Nonlinearity, Weather Prediction, and Climate Deduction

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ABSTRACT

The equations governing the atmosphere are nonlinear. Weather prediction is identified with determining particular solutions of these equations, while climate deduction is identified with determining statistics of the general solution.

The nonlinearity gives rise to small-scale motions and nonperiodicity. The nonperiodicity makes analytic solution of the equations unfeasible. Particular solutions must therefore be determined numerically, and the small-scale motions cannot be properly included. The range at which accurate detailed forecasts can be produced is thus limited.

The nonlinearity also prevents the derivation of closed systems of equations with statistics as unknowns. The statistics must therefore be estimated from particular numerical solutions, which are merely samples.

Numerical methods are not required when only upper and lower bounds of the statistics are sought. The need for numerical methods when precise values are desired is illustrated with a simple quadratic difference equation, while the process of establishing upper and lower bounds is illustrated with a simple partial differential equation.

NONLINEARITY, WEATHER PREDICTION, AND CLIMATE DEDUCTION

1. Consequences of nonlinearity

The physical laws which govern the behavior of the atmosphere may be expressed as a system of mathematical equations. These equations, or various convenient approximations to them, have formed the basis of numerous theoretical studies. Among these studies some have dealt with weather prediction, while others have been concerned with various aspects of the climate.

In principle the problem of weather forecasting may be identified with that of determining particular time-dependent solutions of the equations, starting from specified initial conditions. Likewise, the problem of deducing the climate may be identified with that of finding the long-term statistical properties of solutions of the equations. To the latter problem there are alternative approaches; we may first determine explicit solutions of the equations, and then evaluate their statistical properties, or we may derive an auxiliary system of equations whose unknowns are the desired statistics, and then solve these new equations.

It is common experience that we do not yet produce weather forecasts of sufficiently high quality to satisfy the general public. Likewise, there are many atmospheric phenomena – tropical hurricanes, for example – whose very existence has not yet been satisfactorily explained. In short, neither the problem of weather prediction nor that of climate deduction is close to a complete solution.

A characteristic feature of the governing equations is their nonlinearity. In this study we propose that virtually all of the difficulty encountered in attacking the problems of weather prediction and climate deduction as mathematical problems arises because of the nonlinearity of the equations. There may be other contributing factors, but these by themselves would not be insuperable if the equations were linear.

Although a number of physical factors give rise to nonlinear terms in the equations, one of these — the process of advection — is especially prominent. We further propose that advection by itself is sufficient to give rise to the difficulties which can be specifically assigned to nonlinearity, although, if advection could be suppressed, some of the other nonlinear processes might well have an equally disturbing effect. The process of advection is simply the displacement of the various properties of the atmosphere — heat, momentum, and moisture — by the motion of the atmosphere. Since the field of motion is ordinarily nonuniform, different portions of the field of any property will receive different displacements, and the field as a whole will be distorted as well as displaced. In the governing equations the components of the motion are among the dependent variables, so that the terms representing advection are quadratic, containing products of the motion with the dependent variables representing the various advected properties.

Let us look at the various aspects of nonlinearity. First of all, the atmosphere is a forced dissipative system. The forcing is due to heat received from the sun, while the dissipation is both thermal and mechanical.

The forcing is periodic in time; it consists of a large constant component with superposed annual and diurnal variations. The horizontal pattern of the forcing is large-scale; a major portion of it consists of a contrast between summer and winter hemispheres, day and night hemispheres, and oceanic and continental areas. If the atmosphere behaved as a quasi-linear system, we might expect that the circulation would consist entirely of periodically varying large-scale features.

Such is not the case. A time-space spectral analysis of the atmosphere certainly reveals pronounced periodically-varying large-scale motions, but superposed upon these are motions which are decidedly nonperiodic, ranging in scale from the large circumpolar currents through cyclones and anticyclones, thunderstorms and smaller cumulus clouds, to the smallest turbulent eddies. The general nonperiodic behavior and much of the small-scale structure are direct results of nonlinearity.

