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# GENERATION OF AVAILABLE POTENTIAL ENERGY AND THE INTENSITY OF THE GENERAL CIRCULATION

by

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## Abstract

The problem of deducing the observed intensity of the general circulation from the distribution of solar heating is examined. If this intensity is defined as the amount of kinetic energy in the atmosphere, it is roughly proportional to the rate at which friction dissipates kinetic energy, and hence in the long run to the rate at which nonadiabatic heating generates available potential energy.

A direct explanation of the latter rate requires an explanation of the fields of heating and temperature, and hence an explanation of the large-scale exchange processes which affect the temperature field. The exchange processes are temporarily by-passed by considering the problem of determining the maximum rate of generation of available potential energy corresponding to any temperature field. This problem is made tractable by treating a hypothetical atmosphere with idealized properties. The problem yields to analytic solution if vertical and longitudinal variations are neglected, and is solved numerically if vertical variations are included. The solutions suggest that the general circulation may be operating at nearly its maximum possible intensity.

To complete the problem it is sufficient to explain why the general circulation must operate at nearly its maximum intensity, if indeed it does. A possible explanation is presented in a special case, but a general explanation must await further study.

As by-products, a possible mechanism whereby variations of the albedo may lead to fluctuations of the general circulation is described, and a possible qualitative explanation for the existence of two regimes of flow in the "dishpan" experiments is offered.

1. The energy cycle and its intensity

The general circulation of the atmosphere is characterized by an energy cycle. This cycle consists of a net generation of potential and internal energy by nonadiabatic processes, a net conversion of potential and internal energy into kinetic energy by adiabatic processes, and a net dissipation of kinetic energy by friction.

Strictly speaking, nonadiabatic heating generates only internal energy. In doing so it upsets the hydrostatic balance, and the resulting vertical motions rapidly convert some of this internal energy into potential energy. If the process of readjustment to hydrostatic equilibrium is regarded as being instantaneous, it is both permissible and convenient to regard potential energy and internal energy as together constituting a single form of energy. This form has been called total potential energy by Margules [ 8 ].

In the long run, each step in the energy cycle must take place at the same rate. This rate will be called the intensity of the energy cycle. Temporary differences in the rates at which the different steps in the cycle occur lead to temporary fluctuations in the amount of one or more forms of energy contained in the atmosphere.

One problem which confronts the meteorologist today is that of explaining the observed intensity of the general circulation. A suitable measure of this intensity is the amount of kinetic energy contained in the atmosphere. The rate of dissipation of this amount of kinetic energy depends, although in a rough fashion, upon the amount of kinetic energy itself. Hence we can explain the intensity of the general circulation, at least roughly, if we can explain the intensity of the energy cycle. We shall be more directly concerned with the latter task. We should notice that the intensity of the energy cycle is an important property of the general circulation even aside from its relation to the amount of kinetic energy.

Since the sun is presumably the ultimate source of atmospheric energy, the intensity of the energy cycle must be related in some way to the intensity of solar heating, and, indeed, must be less than the rate at which the earth receives solar energy, since no thermodynamic engine is perfectly efficient. The approximate rate at which solar energy reaches the outer extremity of the atmosphere, which depends upon the solar constant, has been known for many years, and the rate at which kinetic energy is dissipated by friction has also been estimated; according to Brunt [ 1 ] the latter rate is about 2% of the former. However, no satisfactory explanation as to why this ratio should be as low as 2%, nor on the other hand why it should not be much lower, has yet been given. Indeed, it is noteworthy that in spite of the amount of outstanding research on the general circulation performed during the past decade, relatively little attention has been devoted to explaining its intensity. An exception is a recent paper by Wulf and Davis [ 11 ], to which further reference will be made.

It might appear possible, on the basis of our knowledge of the incoming solar radiation, and the laws governing heat exchange by radiation and other processes, to obtain an independent estimate of the intensity of the energy cycle by computing the rate of generation of total potential energy, which is simply the rate at which heat is added to the atmosphere. Unfortunately, if we attempt to do this, we are faced with the inevitable result that the rate of dissipation of kinetic energy by friction also equals the rate of heating by friction, at least to the extent that the heat generated by friction is released in the atmosphere itself. It follows that the net heating by non-frictional processes is in the long run zero. An estimate of the total heating of the atmosphere must therefore include frictional heating, and so will not be an independent estimate of the intensity of the energy cycle, but must *a priori* agree with the estimate computed from frictional dissipation of kinetic energy.

However, we can obtain an independent estimate and at least a partial explanation of the intensity of the energy cycle by considering available potential energy. This form of energy has been discussed by Margules [ 8 ], and its properties have been described in detail by the author [ 7 ]. Some of these properties will be mentioned now.

The *available potential energy* of the atmosphere may be defined as the excess of the total potential energy above the minimum total potential energy which could result from any adiabatic redistribution of mass. As such it does not represent an amount of energy additional to total potential energy, but instead represents the maximum portion of the total potential energy immediately available for conversion into kinetic energy. The difference between total and available potential energy may be called *unavailable potential energy*.

The conversion of total potential energy into kinetic energy may therefore equally well be regarded as a conversion of the same amount of available potential energy into kinetic energy. It follows that the energy cycle may be regarded as consisting of a net generation of available potential energy by nonadiabatic processes, a net conversion of available potential energy into kinetic energy, and a net dissipation of kinetic energy by friction. The intensity of the energy cycle is then equal to the net rate of generation of available potential energy.

At any one time, however, the rate of generation of total potential energy simply equals the total heating, while the rate of generation of available potential energy depends largely upon the correlation between temperature and heating within isobaric surfaces. The effects of different physical processes upon the various forms of energy are shown schematically in fig. 1.

