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Nondeterministic Theories of Climatic Change

EDWARD N. LORENZ

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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A basic assumption in some climatic theories is that, given the physical properties of the atmosphere and the underlying ocean and land, specified environmental parameters (amount of solar heating, etc.) would determine a unique climate and that climatic changes therefore result from changes in the environment. The possibility that no such unique climate exists and that nondeterministic factors are wholly or partly responsible for long-period fluctuations of the atmosphere–ocean–earth system, is considered. A simple difference equation is used to illustrate the phenomena of transitivity, intransitivity, and almost-intransitivity. Numerical models of moderate size suggest that almost-intransitivity might lead to persistence of atmospheric anomalies for a whole season. The effect of this persistence could be to allow substantial anomalies to build up in the underlying ocean or land, perhaps as abnormal temperatures or excessive snow or ice. These anomalies could subsequently influence the atmosphere, leading to long-period fluctuations. The implications of this possibility for the numerical modeling of climate, and for the interpretation of the output of numerical models, are discussed.

1. INTRODUCTION

A problem in fluid dynamics which has attracted the attention of a number of theoreticians is the following. Given a cylindrical vessel of known radius, filled with water to a known depth, mounted on a turntable rotating at a known rate, and subjected to heating of known intensity near its rim and cooling of known intensity near its center; to determine the distributions of temperature and velocity which will establish themselves in the water. As fluid dynamical problems go, it is reasonably tractable. The heating and cooling are ordinarily taken into account by specifying the temperature distribution at the boundary of the liquid; likewise, it is specified that the liquid in immediate contact with the cylinder should rotate with the cylinder. Heat and momentum are transferred from one element of fluid to another by conduction and viscous drag, and are also transferred from one location within the fluid to another by the motion of the fluid. A rather detailed steady solution has been presented by Davies (1956) while the writer (Lorenz, 1962) using very crude approximations to the governing equations, has obtained analytic solutions for some cases where the temperature and motion fields refuse to settle down to a steady state.

To establish the relevance of these studies for the problem of climate, we note first that the vessel which we have described has been used extensively in the laboratory for the purpose of simulating the circulation of the atmosphere. In these familiar "dishpan" experiments (Fultz *et al.*, 1959) the rotating vessel is supposed to simulate a hemisphere of the rotating earth, the water simulates the atmosphere, and the heating and cooling simulate the heating in the tropics and the cooling in the polar regions. In many instances the gross features of the temperature and motion fields in the water prove to bear considerable resemblance to those in the atmosphere. By making a few modifications in the presentation of the dishpan problem as a fluid dynamical problem—notably by replacing the cylindrical geometry by spherical geometry, the incompressible liquid by a compressible gas, and molecular conductivity and viscosity by turbulent conductivity and viscosity—and then repeating the mathematical work, we should obtain solutions representing the fields of temperature and motion in the atmosphere.

The climate of a region is often thought of as the weather which "ordinarily" occurs there. In more readily quantifiable terms, the climate may be identified with the set of long-term statistical properties of the atmosphere. Fluid dynamical investigations of the dishpan circulation, with the modifications indicated above, may therefore be regarded as attempts to deduce the climate; even without the modifications, they may be looked upon as studies of the "climate" of the dishpan.

A little further examination indicates, however, that for the problem of climatic change as it has occurred on the earth, and as it is likely to continue to occur, the theoretical studies of the dishpan are seriously deficient. First of all, the effect of solar heating is certainly not to impose a fixed temperature distribution in the layer of atmosphere adjacent to the earth, with the interior of the atmosphere obtaining its heat from this layer by conduction and convection. A fair portion of the solar radiation is absorbed within the atmosphere, thereby appreciably complicating the problem. Moreover, much of the energy which does penetrate to the earth is returned to the atmosphere by radiation. Since the atmosphere absorbs selectively according to wavelength, it is actually not sufficient to specify the total solar output—the spectrum is also required.