The equations governing the atmosphere presumably have periodic solutions, possessing only the annual and diurnal periods (and their overtones). Indeed, equations otherwise like those governing the atmosphere but with constant forcing possess steady-state solutions. Such steady-state or periodic solutions might be found by analytic means — perhaps by a power-series scheme. But since the observed behavior of the atmosphere is nonperiodic, these periodic solutions are presumably unstable with respect to small perturbations, and are not the solutions in which we are primarily interested.

For the problem of deducing the climate, as opposed to forecasting the weather, there is the alternative procedure of deriving new equations whose unknowns are the statistics themselves. In the case of linear governing equations, this method has proven highly fruitful. However, with nonlinear governing equations it is impossible to close the new system; there are always more unknowns than equations, and any attempt to introduce additional equations containing the unknowns already present will add still more unknowns.

Some of the most promising recent work in the theory of turbulence has attacked the closure problem by combining hypothesized relations between the unknowns with the relations already available, to render the system closed (cf. Kraichnan, 1963). In the present instance, where the "turbulence" may include eddies as large and as non-random as cyclones, the appropriate form for the additional relations is not at all apparent. We must therefore return to the original equations, and determine time-dependent solutions, from which statistics may be compiled.

There is no readily available analytic method for obtaining the general solutions of nonlinear equations, and, in any event, nonperiodic functions are not generally expressible in analytic form. We are therefore forced to introduce certain approximations. The best approximate solutions to nonlinear equations which we now know how to discover are obtained by well-known numerical methods. Ordinarily we represent the field of each dependent variable by its values at a standard grid of points, and replace the partial derivatives by finite differences. But in so doing we must omit the details of the small-scale features, which become lost between the grid points. We can reduce the scale of features retained by increasing the number of points, but no digital computer available at present or likely to be available in the near future will allow us to represent the details of every thunderstorm, not to mention smaller cumulus clouds.

Thus the same nonlinearity which gives rise to small-scale features forces us, by also giving rise to nonperiodicity, to omit the details of these features. But we must still try to incorporate their effects upon the features of larger scale. This we can do to some extent through coefficients of turbulent viscosity and conductivity; however, we certainly do not know the proper values for these coefficients to within a factor of two. To determine these coefficients theoretically would require solving the same nonlinear equations whose intractability required our determining the coefficients in the first place; to determine them empirically would require the recording and processing of far more data than we can readily accumulate and handle at present.

We must therefore be content to work with approximate equations. In the final report of an earlier contract under which the Statistical Forecasting Project operated (Lorenz, 1963), we discussed some of the implications of this state of affairs with regard to predictability. In particular, we demonstrated that any nonperiodically varying system is unstable, in the sense that two solutions originating from slightly different initial conditions must ultimately diverge from one another. From this we concluded that if any errors whatever are present in observing the initial conditions, no system of forecasting can give acceptable predictions at sufficiently long range.

The same situation occurs if there are any errors whatever in formulating the governing equations. For even if the initial conditions are known exactly, and the equations are integrated by a stepwise procedure, the conditions are no longer exactly known after the first time step.

Just as the earlier final report was concerned mainly with the extent to which the weather may be predicted, this report will be concerned mainly with the extent to which the climate may be deduced. We have noted that when the general solution of the equations is nonperiodic, we must obtain explicit solutions to the equations, and compile statistics from them. Furthermore, the equations can at best be expressed in approximate form, and the solutions must be obtained by numerical procedures.

The solutions will then soon diverge from the appropriate solutions of the exact equation. This may not pose a serious problem, because the exact solution and the diverging approximate solution may possess nearly the same statistics. More serious is the fact that the numerical solution is a particular solution, and its behavior may differ considerably from that of other particular solutions. Thus the statistics will be computed from a sample, and will be subject to all the errors (except those due to missing data) which arise when the climate of the real atmosphere is computed from observational samples.

It is commonly observed that no matter how long a period of actual atmospheric records is chosen for computing climatological statistics, the statistics evaluated from a different period of similar length may have considerably different values. We can therefore never be certain that, in computing statistics from a numerical solution, we have chosen a representative solution.