Frictional heating contributes very little to the generation of available potential energy, which must then depend primarily upon other forms of heating. It should therefore be possible to estimate the intensity of the energy cycle independently of any estimates based upon friction. This has been done by the author [ 7 ]; the result that this rate equals about 2% of the incoming solar energy agrees with earlier estimates of frictional dissipation, to within the limits of accuracy of the computations.

The introduction of available potential energy does not automatically solve the problem of explaining the intensity of the energy cycle, since, to solve this problem directly, we must explain the temperature distribution. The temperature is influenced by the large-scale exchange processes as well as by local effects, and the effectiveness of the exchange processes depends partly upon the intensity of the general circulation.

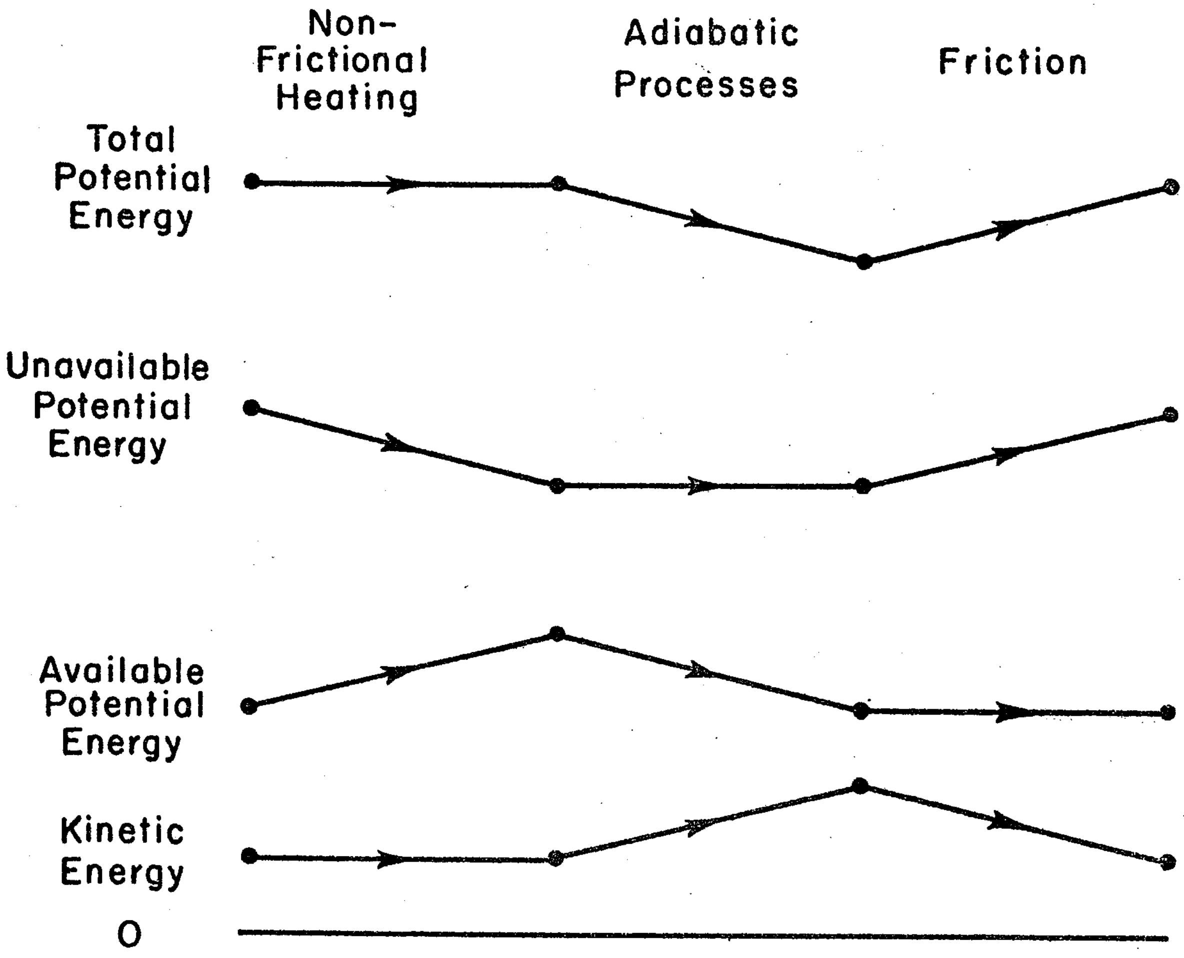


Fig.1 The net effect of the indicated physical processes upon indicated forms of energy.

We have therefore concentrated upon the less involved problem of finding the maximum possible rate of generation of available potential energy corresponding to any temperature distribution. This problem does not require a knowledge of the particular exchange process leading to the temperature distribution. As we shall see, our results indicate that the maximum rate of generation of available potential energy does not greatly exceed the observed rate of about 2% of the incoming solar energy. We have therefore partially explained the intensity of the energy cycle, by showing that it cannot be much greater.

At present, it appears to the author that the remaining problem, namely, that of explaining why the intensity of the energy cycle is not considerably less, must involve consideration of the exchange process. A possible explanation will be presented in a special case, but a more general result must await further study.

## 2. Available potential energy and its generation

In the following paragraphs, some of the ideas already presented will be put upon a more rigorous foundation. The expressions for available potential energy and its rate of generation will first be introduced. In the equations which follow, these familiar symbols appear:

$t$ : time

$z$ : elevation above earth's surface, regarded as horizontal

$p$ : pressure

$p_o$ : pressure at earth's surface

$p_{oo}$ : a standard pressure, 100 cb.