A perhaps more serious deficiency of the dishpan as a climate model is that the surface underlying the atmosphere is not a rigid bottom with a fixed temperature; in exchanging heat and momentum with the atmosphere it undergoes temperature variations comparable to those of the atmosphere itself, while the oceanic portion, at least, acquires momentum in the form of currents and waves. Equally important, the surface exchanges mass in the form of water, and, to a lesser but not negligible extent, in the form of dust and other atmospheric "impurities."

It therefore appears that if we are not to underspecify the problem, we must not be content to treat the total amount and spectral distribution of energy received from the sun as the only significant "environmental" parameters, i.e., quantities external to the atmosphere which influence the atmosphere but are not in turn influenced by the atmosphere. We must specify the geographical distribution of the properties of the underlying surface—primarily, the location of land and water, and, for the land, the location of mountains and plains, but also, if we desire precision, the composition of each portion of the land, which influences the depth through which the absorbed solar heat becomes distributed. Since some of these quantities vary with time, we need to know their variations—not simply the seasonal and other normal variations associated with the earth's orbital parameters, but also the irregular variations, including irregular solar activity which plays a central role in some theories of climatic change. If we are interested in variations over millions of years, we must specify how the continents and oceans have moved, and how the great mountain ranges have risen and decayed. Finally, the required environmental parameters include volcanic activity, which may inject quantities of matter into the atmosphere, and, to an ever increasing degree, human activity.

In contrast to the environmental parameters, the variables whose statistical properties logically constitute the climate include not only the fields of temperature and motion in the atmosphere, but also the distribution of water in its gaseous, liquid, and solid phases. They also include many variable properties of the ocean and earth which are in turn influenced by the atmosphere, and which must therefore appear as dependent variables in a closed mathematical system. The most prominent of these are the ocean-surface temperature, and the presence or absence of ice covering the ocean and snow or ice covering the land. Less prominent but also to be included are the particulate matter in the atmosphere, the salinity of the ocean, and the moisture content of the soil. In a somewhat ambiguous category is vegetation; over periods of years its variations may be mainly seasonal, but over centuries it is profoundly influenced, even if not controlled, by climatic factors.

Our principal reason for describing the dishpan problem, which by now may appear rather simple, together with the more intricate climate problem is to point out that no matter which of these problems we are attacking, we are, if we truly expect to find a solution, effectively adopting a *deterministic* point of view. That is, we are assuming that the environmental parameters, together with the physical nature of the materials which make up the atmosphere–ocean–earth system, determine the climate. It would appear to follow as a corollary that changes in the climate are caused by changes in the environmental parameters, e.g., in the energy emitted from the sun, or perhaps in the composition of the atmosphere itself. The purpose of this discussion is to point out that such a conclusion is not necessarily justified, and to consider some of the alternative possibilities.

Many current theories of climatic change do not, in fact, include changes in environmental parameters as essential parts. It is not our purpose here to enumerate specific theories of this sort; they are readily found elsewhere (Mitchell, 1965; Sellers, 1965). We shall instead concern ourselves with observations and speculations which are relevant to nondeterministic theories of climatic change in general.

2. NONDETERMINISTIC CONSIDERATIONS

To proceed with our task, we must first be more specific about what we usually mean by climate. "Long-term statistical properties" might be interpreted as averages and other statistics taken over infinite time spans, and, indeed, such a concept is extremely useful in some theoretical work. However, climatic change has acquired its present status as a challenge to the scientific mind not because changes of climate are theoretically possible, but because the record shows that they have taken place. Observed "climatic" changes are of necessity changes in statistics taken over finite intervals, although sometimes very long ones. Changes from week to week or month to month are seldom regarded as climatic; year-to-year changes occasionally, decade-to-decade changes frequently, and century-to-century changes usually are considered climatic.