The situation is less discouraging if we are content with qualitative or semi-quantitative deductions. It is frequently possible to deduce certain constraints upon the values which various statistics may assume, even when exact numerical values cannot easily be found. Such constraints might, for example, be deduced from an incomplete system of equations whose unknowns are some of the statistics. It would appear easier to establish such constraints for some of the large-scale over-all features of the atmosphere, than for some of the more local climatological properties.

For example, consideration of the rate of incoming solar energy establishes an upper limit for the average amount of kinetic energy which the atmosphere may contain, provided that certain reasonable assumptions concerning the rate of dissipation are accepted. Except instantaneously, an atmosphere at rest is clearly incompatible with the incoming energy, and it should be possible to deduce some lower bound for the kinetic energy. If this lower bound does not differ too widely from the upper bound, the order of magnitude of the kinetic energy will be determined.

The real atmosphere is an extremely complicated system. One of the complicating factors is the presence of oceans and continents, with their contrasting heat capacities and degrees of roughness. Mountains, hills, and smaller irregularities add to the asymmetry. A rigorous derivation of certain results for an atmosphere with a homogeneous underlying surface might lose its validity when the true nature of the surface is considered. If however the results are of a semi-quantitative nature — a demonstration, for example, that the low-latitude surface winds must be easterly and of moderate strength — they might be made rigorous for a non-uniform earth by demonstrating that the effect of the asymmetries cannot exceed some critical value. Indeed, it is such semi-quantative results which seem to offer the best prospects of eventually being rigorously established.

In the following sections we shall illustrate the problem of climate deduction with two examples. The equations which we shall use are not the equations governing the behavior of the real atmosphere, but are vastly simpler. They are, however, nonlinear, and serve to illustrate some of the complications which arise.

The first example uses the simplest possible nonlinear equation — a first order quadratic difference equation in one variable. This equation does not govern any physical system resembling the atmosphere. It serves mainly to illustrate the necessity for using numerical procedures to determine precise values of the statistics. The equation has been discussed in greater detail in a previous article (Lorenz, 1965), which is included as an appendix to this report.

The second example uses one of the simplest nonlinear partial differential equations in three independent variables. This equation represents an idealized two-dimensional forced viscous flow. It is used primarily to illustrate the process of establishing upper and lower bounds for the statistics.

2. A simple difference equation

In this section we consider a single dependent variable governed by the quadratic difference equation

$$X_{n+1} = a X_n - X_n^2 \quad . \tag{1}$$

The equation is not intended to describe any real physical process. Here X_n and X_{n+1} are the n^{th} and $(n+1)^{th}$ terms of a sequence $X_0, X_1, X_2, ---$ generated by equation (1), and a is a constant lying in the range $0 \leqq a \leqq 4$. If X_0 lies within the range $0 \leqq X_0 \leqq a$, X_n will lie within this range for all values of n .

A detailed treatment of this equation has appeared as a published article (Lorenz, 1965). Since this article is included in this report as an appendix, only the principal results will be presented at this time.

Upon averaging both sides of (1), we find that

$$(a-1)\,\overline{X} - \overline{X}^2 = \sigma^2 \quad , \tag{2}$$

where $\bar{X}$ is the average value of X_n over all values of n, and σ is the standard deviation of X_n. Equation (2) requires that $\bar{X} = 0$ if $0 \leqq a \leqq 1$; it restricts $\bar{X}$ to the interval $0 \leqq \bar{X} \leqq a-1$ if $1 \leqq a \leqq 4$.

For the interval $0 \leqq a \leqq 1$, the solution $X_n = 0$ is stable, whence, as already noted, $\bar{X} = 0$. For $1 < a \leqq 3$, this solution is unstable, but the solution $X_n = a-1$ is stable, and $\bar{X} = a-1$. For $3 < a \leqq 3.449$, this solution is also unstable, but a periodic solution of period 2 proves to be stable, with $\bar{X} = \frac{1}{2}(a+1)$. For $a=4$, the general solution is nonperiodic, but symmetry arguments show that $\bar{X} = 2$.

