dc_pa^{-2}T_n + r_n, \tag{28}$$

where $\bar{r}$ or r_n is generated in some manner by a random number generator. The potentialities of formulas of this general sort have been examined in detail by Hasselmann (1976), and specific formulas have been used by Robock (1978) to produce year-to-year free temperature variations having the order of magnitude of the observed variations. It is interesting to note that, despite the negative covariance of C_n and T_n seemingly implied by (28), the covariances in the unforced scales would turn out to be positive, as we have already demonstrated, since the sign of the covariance does not depend on the nature of the parameterization.

The transport of sensible heat effectively transfers energy from one scale to another. The effect is nonlinear and cannot readily be captured by an essentially linear parameterization. The atmospheric currents which accomplish the transports, or at least the variations of these currents, owe their existence to baroclinic instability, at least in middle and higher latitudes. A suitable parameterization ought therefore to represent the effects of baroclinic instability, whether or not it is derived from baroclinic theory. In the parameterization (27) the random term $\tilde{r}$ succeeds in simulating these effects. Without explicit randomness, it would appear that a parameterization of the form (21) or (22) ought to be nonlinear. We suspect that the form (22) is the more realistic.

Acknowledgments. The author is grateful to Cecil E. Leith and Maurice L. Blackmon for the opportunity to engage in many fruitful discussions concerning this topic, and for the use of their data tapes. This work has been supported by the Climate Dynamics Program, Division of Atmospheric Sciences, National Science Foundation, under Grant NSF-g 77 10093 ATM. Some of the work, including the computing, was performed while the author was visiting the National Center for Atmospheric Research; NCAR is supported by the National Science Foundation.

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