In seeking to learn how the climate can change if the environmental effects remain constant, we turn to a more fundamental question: why should the *weather* change if the environment remains constant? In treating the simple dishpan or the more complicated atmosphere–ocean–earth system as a fluid dynamical problem, we find that, if the environment does not change, there are indeed solutions where the weather does not change. At any given location it is raining all the time, or else it never rains at all. A preponderance of jungles and deserts and a relative scarcity of intermediate climates characterize the warmer part of the world.

The principal reason why such a situation does not actually prevail, and would not prevail in nature even if the environment never varied, is the instability of all steady-state weather patterns with respect to disturbances of small amplitude. Finite-amplitude disturbances will therefore develop; prominent among these will be the cyclones and anticyclones which travel across the oceans and continents in middle latitudes and bring alternate intervals of rain and sunshine, warmth and cold. Accompanying these will be fluctuations in the temperature and salinity of the oceans, the dampness of the ground and the run-off of rivers.

It follows that any finite time segment of the history of the atmosphere is no more than a statistical sample, and, consequently, that climatological statistics obtained from one segment will not be identical with those obtained from another. If the fluctuations of the atmosphere are primarily of short period, the separate samples may be hardly distinguishable from one another. If, however, a considerable portion of the variance of the atmosphere is in periods comparable to the duration of a sample, the separate samples may possess markedly differing statistics, and the changes from one sample to the next may be interpreted as changes of climate. The question we face is, therefore, what periods can occur if the environment is constant, and, in particular, can very-long-period fluctuations occur?

To a first approximation the cyclones and anticyclones which form because of instability are carried along with the large-scale currents. Typical periods which ensue are therefore comparable to the time required for a typical current to carry a system through a full wavelength, say a few days. Indeed, there are mathematical solutions in which the cyclones follow one another at fixed intervals, and each is an exact replica of its predecessor; here the system fluctuates with a single period. But these solutions are unstable with respect to still further disturbances, and the end result is a rather irregular behavior, with continual fluctuations in the number of cyclones and in their intensity and shape.

When the cyclones increase in intensity or number, they gain their energy from the currents on which they are superposed, thus temporarily rendering the latter less capable of producing new cyclones. When as a consequence the cyclone activity declines, the large-scale currents are able to regain their strength. This leads to periods comparable to or exceeding the life span of individual cyclones, say, a few weeks.

Features of the atmosphere-ocean-earth system which favor still longer periods are those which by their nature are constrained to vary rather slowly. Perhaps the most conspicuous of these is the ocean-surface temperature, which may change only slightly in response to a large gain or loss of heat, in view of the large heat capacity of the layer of water which takes part in the change. The potential importance of oceanic anomalies in inducing subsequent changes in atmospheric circulation has been stressed by Bjerknes (1969) and Namias (1969). Ocean-surface temperature variations may likewise be produced by contact with a variable atmosphere; in some instances, however, they result from the instability of an ocean current, such as the Gulf Stream. Periods of a few months or longer may be anticipated.

It is therefore not difficult to account for a variety of periods ranging from about a day up to a year or so, with a completely constant environment. It is less obvious how fluctuations with periods of many years or centuries may be produced, and less certain that they can be produced at all. To investigate this question, we turn to some of the results of ergodic theory.

We begin by noting that according to these results the climate need not be unique. That is, some systems of differential equations have the property that the long-term statistics of certain time-dependent solutions are not the same as those of other time-dependent solutions. Stated otherwise, some physical systems have the property that there are two or more distinct sets of statistics, any one of which could constitute the climate of the system without violating any physical laws. The particular climate which prevails then depends upon the conditions which happened to exist when the system first became established. Such systems are called *intransitive.* Systems for which only one climate is physically possible are called *transitive.*

Both transitive and intransitive systems occur in nature. We have no means at present of determining whether the atmosphere–ocean–earth system is transitive or intransitive. To do so would require establishing much more precise mathematical models of the system than we can presently construct, or temporarily disrupting the system to a far greater extent than we already do when we discharge large amounts of energy and waste matter, or when we create large artificial lakes or remove large stands of timber. We do know that both transitive and intransitive systems are to be found among the dishpan experiments.