$T$ : temperature

$\mathbb{V}$ : horizontal velocity vector

$g$ : acceleration of gravity

$c_v, c_p$ : specific heats of air at constant volume and constant pressure

$\kappa$ : ratio $(c_p - c_v)/c_p$ approximately $2/7$

$\theta$ : potential temperature, $T p^{-\kappa} p_{00}^{\kappa}$

$\Gamma$ : lapse rate of temperature, $-\partial T/\partial z$

$\Gamma_d$ : dry adiabatic lapse rate, $g/c_p$

If $\theta$ is chosen as a vertical coordinate, the average available potential energy, per unit area of the earth's surface, is (see [7])

$$\bar{A} = (1+\kappa)^{-1} c_p g^{-1} p_{00}^{-\kappa} \int_0^{\infty} \left( \overline{p^{1+\kappa}} - \bar{p}^{1+\kappa} \right) d\theta . \qquad (1)$$

In (1), a bar in the integrand denotes an average value within an isentropic surface, per unit area of the horizontal projection of the surface. The integrand is defined for all values of $\theta$ from $0$ to $\infty$ by adopting the convention that $p = p_0$ when $\theta < \theta_0$, where $p_0$ and $\theta_0$ are the values of $p$ and $\theta$ at the earth's surface. Expression (1) is made valid even when superadiabatic lapse rates are present, by regarding as negative the area of the projection of the portion of an isentropic surface in the region where $\partial\theta/\partial z < 0$ .

The first term in the integrand in (1), together with the factors outside the integral, gives the average total potential energy. The second term gives the average unavailable potential energy. It is thus evident that $\bar{A} > 0$ unless $p \equiv \bar{p}$.

A good approximation to (1), expressing $\bar{A}$ in terms of the variance of pressure within isentropic surfaces, is

$$\bar{A} \sim \tfrac{1}{2} \kappa c_p g^{-1} p_{00}^{-\kappa} \int_0^{\infty} \bar{p}^{(-1+\kappa)} \overline{p'^2} \, d\theta . \qquad (2)$$

Expression (2) is obtained by letting $p = \bar{p} + p'$, and applying the binomial theorem. A fair approximation to (2), expressing $\bar{A}$ in terms of the variance of temperature within isobaric surfaces, is

$$\bar{A} \sim \frac{1}{2}\int_0^{p_0} \bar{T}^{-1}(\Gamma_d - \bar{\Gamma})^{-1}\,\overline{T'^2}\,dp \qquad (3)$$

where the bars in the integrand denote average values within isobaric rather than isentropic surfaces, and $T = \bar{T} + T'$. Expression (3) is obtained from (2) by introducing the approximation

$$p' \sim -\left(\frac{\partial\bar{\theta}}{\partial p}\right)^{-1}\theta' = \kappa\, p\, \bar{T}^{-1}\,\Gamma_d\,(\Gamma_d - \bar{\Gamma})^{-1}\,T' \qquad (4)$$

The approximation becomes rather poor if $\bar{\Gamma}$ is close to $\Gamma_d$, but this situation is not common in the actual atmosphere. More detailed derivations and discussions of (1), (2), (3) and (4) appear in [ 7 ].

The rate at which $\bar{A}$ varies is obtained by differentiating expression (1) and using the appropriate expression for $\partial p/\partial t$ in a system where $\theta$ is the vertical coordinate, namely

$$\frac{\partial p}{\partial t} = \int_\theta^\infty \nabla\cdot\frac{\partial p}{\partial\theta}\mathbb{V}\,d\theta_1 - \frac{\partial p}{\partial\theta}\frac{d\theta}{dt} \qquad (5)$$

where the dummy variable $\theta_1$ is the argument of the integrand, and the operator $\nabla$ denotes differentiation with $\theta$ held constant. The individual derivative $d\theta/dt$ depends upon nonadiabatic processes; specifically

$$\frac{d\theta}{dt} = c_p^{-1}\,p_{00}^{\kappa}\,p^{-\kappa}\,Q$$

where $Q$ is the rate of addition of heat to the atmosphere, per unit mass and per unit time. Upon eliminating $d\theta/dt$ and $\partial p/\partial t$

between (6), (5), and the time derivative of (1), we obtain

$$\frac{\partial \bar{A}}{\partial t} = -\bar{C} + \bar{G} \tag{7}$$

where

$$\bar{C} = -c_p g^{-1} p_{00}^{-\kappa} \int_0^\infty \left( \overline{p^\kappa \nabla \cdot \frac{\partial p}{\partial \theta} \mathbb{V}} - \bar{p}^\kappa \overline{\nabla \cdot \frac{\partial p}{\partial \theta} \mathbb{V}} \right) d\theta \tag{8}$$

and

$$\bar{G} = -g^{-1} \int_0^\infty \left( \overline{\frac{\partial p}{\partial \theta} Q} - \bar{p}^\kappa \overline{p^{-\kappa} \frac{\partial p}{\partial \theta} Q} \right) d\theta \tag{9}$$

The quantity $\bar{C}$ represents the rate of <u>conversion</u> of available potential energy into kinetic energy, by adiabatic processes. Since $\overline{\nabla \cdot (\partial p/\partial \theta) \mathbb{V}} = 0$ the second term in the integrand of (8) vanishes, so that $\bar{C}$ also represents the rate of conversion of total potential energy into kinetic energy. The quantity $\bar{G}$ represents the rate of <u>generation</u> of available potential energy, by nonadiabatic processes. The second term in the integrand in (9) does not in general vanish, so that the effect of heating upon available potential energy is considerably different from its effect upon total potential energy.