For the range $3.449 < a < 4$, and particularly for those values of a for which the general solution of (1) is nonperiodic, the value of $\bar{X}$ is not readily determined by analytic means. The restriction $0 \leqq \bar{X} \leqq a-1$ still holds, but to determine an exact or nearly exact value of $\bar{X}$ it is necessary to solve (1) by numerical means, and obtain $\bar{X}$ by averaging the successive values of X_n.

The process of solving (1) numerically is extremely simple. Figure 3 in the Appendix shows a graph of $\bar{X}$ as a function of a, for the range $3.4 \leqq a \leqq 4$. The irregularity of the graph clearly indicates that $\bar{X}$ is not an analytic function of a, and it strongly suggests that any attempt to obtain a closed system of equations, with statistics as unknowns, by adding hypothesized relations to equation (2) would not yield the proper result.

It is to a large extent this particular example which has led us to conclude that usually when the general solution of a nonlinear equation is nonperiodic, the statistics of the solution of this equation cannot be found except by first solving the equation numerically. With the development of faster and faster computing machines, it is becoming possible to obtain reasonably accurate numerical solutions to more and more complicated systems of equations. If, however, one is interested in a system of equations for which it is not possible or convenient to obtain a numerical solution which represents a fairly large statistical sample, the most fruitful research may be that which is devoted to obtaining upper and lower bounds for the statistics, rather than precise numerical values.

3. A simple partial differential equation

In this section we consider an idealized physical system governed by the equation

$$\frac{\partial}{\partial t} \nabla^2 \psi = -J(\psi, \nabla^2 \psi) - k \nabla^2 \psi + k \nabla^2 \psi^* . \qquad (3)$$

The single dependent variable ψ may be regarded as a stream function for a two-dimensional incompressible flow, whose vorticity is then given by $\nabla^2 \psi$, ∇^2 being the Laplacian. The terms on the right represent respectively the effects of <u>advection</u>, J being the Jacobian with respect to horizontal variables, <u>friction</u> k being the coefficient of friction, and mechanical forcing, ψ^* being a stream function for a hypothetical prespecified flow, toward which the existing flow would tend if advection were not present.

Equations of this general form have been used to study the flow in an ocean basin, when the forcing is due to wind stress (e.g., Veronis, 1963).

In this particular study the geometry of the two-dimensional region occupied by the fluid need not be completely specified. The region could be a spherical shell or an infinite plane, or it could be a bounded plane region with no flow across the boundaries.

We shall be interested mainly in determining constraints placed upon the motion by equation (1), rather than in explicit solutions. In particular, we shall be interested in obtaining upper and lower bounds for the mean kinetic energy

$$E = \frac{1}{2} \overline{\nabla\psi \cdot \nabla\psi} \quad , \tag{4}$$

which may be expressed in terms of the known mean kinetic energy of the "equilibrium" flow

$$E^* = \frac{1}{2} \overline{\nabla\psi^* \cdot \nabla\psi^*} \quad . \tag{5}$$

Here the bars denote averages over time and space.

The simplest case occurs when the equilibrium flow field is independent of time, and contains a single scale of motion, i.e., when

$$\nabla^2\psi^* = -c^2\psi^* \quad . \tag{6}$$

We shall confine our attention to this case. In this event, equation (1) always possesses the steady-state solution $\psi = \psi^*$. Other solutions always exist, since initial conditions may be chosen arbitrarily, but in some cases all solutions approach the known solution ψ^* asymptotically. The long-term statistics of any solution are then the statistics of the known solution, and the problem of determining the climate becomes trivial. In other cases the known solution ψ^* may be unstable. The general solution then need not be steady-state or even periodic, and its statistical properties may differ considerably from those of the known solution.