However interesting the phenomenon of intransitivity may be, its importance is not immediately obvious, because it cannot be a *sole* cause of climatic change. If the system is truly intransitive, changes from one possible climate to another will never occur of their own accord. We therefore note the possibility that if the system should be intransitive under a constant environment, a temporary change in the environment, perhaps due to some anomalous solar activity, could cause a new climate to develop. If the environment should then return to its original state, one of the alternative climates compatible with this state might become established instead of the original climate. Here, even though the environmental parameters would not determine the climate, the conclusion that a change in some parameter is needed for a climatic change would not be negated.

Returning to ergodic theory, we note that certain transitive systems of equations may be converted into intransitive systems simply by changing the numerical value of a single constant. In the case of the dishpan this constant might be the rate of rotation; for the atmosphere–ocean–earth system it might be a coefficient of turbulent viscosity or conductivity, whose most appropriate value in the atmosphere or ocean is uncertain in any case. If in a transitive system we do alter such a constant, but by an amount not quite enough to make the system intransitive, we may observe another form of behavior. Two particular time-dependent solutions of the system may appear to have considerably different sets of statistics if the solutions are extended over only a moderate time span, i.e., the system may appear to be intransitive. However, when the time span is made sufficiently long, the solutions will be found to have similar statistics. This means also that a single solution will exhibit different statistical properties within different segments of a long time span. We have called systems of this sort *almost intransitive* (Lorenz, 1968).

We have already noted that when a physical system possesses a component which is constrained to behave rather sluggishly, long-period fluctuations will be favored, and separate segments of a time-dependent solution may possess rather different statistics. We prefer not to regard the phenomenon as almost-intransitivity in this case. We prefer to consider a system almost intransitive only when there is nothing which obviously favors periods as long as those which actually ensue.

There does not appear to exist any well-developed theory of almost-intransitivity. Certainly we are not in a position to say whether the atmosphere–ocean–earth system is almost intransitive, or whether the atmosphere by itself would be almost intransitive if conditions in the underlying surface were held fixed. From experience with numerical solutions of a number of simple models, including some designed to simulate atmospheric behavior, it appears that almost-intransitivity may be a fairly common phenomenon, and may not actually require for its existence that some constant have a nearly critical value.

In any event, it is obvious that if the atmosphere–ocean–earth system should be almost intransitive, periods of virtually any length could occur even with a constant environment. In the following section we shall examine a simple equation which illustrates some of the mathematics of almost-intransitivity. Subsequently, we shall consider the possibility that almost-intransitivity may be of direct importance to climatic change.

3. AN EXAMPLE OF ALMOST-INTRANSITIVITY

The phenomenon of almost-intransitivity may be illustrated with very simple systems of equations. We have chosen the simplest such system that we have been able to discover—the single first-order cubic difference equation

$$X_{n+1} = a(3X_n - 4X_n^3), \qquad (1)$$

where a is a positive constant. This equation can govern a transitive or intransitive system, according to the value of a. It appears (Lorenz, 1964) that a quadratic difference equation, which would be even simpler, cannot govern an intransitive system.

Our equation is not the result of simplifying the equations governing the atmosphere–ocean–earth system or any other familiar physical system. We should note, however, that in many numerical models of the atmosphere, the right-hand sides of the governing differential equations are quadratic functions of the dependent variables. Under many standard time-differencing schemes, such as a low-order Runge–Kutta scheme, these equations become converted into difference equations whose right-hand sides are polynomials of relatively low degree. With this feature in common with the atmospheric models, Eq. (1) will serve to illustrate some of the *mathematical* aspects of transitivity and intransitivity.