In the following discussion, we shall use the symbol $\mathcal{P}$ instead of $\bar{p}$ to denote the quantity whose value at any point equals the average pressure on the isentropic surface through the point. If we introduce the dimensionless quantity

$$N = 1 - p^{-\kappa} \mathcal{P}^\kappa \tag{10}$$

we can replace (9) by

$$\bar{G} = -g^{-1} \int_0^\infty NQ \frac{\partial p}{\partial \theta} d\theta \tag{11}$$

If we denote an element of mass of the atmosphere by $dM$, the rate of generation of available potential energy becomes

$$G = \int N Q \, dM \tag{12}$$

where the integration extends over the entire mass of the atmosphere. In contrast, the rate of generation of total potential energy is

$$G_0 = \int Q \, dM \, . \tag{13}$$

To understand the physical nature of the generation of available potential energy, as given by (12), we shall investigate some of the properties of $N$, which depend upon the properties of $\rho$. We first observe that by definition $\rho$ must be a monotone decreasing function of $\Theta$. Hence, along any isobaric surface, $\rho$ is also a monotone decreasing function of $T$. Also, from the definition of $\rho$, the statistical distribution of $\rho$ throughout the atmosphere, with respect to mass, is identical with the statistical distribution of $p$, so that the average value of any function of $\rho$ equals the average value of the same function of $p$. However, along the surface $p = p_0$, $\rho \leqq p_0$, so that near the earth's surface $\bar{\rho} < p$. Hence at some other level, apparently in the upper troposphere, $\bar{\rho} > p$. Here the bars denote averages on isobaric surfaces.

It then follows that, along any isobaric surface, $N$ is a monotone increasing function of $T$. It also follows that $\bar{N} > 0$ at low levels, while $\bar{N} < 0$ in the upper troposphere.

These properties of $N$ are apparent in fig. 2. Diagram (a) shows a hypothetical temperature field, constructed from observed average temperatures for the four months January, February, July, and August 1949, as tabulated at standard upper levels by Mintz [ 9 ]. Except within the

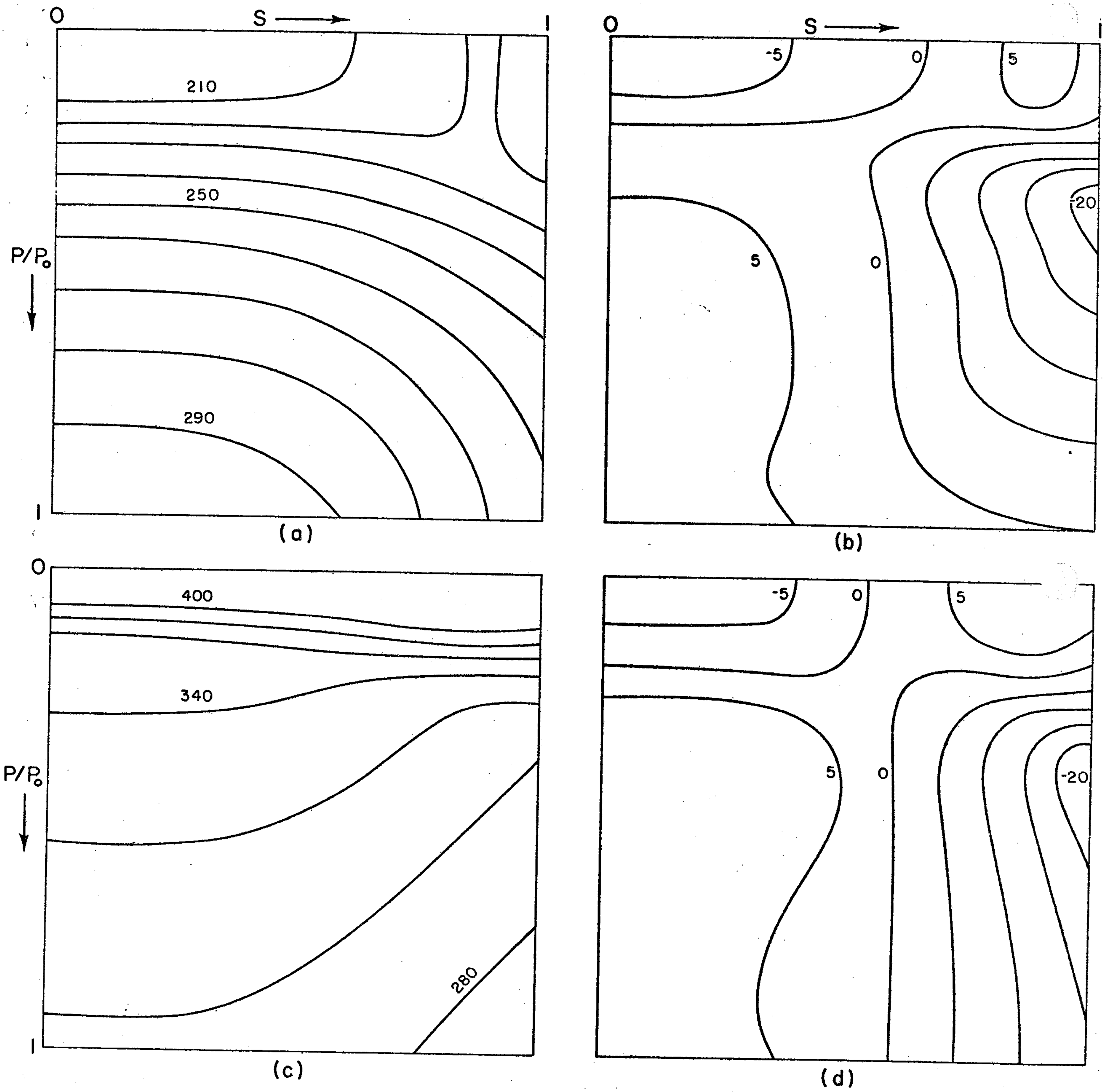


Fig. 2. (a): Hypothetical field of T in °K. (b), (c), (d): Corresponding fields of in °K, N in hundredths, and $\overline{\overline{T}}^{-1}\,\Gamma_d\,(\Gamma_d - \overline{\Gamma})^{-1}\,T'$ in hundredths.