For the general solution we shall let

$$\psi = a\psi^* + \psi_1 \quad , \tag{7}$$

where the constant a is to be chosen so that ψ_1 is uncorrelated with ψ^* in space-time, i.e.,

$$\overline{\psi^*\psi_1} = 0 \quad . \tag{8}$$

It follows upon multiplying equation (6) by ψ^* and averaging that

$$a = \overline{\psi^*\psi} \,/\, \overline{\psi^{*2}} \quad . \tag{9}$$

In the special case where the solution ψ^* is stable, $a = 1$, and ψ_1 vanishes. In the general case, a must be regarded as an unknown constant. With the aid of (7), equation (1) may be rewritten

$$\frac{\partial}{\partial t}\nabla^2\psi_1 = -J(\psi_1,\nabla^2\psi_1) - aJ(\psi_1,\nabla^2\psi^*) - a\,J(\psi^*,\nabla^2\psi_1)$$
$$- k\nabla^2\psi_1 + (1-a)\,k\,\nabla^2\psi^* \quad . \tag{10}$$

From (10) we shall derive a system of three algebraic equations whose unknowns are statistics of the solutions of (10). From our remarks in the first section, we can infer that there will be more unknowns than equations; actually there will be four unknowns, $a, E_1, V_1, B,$ where

$$E_1 = \tfrac{1}{2}\,\overline{\nabla\psi_1\cdot\nabla\psi_1} \quad , \tag{11}$$

$$V_1 = \tfrac{1}{2}\,\overline{(\nabla^2\psi_1)^2} \quad , \tag{12}$$

$$B = -\tfrac{1}{2}\,\overline{\nabla^2\psi_1\,J(\psi^*,\psi_1)} \quad . \tag{13}$$

Multiplying (10) by ψ_1 and averaging, we obtain

$$\overline{\psi_1\frac{\partial}{\partial t}\nabla^2\psi_1} = -\overline{\psi_1 J(\psi_1,\nabla^2\psi_1)} - a\overline{\psi_1 J(\psi_1,\nabla^2\psi^*)} - a\overline{\psi_1 J(\psi^*,\nabla^2\psi_1)}$$
$$- k\,\overline{\psi_1\nabla^2\psi_1} + (1-a)\,k\,\overline{\psi_1\nabla^2\psi^*} \quad . \tag{14}$$

To simplify this equation we note that time-space averages of time derivatives and Jacobians are zero, we use equations (6) and (8), and we make frequent use of integration by parts. Thus

$$\overline{\psi_1 \frac{\partial}{\partial t} \nabla^2 \psi_1} = -\frac{1}{2} \frac{\partial}{\partial t} \overline{\nabla\psi_1 \cdot \nabla\psi_1} = 0 \quad ,$$

$$\overline{\psi_1 J(\psi_1, \nabla^2\psi_1)} = \frac{1}{2} \overline{J(\psi_1^2, \nabla^2\psi_1)} = 0 \quad ,$$

$$\overline{\psi_1 J(\psi_1, \nabla^2\psi^*)} = \frac{1}{2} \overline{J(\psi_1^2, \nabla^2\psi^*)} = 0 \quad ,$$

$$\overline{\psi_1 J(\psi^*, \nabla^2\psi_1)} = \overline{J(\psi^*, \psi_1\nabla^2\psi_1)} - \overline{\nabla^2\psi_1 J(\psi^*, \psi_1)} = 2B \quad ,$$

$$\overline{\psi_1 \nabla^2\psi_1} = -\overline{\nabla\psi_1 \cdot \nabla\psi_1} = -2E_1 \quad ,$$

$$\overline{\psi_1 \nabla^2\psi^*} = -c^2 \overline{\psi_1 \psi^*} = 0 \quad .$$

Thus, upon dividing by two, (14) becomes

$$-aB + kE_1 = 0 \quad . \tag{15}$$

Multiplying (10) by $-\nabla^2\psi_1$, averaging, and simplifying in a similar manner, we obtain the second algebraic equation

$$-c^2 aB + kV_1 = 0 \quad , \tag{16}$$

while, multiplying (10) by ψ^*, we obtain

$$B - (1-a)kE^* = 0 \quad . \tag{17}$$

Equations (15), (16), and (17) form our algebraic system.

From (15) and (16)

$$V_1 = c^2 E_1 \quad , \tag{18}$$

This implies that the flow field defined by ψ_1 contains the same average scale as the known field defined by ψ^*. The field defined by ψ_1 cannot contain just one scale, for then the advective terms in equation (1) would drop out, and ψ_1 would vanish altogether. Hence the field defined by ψ_1 contains a superposition of scales, some larger and some smaller than the scale of the known field.