If $a > 1$, the solution of (1) will generally blow up. If, however, $a \leqslant 1$, and $|X_n| \leqslant a$, it is readily demonstrated that $|X_{n+1}| \leqslant a$. A choice of an initial value X_0 with $|X_0| \leqslant a$ will therefore determine an infinite sequence $X_0, X_1, X_2, \ldots,$ with each term lying between $-a$ and a. The long-term statistical properties of such a sequence constitute the "climate" governed by (1).

Further examination reveals that if $a^2 < 3/4$, X_{n+1} and X_n must have the same sign. It follows that every sequence (except a sequence of zeros) consists entirely of positive numbers or entirely of negative numbers. Since positive and negative sequences have different statistical properties, including different "climatic means," the system governed by the equation is clearly intransitive.

If, however, $a^2 > 3/4$, changes of sign within a sequence are possible. In fact, if X_n is sufficiently close to $1/2$, then $X_{n+1}^2 > 3/4$, and X_{n+2} and X_{n+1} have opposite signs. For certain special values of $a^2 > 3/4$, some sequences contain mostly positive terms and others mostly negative, and again there are two climates. However, for "most" values of $a^2 > 3/4$ (and <1), the system seems to be transitive.

When a^2 exceeds $3/4$ only slightly, the range of values of X_n about $1/2$ for which $X_{n+1}^2 > 3/4$ is very small. The probability that an arbitrary member of an arbitrary sequence will lie in this range is therefore small. Hence it may be anticipated that most sequences will be characterized by large numbers of successive terms of one sign, and that transitions from one sign to another will be relatively rare events. Examination of a few short segments might fail to reveal any sign changes at all. The system therefore qualifies as being almost intransitive.

To illustrate the almost intransitive behavior of Eq. (1), we present some numerical solutions with $a = 7/8$, whence a^2 slightly exceeds $3/4$. Solving this equation numerically is, incidentally, an easy task which anyone can perform in a few spare moments; it does not even require a programmable computer except when very long sequences are needed. Although the equation is not derived from any meteorological equation, it will be convenient to think of X_n as the value of some weather index on the nth "day," since weather data from which climatological statistics are compiled often consist of daily observations.

Equation (1), like the system of equations governing the atmosphere–ocean–earth system with a constant environment, possesses steady-state solutions; with $a = 7/8$, they occur when $X_n = 0$ or $X_n^2 = 13/28$. These solutions, again like the atmospheric solutions, are unstable with respect to small disturbances. Equation (1) also possesses solutions with

TABLE 1
A Particular Solution of Eq. (1)[a]

n	X_n	n	X_n	n	X_n	n	X_n
0	0.6814	30	0.8659	60	-0.1690	90	0.3921
1	0.6814	31	0.0007	61	-0.4268	91	0.8182
2	0.6815	32	0.0019	62	-0.8482	92	0.2305
3	0.6812	33	0.0050	63	-0.0906	93	0.5622
4	0.6818	34	0.0132	64	-0.2353	94	0.8538
5	0.6805	35	0.0348	65	-0.5720	95	0.0627
6	0.6833	36	0.0911	66	-0.8465	96	0.1637
7	0.6770	37	0.2365	67	-0.0993	97	0.4144
8	0.6910	38	0.5746	68	-0.2571	98	0.8387
9	0.6590	39	0.8443	69	-0.6154	99	0.1368
10	0.7283	40	0.1097	70	-0.7997	100	0.3501
11	0.5598	41	0.2833	71	-0.3094	101	0.7688
12	0.8555	42	0.6641	72	-0.7085	102	0.4278
13	0.0544	43	0.7181	73	-0.6152	103	0.8489
14	0.1424	44	0.5890	74	-0.8000	104	0.0871
15	0.3636	45	0.8309	75	-0.3080	105	0.2262
16	0.7862	46	0.1733	76	-0.7062	106	0.5533
17	0.3629	47	0.4368	77	-0.6212	107	0.8595
18	0.7854	48	0.8549	78	-0.7917	108	0.0337
19	0.3661	49	0.0574	79	-0.3415	109	0.0882
20	0.7892	50	0.1499	80	-0.7570	110	0.2292
21	0.3511	51	0.3817	81	-0.4687	111	0.5596
22	0.7702	52	0.8074	82	-0.8700	112	0.8556
23	0.4228	53	0.2774	83	0.0208	113	0.0536
24	0.8454	54	0.6535	84	0.0546	114	0.1401
25	0.1047	55	0.7387	85	0.1428	115	0.3583
26	0.2708	56	0.5284	86	0.3647	116	0.7795
27	0.6413	57	0.8707	87	0.7876	117	0.3885
28	0.7602	58	-0.0247	88	0.3575	118	0.8146
29	0.4577	59	-0.0647	89	0.7785	119	0.2464