Arctic stratosphere, these temperatures agree very closely with the average temperatures for the year 1950 tabulated by Starr and White [ 10 ]. The latter temperatures were used to extend the diagram to sea level, which was assumed to coincide with the surface $p = 100$ cb. The corresponding fields of $\Theta$ and $N$ are given in diagrams (b) and (c). Diagram (d) will be discussed later.

The use of average temperatures obliterates some typical features, notably the tropopause. The introduction of a tropopause into fig. 2 would intensify the negative values of $N$ at that level. No attempt has been made to incorporate what is known about the temperature field above 10 cb.

With these properties of $N$ in mind, we observe that it is convenient to resolve the total generation of available potential energy into two forms, represented by the two terms in the integrand of the expression.

$$G = \int \left( \overline{N'Q'} + \bar{N}\bar{Q} \right) dM \qquad (14)$$

Here the bars denote averages on isobaric surfaces, and the primes denote departures from these averages.

The first term represents a generation resulting from a positive correlation between $N$ and $Q$ within isobaric surfaces. In general this implies a positive correlation between temperature and heating. This term was considered in detail by the writer [ 7 ] in studying the maintenance of the general circulation. An approximate expression, involving the correlation between $T$ and $Q$, namely.

$$G \sim \int \bar{T}^{-1} \Gamma_d (\Gamma_d - \bar{\Gamma})^{-1} \overline{T'Q'} \, dM \qquad (15)$$

was derived by differentiating the approximation (3), but could also be obtained by substituting the approximation (4) into (9), and making suitable simplifications. The estimate of the net rate of generation of available potential energy referred to earlier was based upon (15).

Diagram (d) in fig. (2) shows the field of $\bar{T}^{-1}\Gamma_d(\Gamma_d-\bar{\Gamma})^{-1}T'$, which serves as an approximation for $N'$ in (15), as deduced from diagram (a). The approximation should lead to good estimates of $G$ , since it resembles $N'$ rather closely except near the ground.

The second term in the integrand of (14) represents a generation resulting from heating where $\bar{N}$ is positive or cooling where $\bar{N}$ is negative. In general this implies heating at low levels or cooling near the the tropopause. The presence of such a distribution of heating has often been mentioned.

It should be noted that there are other equally logical schemes for resolving the integrand in (12) into two or more terms. For the present study, the scheme used in (14) seems to be the most convenient.

It is now possible to observe the effects of different physical processes upon $A$. Considering friction first, we observe that there is no pronounced tendency for friction to be greater in warm air than in cold air, so that the generation given by the first term in (14) is probably negligible. There may be a tendency for frictional dissipation to be most intense near the ground, in which case the generation by the second term in (14) is positive. However, this effect is partly canceled by whatever dissipation may occur near the tropopause, and in any case the values of $N$ are so small that the total generation of available potential energy by friction is probably less than 1% of the total frictional heating, or less than $2 \times 10^{-4}$ times the solar radiation.

Most of the generation must therefore be accomplished by other physical processes. Among these, radiation appears to be dominant. The surplus of radiational heating in low latitudes and the deficit in high latitudes lead to a positive correlation between $Q$ and $N$, and a large generation of available potential energy.

3. <u>The maximum intensity of the energy cycle</u>.

The problem of explaining the rate of generation of available potential energy involves additional considerations, since it requires an explanation of the temperature field and the distribution of heating. The heating at a given point is fairly well determined by conditions along the vertical through the point in question, but the temperature depends also upon the large-scale exchange processes. The motions which accomplish the exchanges are none other than the motions which constitute the general circulation itself. Thus in attempting to solve our problem in a direct manner we apparently encounter another problem at least as difficult as the one which we wish to solve.

We shall therefore attack the problem indirectly, by first considering a somewhat less involved problem, suggested by the following considerations. If there were no horizontal temperature contrast there would be no generation of energy, in spite of the strong heating and cooling which would then prevail. At the other extreme, if the atmosphere were everywhere in radiative equilibrium, there would again be no generation of energy, in spite of the large horizontal temperature contrast which would prevail. If however, the horizontal temperature contrast is intermediate to these two cases, the contrast in heating is also intermediate, and temperature and heating are positively correlated, so that there is positive generation of energy. Thus, among the different temperature distributions in this last category, there may be one which leads to the maximum possible rate of generation of available potential energy. Our problem will be to determine this maximum rate, and to find the distribution of temperature which leads to it.

Our problem is still very complicated. For one thing, even the distribution of radiative heating alone is not determined solely by the incoming solar radiation and the temperature field, but depends also upon the location of cloud layers which reflect short-wave radiation, and the distribution of the constituents of the atmosphere which absorb long-wave radiation. These quantities are affected by the exchange processes. To eliminate the effect of the exchange processes, we should, ideally, find the combination of temperature field, cloudiness distrubution, and distribution of absorbing constituents which maximizes the rate of generation of available potential energy.