If the geometry is such that there is a largest possible scale — e.g., if the domain is a spherical surface — and if this largest scale is the scale of the known field, the only possibility is for ψ_1 to vanish. The solution ψ^* is then stable. If the scale of the known field is not the largest possible scale, the known solution may be unstable.

Returning to (15) and (17) we find that

$$E_1 = a\,(1-a)\,E^* \tag{19}$$

Since E_1 and E^* are by their nature non-negative, $a\,(1-a) \geqq 0$, whence $0 \leqq a \leqq 1$. From (7) it follows directly that

$$E = a^2 E^* + E_1 \quad , \tag{20}$$

whence, in view of (17),

$$E = a\,E^* \quad . \tag{21}$$

Thus a may be interpreted as the ratio of the kinetic energy to the kinetic energy of the equilibrium flow. Since $a \leqq 1$, E^* is an upper bound for E. The case $a = 0$ is not possible, since this would imply to motion at all, a condition incompatible with equation (1).

It seems reasonable, moreover, that a state of extremely weak motion should also be incompatible with the governing equation. That is, there should be some lower bound for E considerably greater than zero. Such a bound may be obtained by referring to the definition of B, and noting that, like E_1 and V_1, it is quadratic in the variable ψ_1.

Squaring equation (11), we find that

$$\begin{aligned} B^2 &= \tfrac{1}{4}\left(\overline{\nabla^2\psi_1 J(\psi^*, \psi_1)}\right)^2 \\ &\leqq \tfrac{1}{4}\overline{(\nabla^2\psi_1)^2}\ \overline{J^2(\psi^*, \psi_1)} \\ &\leqq \tfrac{1}{2} V_1\ \overline{(\nabla\psi^*\cdot\nabla\psi^*)(\nabla\psi_1\cdot\nabla\psi_1)} \\ &\leqq V_1 M^2 E_1 \\ &= c^2 M^2 E_1^2 , \end{aligned}$$

where M^2 is the maximum value of $\nabla\psi^*\cdot\nabla\psi^*$, and is thus a known quantity. Hence

$$B \leqq cME_1 . \tag{23}$$

Combining (23) with (15), we find that $a \geqq k/cM$, provided that $E_1 \neq 0$. Since $a \leqq 1$ in any case, we obtain the alternative results

$$a = 1 \qquad \text{if} \qquad M \leqq M_c , \tag{24}$$

$$M_c/M \leqq a \leqq 1 \qquad \text{if} \qquad M > M_c , \tag{25}$$

where $M_c = k/c$ may be interpreted as a critical value for M.

Now M is simply the maximum velocity occurring in the equilibrium flow field. Equations (25) and (26) therefore say that if the equilibrium flow falls short of some critical strength, it will be stable, and no other flow will develop. If the equilibrium flow exceeds its critical strength, it may be unstable, but will not necessarily be so. If it is unstable, the energy of the resulting flow will be less than that of the equilibrium flow, but the ratio of the energy of the flow to that of the equilibrium flow will be at least as great as the ratio of the critical strength to the actual strength of the equilibrium flow.

In passing we note a distinction between the immediate and ultimate effects of advection, in the case when the equilibrium flow is unstable. The immediate effect of advection upon the total kinetic energy is nil; advection simply distorts the flow field without altering its total energy. In this sense it is a conservative process. The ultimate effect of advection, however, is to bring about a lower average kinetic energy than would prevail if advection were absent. This it does by distorting the flow field into a form where it makes less efficient use of the forcing process in maintaining itself against dissipation.

It must be stressed that this conclusion applies only to the idealized system governed by equation (1). Analogous conclusions may be valid for the atmosphere or other physical systems, but we have not demonstrated that this is so.

In summary, we have seen that when physical systems are governed by partial differential equations whose solutions cannot be readily obtained, it may still be possible to obtain useful qualitative or semi-quantitative results. These results may appear in the form of upper and lower bounds for certain statistics. Rigorous determination of these bounds appears most feasible when the statistics are characteristic of the entire flow — in the case of the atmosphere, when they are general-circulation parameters, rather than features of the local climate.

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