[a]The initial value represents a small departure from a steady state.

perfect "two-day" periodicity; these are analogous to the atmospheric solutions where the cyclones pass by at fixed intervals, and, like these solutions, they also are unstable with respect to still further disturbances.

For the first numerical solution we choose $X_0 = 0.6814$, a slight departure from steady conditions. Table 1 presents the values of X_n for the first four "months." The initial state is so close to the steady state that X_n does not vary appreciably during the first week, but during the second week all resemblance to the initial state disappears. Quite accidentally, on day 15 a perfect two-day periodicity is nearly but not exactly established; this also disappears before the end of the first month. Close approaches to periodic solutions with four-day periodicity occur during the third month, but evidently all periodic solutions are unstable.

The most striking feature of Table 1 is that only two changes of sign or "transitions" occur in four months. The almost-intransitivity of the system is thereby clearly exhibited.

Let us consider what might happen if the numbers in Table 1 were real observations of some weather element, in which case the governing equations would not be known with sufficient accuracy to indicate whether the observed behavior was normal or exceptional. If no further data were available, we might easily conclude that in the long run positive values of X_n predominate over negative values. If our data sample had consisted of only the first 50 days of observation, we might even have concluded that negative values never occur at all. The implications regarding the reliability of climatological statistics, if the atmosphere–ocean–earth system should be almost intransitive, are obvious.

Figure 1 compares three additional solutions of (1), having the nearly equal initial values of 0.0999, 0.1000, and 0.1001. The transitions happen to occur more frequently than in Table 1, in some cases as close together as five days, but

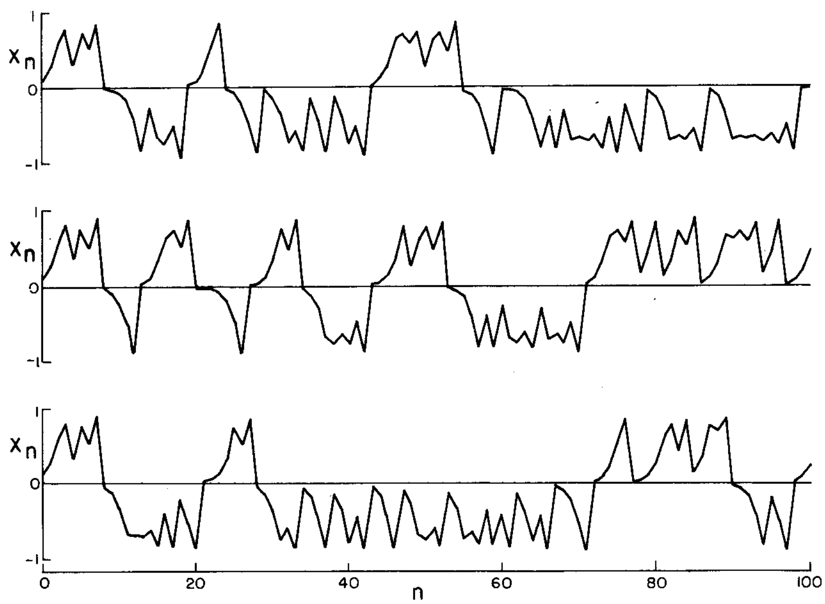

FIG. 1. Solutions of Eq. (1) extending for 100 time steps, starting from initial values of 0.0999 (upper), 0.1000 (middle), and 0.1001 (lower). The straight-line segments joining consecutive points are solely for the purpose of making the chronological order easier to see.