Instead, we shall simplify the problem along the following lines. We shall deal with a hypothetical atmosphere containing a single absorbing constituent, which occurs with a constant mixing ratio, and whose absorption coefficient is indepentent of wave length. All the solar radiation which is not reflected back to space will be absorbed by the ground. The ground will radiate as a black body. Transfer of heat between the ground and the atmosphere, and within the atmosphere, will be accomplished by long-wave radiation. Within the atmosphere there will also be adiabatic large-scale radiation. To make the atmosphere dynamically consistent, friction will also be included, but will generate no available potential energy.

We are therefore assuming that the radiational heating, and hence $\int N Q\, dM$, is completely determined by the temperature field and the incoming solar radiation. In finding the temperature field which maximizes this integral, we must limit consideration to those fields for which $\int Q\, dM$ is zero, since the net incoming and outgoing radiation must balance.

The differencesbetween this hypothetical atmosphere and the actual terrestrial atmosphere are evident. The principal absorbing constituent of the latter, namely water vapor, has neither a constant mixing ratio nor a constant absorption coefficient. The transfers of heat are accomplished also by eddy-conduction, which does not obey the laws of radiation, and in the form of latent heat. Nevertheless, the model atmosphere is a dynamically consistent atmosphere in which heat is supplied directly by the ground, which has received the heat directly from the sun, while the long-wave radiation always heats cooler regions at the expense of warmer regions, in agreement with the second law of thermodynamics. It is therefore to be hoped that the model will exhibit some of the features of the temperature field which must prevail if the energy cycle is to operate at its maximum rate, and that it will give a plausible estimate of this maximum rate.

Some of these features can be anticipated without computation. There must be a horizontal temperature contrast if there is to be any generation at all. The temperature must be higher at low latitudes than at high latitudes if there is to be gain of energy. Hence, at upper levels in the polar regions, $N < 0$. A greater loss of heat here is aided by a lower surface temperature. This leads to a smaller loss of heat at the surface, but this is of no consequence, since $N = 0$ there. It is therefore to be anticipated that the lapse rate will be relatively stable in the polar regions, whereas the same considerations do not point to a stable lapse rate in the tropics. As a result, the horizontal temperature contrast should decrease with height.

It is well worth noting that these features are found in the actual atmosphere.

## 4. The one-dimensional model

In the computations which follow, we shall first simplify the problem by regarding the atmosphere as a single layer. At the same time we shall ignore variations with longitude. The position of a point is then given by its latitude $\phi$, or alternatively by the coordinate $S = \sin\phi$ chosen so that equal increments of $S$ represent equal increments of area. The incoming solar radiation per unit area actually reaching the ground, the temperature of the atmosphere, and the temperature of the underlying earth will be given by $F(s)$, $T(s)$, and $T_0(s)$, respectively.

We assume that the long-wave radiation emitted by the ground is proportional to $T_0^4$ while that emitted both upward and downward by the atmosphere is proportional to $T^4$. The balance between radiational heating and cooling at the ground, and the lack of balance in the atmosphere, are then expressed by the relations

$$F + \sigma T^4 - \sigma T_0^4 = 0 \qquad (16)$$

$$\sigma T_0^4 - 2\sigma T^4 = mQ \qquad (17)$$

where $\sigma$ is the Stefan-Boltzmann constant, and $m$ is the mass of the atmosphere per unit area. Eliminating $T_0$, we have

$$m Q = F - \sigma T^4 \tag{18}$$

It is interesting to note that we should have reached this same equation if we had assumed the solar radiation to be absorbed by the atmosphere, and the atmosphere to radiate upward only, at a rate proportional to $T^4$.

With this model we may use the approximation (15) for $\bar{G}$. However we should note that (15) was obtained by approximating the departure of $p$ from $P$ in terms of the departure of $T$ from $\bar{T}$. It is just as logical, and in the present case much more practical, to approximate the departure of $p$ from $P$ in terms of the departure of $T^4$ from $\overline{T^4}$. Thus (15) may be replaced by

$$\bar{G} \sim \frac{1}{4} \int \overline{T^4}^{-1} \Gamma_d (\Gamma_d - \bar{\Gamma})^{-1} \overline{(T^4)' Q'} \, dM \tag{19}$$

Since the model does not describe vertical variations, a standard value may be chosen for $\Gamma_d / (\Gamma_d - \bar{\Gamma})$. The value $3$, corresponding to $\bar{\Gamma} = 2/3 \, \Gamma_d$, seems suitable. From (19) it follows that

$$\bar{G} = \frac{3}{4} \, \overline{T_4}^{-1} \left( \overline{F'(T^4)'} - \sigma \overline{(T^4)'^2} \right), \tag{20}$$

We now employ the calculus of variations to maximize $\bar{G}$, subject to the additional condition that $\bar{Q} = 0$. Denoting the variation of a quantity by $\delta$, we find that

$$\delta \bar{G} = \frac{3}{4} \, \overline{T_4}^{-1} \overline{\left( F' - 2\sigma (T^4)' \right) \delta (T_4)'} \tag{21}$$

while

$$m \delta \bar{Q} = - \sigma \, \overline{\delta (T_4)} = 0 \tag{22}$$

In order to maximize (20), the right side of (21) must vanish for all variations $\delta (T_4)$ which satisfy (22). This will occur only if

$$F - 2 \sigma T^4 = c \tag{23}$$

where $C$ is a constant. From (18), it follows that $C = -\overline{F}$, so that

$$\sigma T^4 = \tfrac{1}{2}\left(\overline{F} + F\right) . \tag{24}$$

Thus

$$\overline{G} = 3/16 \; \overline{F'^2} / \overline{F} \tag{25}$$

while the fraction of the solar energy converted into available potential energy is

$$\overline{G}/\overline{F} = 3/16 \; \overline{F'^2} / \overline{F}^2 . \tag{26}$$

If some of the solar energy is reflected to space, the maximum fraction of the solar energy reaching the extremity of the atmosphere which enters the energy cycle is further diminished. If $\alpha$ is the albedo, this fraction is

$$(1-\overline{\alpha})\,\overline{G}/\overline{F} = 3/16\,(1-\overline{\alpha})\,\overline{F'^2}/\overline{F}^2 . \tag{27}$$

If $F$ is taken as the distribution of solar energy at the time of the equinoxes, it is proportional to $\cos\phi$ so that

$$F = 4/\pi \; \overline{F}\sqrt{1-s^2}$$

whence, from (26)

$$\overline{G}/\overline{F} = 2/\pi^2 - 3/16 = 0.015 .$$

If $\overline{\alpha}$ is taken as

$$(1-\overline{\alpha})\,\overline{G}/\overline{F} = 0.010 .$$

The corresponding temperature distribution is given by

$$T^4 = \overline{T^4}\left(\frac{1}{2} + \frac{2}{\pi}\sqrt{1-s^2}\right) .$$

If we assume that $\overline{T^4} = (250^\circ K)^4$, the fourth power of the planetary temperature of the atmosphere (see [ 4 ], for example), we find that $T$ varies from $258^\circ K$ at the equator to $210^\circ K$ at the pole.

We now observe that the computed fraction 1% of solar energy entering the energy cycle is less than the estimated fraction for the actual atmosphere. It follows, if the estimates for the atmosphere are valid, that we have failed to take some factor into account. The discrepancy could arise from our neglect of the vertical dimension, or from our neglect of variations of the albedo. These possibilities will be considered later.

The temperature distribution shows a decided resemblance to the observed mean temperature distribution in the atmosphere, although the polar temperature seems to be too low. The results suggest that the energy cycle may be progressing at nearly its maximum possible rate. This conclusion, if correct, is so important that it is imperative that an attempt be made to take into account the vertical distribution of temperature and heating in computing the maximum intensity of the energy cycle.

## 5. The two-dimensional model

In this model we shall take the vertical extent of the atmosphere into account, but again ignore variations with longitude. We shall use pressure as the vertical coordinate, so that the incoming solar radiation per unit area, the air temperature, and the ground temperature may be given by $F(s)$, $T(s,p)$, and $T_0(s)$. We shall treat $p_0$ as a constant.

We assume that the long-wave radiation emitted by the ground is proportional to $T_0^4$ while that emitted by a layer of air whose thickness is one unit of pressure is proportional to $k T^4$, where $k$ is the absorption coefficient. Likewise the fraction of the long-wave radiation reaching such a layer which is absorbed by the layer is also $k$. The balance between radiational heating and cooling at the ground, and the lack of balance in the atmosphere, are then expressed by the relations

$$F + \sigma k \int_0^{p_0} h(p_0 - p) T^4(p,s)\, dp - \sigma T_0^4 = 0 \tag{28}$$

$$\sigma k h(p_0 - p) T_0^4 + \sigma k^2 \int_0^{p_0} h(p - p') T^4(p',s)\, dp' - \sigma k T^4(p,s) = Q \tag{29}$$

where $h(p-p')$ is the fraction of the radiation leaving the level $p'$ which is not absorbed before reaching the level $p$, assumed to depend only upon the difference $|p-p'|$. Eliminating $T_0$, we have

$$\begin{aligned} Q = {} & k h(p_0 - p) F - \sigma k T^4(p,s) \\ & + \sigma k^2 \int_0^{p_0} \left[ h(p-p') + h(p_0 - p) h(p_0 - p') \right] T^4(p',s)\, dp' . \end{aligned} \tag{30}$$

If we assume that long-wave radiation travels only directly upward or downward, we have

$$h(p-p') = e^{-k|p-p'|} \tag{31}$$

A more realistic assumption is that the radiation travels in all directions, in which case (see [4], for example)

$$h(p-p') = E_2(k|p-p'|) \tag{32}$$

where the exponential integral $E_2$ is defined as

$$E_2(x) = \int_1^{\infty} \eta^{-2} e^{-x\eta}\, d\eta . \tag{33}$$

In the two-dimensional model we cannot use the approximations (15) or (19), which proved so convenient in the one-dimensional model, to obtain the maximum value of $G$, because of the occurrence of the factor

$(\Gamma_d - \bar{\Gamma})^{-1}$. Simply by choosing at some level an average lapse rate as close to $\Gamma_d$ as we wish, we can make $G$, as given by (15) or (19), as large as we wish, obtaining a physically unrealistic result. This difficulty does not detract from the use of (15) or (19) to determine existing values of $G$, using existing average lapse rates.

We must therefore return to the more exact expression (12) for $G$, with $Q$ given by (30). If we employ the calculus of variations to maximize the integral in (12), subject to the condition that the total heating vanishes, we are led to a complicated integral equation to be solved for $T(p,s)$ (whose derivation will be omitted), namely

$$k\int_0^{p_0}\left[h(p-p') + h(p_0-p)\,h(p_0-p')\right]N(p',s)\,dp' - N(p,s)$$
$$+\frac{1}{4}\frac{\chi}{\sigma k}\frac{\theta\rho^{-1}}{T^4}\left(\overline{(1-N)Q\left(\frac{\partial\theta}{\partial p}\right)^{-1}} - \overline{(1-N)Q\left(\frac{\partial\theta}{\partial p}\right)^{-1}\frac{\partial\rho}{\partial p}}\right)$$
$$= c\left(k\int_0^{p_0}\left[h(p-p') + h(p_0-p)\,h(p_0-p')\right]dp' - 1\right), \qquad (34)$$

where $c$ is a constant, and the bar denotes an average on an isentropic surface. Equation (34) replaces the simple algebraic equation (23) occurring in the one-dimensional model. There is no evident easy method to solve (34) analytically. Thus it has appeared most feasible to maximize the integral (12) by numerical means.