again sometimes a month apart. The lack of predictability at extended range, in the absence of perfect observations, is also apparent; even with perfect knowledge of the governing equations, mistaking an initial value of 0.1000 for 0.0999 or 0.1001 would render the forecast useless after two weeks. This is another feature of Eq. (1) which appears to be shared with the atmosphere–ocean–earth system.

For good measure we have extended the solution of (1) with $X_0 = \frac{1}{2}$ for 1,000,000 days, in order to observe the distribution of intervals between transitions. Altogether there are 56,354 transitions, making the average interval 17.7 days. The most frequent interval is 5 days, with 5440 occurrences, while there are 84 intervals of 90 days or longer. In one instance two successive changes of sign were separated by 150 days. We suspect that if the atmosphere–ocean–earth system is almost intransitive, the intervals between transitions will show similar variability.

4. IMPLICATIONS FOR CLIMATIC CHANGE

Having noted some properties of almost-intransitivity, we now consider what bearing they may have on the question of climatic change. Although it is a matter of record that the atmosphere and its surroundings undergo fluctuations ranging in duration from fractions of a day to many centuries, this information does not assure us that almost-intransitivity is involved, since the fluctuations could result from a variable environment, in line with a deterministic theory. With our present state of knowledge, indications that the atmosphere–ocean–earth system is almost intransitive, or that the atmosphere by itself would be almost intransitive, are best sought by means of numerical models. The following considerations are necessarily somewhat speculative.

Some of the simpler models suggest that the atmosphere, if subjected to constant forcing, would be almost intransitive, or perhaps completely intransitive. The models indicate two or more distinct regimes of behavior, and the model atmosphere tends to become trapped in one regime or another for extended periods. There is, however, one aspect of the real atmosphere which we have not so far considered, and which would appear to hinder the production of periods of several years or longer. This is the strong normal seasonal variation of the thermal forcing.

Let us suppose that the atmosphere does possess two distinct regimes of winter behavior, and that within a winter it will pass from one regime to the other with difficulty or not at all. As the winter ends and the solar heating intensifies, the atmosphere will acquire a new form of behavior which is not characteristic of either winter regime. When the next winter arrives, the atmosphere will again enter one of the winter regimes, but the choice between these may be more or less a matter of chance. Almost intransitivity might thus favor persistence throughout a season, but not persistence from one year to another. Somewhat similar probable effects of the passage of the seasons have been noted by Lamb (1972).

It might then appear that almost intransitivity of the atmosphere would be of little importance for long-period climatic fluctuations. To capture the potential importance, we must consider the interaction of the atmosphere with the ocean and earth, and particularly with those parts which tend to vary rather slowly.

One such feature which we have already noted is the ocean surface temperature. Local atmospheric temperature anomalies of a few months' duration, for example, have often been attributed to the influence of abnormally warm or cold regions of the ocean, which act to heat or cool the atmosphere. In giving its heat to the atmosphere an ocean–surface temperature anomaly tends to destroy itself, but, because the ocean has such a high heat capacity, it is assumed that the destruction of the anomaly may require a month or several months. The initial appearance of an ocean anomaly is in turn sometimes attributed to contact with an abnormally warm or cold atmosphere.

Barring some process by means of which an ocean-surface temperature anomaly is self-amplifying rather than simply self-preserving, it is difficult to see how an atmospheric temperature anomaly could produce an ocean temperature anomaly which in turn could maintain another atmospheric temperature anomaly for a matter of months, unless the former atmospheric anomaly lasts considerably longer than the latter one, or is considerably more intense. Somewhere in the chain of events there would seem to be the necessity for an atmospheric anomaly not produced by by the ocean.

An anomaly resulting from almost-intransitivity of the atmosphere would appear to satisfy the requirements. If such an anomaly persists for the majority of a season, it could enable a substantial ocean surface anomaly to develop, despite the ocean's great heat capacity. This oceanic anomaly could subsequently lead to a new atmospheric anomaly.

It would also appear that if an atmospheric anomaly resulting from almost intransitivity has been present for long enough to enable a significant oceanic anomaly to develop, the latter might contribute to further persistence of the *same* atmospheric anomaly. That is, the presence of the ocean should decrease the likelihood of a change of regime. Similar remarks should apply to other anomalies of the ocean–earth system which in turn affect the atmosphere, such as anomalous snow cover. In short, almost-intransitivity of the atmosphere, if present, should increase the likelihood that the atmosphere–ocean–earth system is almost intransitive.

Since some anomalies of the ocean or earth, once established, seem capable of persisting through several seasons, the seasonal variations of the environment need not prevent almost-intransitivity of the complete atmosphere–ocean–earth system, if present, from leading to long-period fluctuations. If the latter system possesses several winter regimes, and one winter regime, characterized, say, by abundant polar ice, gives way to a summer regime as the seasons advance, persistence of some of the ice through the summer might favor the reestablishment of the same winter regime, when the following winter arrives. Almost-intransitivity of the complete system may therefore be a contributing factor to long-period climatic change. More substantial evidence in favor of such a hypothesis, however, will apparently at the very least have to await the development of suitable numerical models, for which solutions extending over many years can be readily obtained.

A phase of climate research somewhat different from that of explaining the observed fluctuations of climate has been the testing of hypotheses, or in some instances merely seeking hypotheses, through the use of numerical climatic models. A familiar example is the attempt to determine the effect upon the climate of a substantial increase in the carbon dioxide content of the atmosphere. Among the most recent and most detailed results are those of Manabe and Weatherald (1975). In studies of this sort the potential importance of almost-intransitivity is less speculative, because we know that some models are almost intransitive, even though the physical system which they represent may not be.

The procedure would consist of making one or more control runs with the model, where the carbon dioxide content of the model atmosphere equals that of today's atmosphere, and also one or more perturbed runs, which are similar to the control runs except that the carbon dioxide content is, say, twice as great. The statistical properties of the perturbed runs are then compared with those of the controls.

Since each run is a statistical sample, it is inevitable that the properties of separate runs will not be identical. The crucial question is then how much the statistics of the perturbed runs would be expected to differ from those of the controls if changing the carbon dioxide actually had no effect at all. This sort of question has been discussed, for example, by Warshaw and Rapp (1973), who used three perturbed runs and three controls in a recent numerical experiment.

If the *model* is almost intransitive and there are two regimes of behavior, it is possible that some statistic, perhaps a temperature averaged over a large region, will differ from one regime to the other by an amount exceeding the change to be expected from increasing the carbon dioxide content. If the total duration of the runs is not too great—a situation often necessitated by the costliness of computations—the likelihood that the perturbed runs will stay largely in one regime, while the control runs stay largely in the other, is fairly high, and so therefore is the likelihood that the "noise" will exceed the "signal." A problem arises even when the noise is only a moderate fraction of the signal, as discussed by Leith (1973).

If the distinction between the regimes is so obvious that one can tell by inspection that the model is almost-intransitive, one might determine the effect of increasing the carbon dioxide by comparing the perturbed and control runs separately for each regime. If instead, as seems more likely, the distinction between the regimes is more subtle, it would appear necessary to continue the computations to the point where the perturbed runs, and also the control runs, are likely to have spent a representative amount of time in each regime, thereby reducing the noise to an acceptable level.

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