The numerical analysis and computations necessarily to solve this problem have been performed by Mr. Hans Reichenbach. The computations have been carried out with the aid of SWAC, the electronic digital computer on the U.C.L.A. campus. This computer is owned by the Office of Naval Research, and has been made available through the courtesy of that organization.

The numerical method employed by Reichenbach makes no direct use of the calculus of variations, but consists of a systematic trial-and-error procedure. The temperature field $T(p,S)$ is represented by the values of $T$ at 25 grid points, which form the intersections of two sets of five lines, namely $p/p_0 = 0, 1/4, 1/2, 3/4, 1$ and $S = 1/10, 3/10, 5/10, 7/10, 9/10$. The values of $T$ at points other than these 25 are obtained by polynomial interpolation and extrapolation. The procedure consists of first estimating the values of $T(p,s)$ at the 25 grid points, and computing the corresponding value of $G$. The estimated values of $T$ are then modified at a specified set of grid points in such a way that $G$ is increased, while $\int Q\, dM$ remains zero. The values are then modified at another set of grid points in a similar fashion. This operation is continued until it is no longer possible to increase the value of $G$. The maximum value of $G$ and the corresponding temperature distribution are then determined.

If a more precise determination of $G$ and $T(p,S)$ is required, the number of grid points may be increased, and the procedure may be repeated. In this case the solution obtained using the smaller number of grid points serves as the initial estimate for $T$.

The preliminary results* obtained from the two-dimensional model are summarized in table 1. Two separate functions $F(S)$ have been used; in each case equation (30) was used for $Q$, with $h$ given by (31), and with $k p_0 = 2$.

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* Further results are not anticipated for several months, because SWAC is temporarily failing to operate perfectly.

Table 1. Preliminary results obtained from the two-dimensional model.

| | Case 1 | Case 2 |
|---|---|---|
| $F$ | $\left(1+\sqrt{1/3}-\sqrt{4/3}\,s\right)\overline{F}$ | $\left(3/2-3/2\,s^2\right)\overline{F}$ |
| Maximum $\overline{G}/\overline{F}$ | 0.0094 | 0.0375 |
| $3/16\ \overline{F'^2}/\overline{F}^2$ | 0.0208 | 0.0390 |

Temperature field (in $^\circ K$) leading to maximum $\overline{G}/\overline{F}$.

| $s$ | .1 | .3 | .5 | .7 | .9 | .1 | .3 | .5 | .7 | .9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $p/p_0 =$ .00 | 234 | 231 | 229 | 189 | 178 | 211 | 200 | 186 | 161 | 158 |
| .25 | 255 | 250 | 250 | 234 | 237 | 256 | 250 | 253 | 231 | 236 |
| .50 | 272 | 271 | 270 | 253 | 247 | 281 | 283 | 283 | 258 | 250 |
| .75 | 279 | 279 | 277 | 263 | 255 | 306 | 303 | 297 | 233 | 222 |
| 1.00 | 287 | 287 | 285 | 191 | 159 | 317 | 308 | 303 | 147 | 139 |

The computed ratio $\overline{G}/\overline{F}$ ranges from about 1% to 4% in the cases treated. The quantity $3/16\ \overline{F'^2}/\overline{F}^2$, which is the value of $\overline{G}/\overline{F}$ obtained from the one-dimensional model, is included for comparison; there is close agreement in the second case, but not in the first.

The horizontal temperature contrast and the stable lapse-rate in the polar regions near the ground, both anticipated in the earlier discussion, are evident. It was not anticipated that the ground layer in the polar regions would be so extremely cold. In retrospect, it is evident that such low temperatures lead to large values of $\overline{G}$ by intensifying the cooling in middle levels, while the resultant heating at the ground contributes very little to $\overline{G}$.

It is, however, unrealistic to have radiational heating where the lowest potential temperature occurs, since no adiabatic large-scale exchange process can carry away the heat which would accumulate there. Therefore, even in the problem of maximizing $\overline{G}$ , we cannot completely ignore the exchange process. To specify that the exchange process must not cool the potentially colder portion of the atmosphere while heating the potentially warmer, we should impose the condition

$$\int_{\theta < \theta_1} Q \, dM \leqq 0 \quad \text{for all } \theta_1 \, , \tag{35}$$

where the integral is taken over the portion of the atmosphere lying beneath any given isentropic surface. This condition will assure us that more reasonable temperatures will occur at low levels in the polar regions.

The absence of a definite tropopause, and the very low temperatures at high levels in the polar regions, are worth noting. Possibly these features would be common to any model which neglects the ozone layer.

The failure of $\overline{G}/\overline{F}$ to equal $3/16\ \overline{F'^2}/\overline{F}^2$ in case 1 is perhaps related to the relatively stable lapse rates appearing in that case. Apparently the estimated value $\Gamma_d/(\Gamma_d - \overline{\Gamma}) = 3$ was too large in case 1 but acceptable in case 2.

The effect of imposing the additional restriction (35) on the heating function can only be to lower the computed values of $\overline{G}/\overline{F}$ . The two-dimensional model therefore gives us little evidence for modifying our earlier conjecture that the energy cycle may be taking place at nearly its maximum possible intensity.

It is well at this point to compare our results with those of Wulf and Davis [ 11 ]. These authors use the principles of thermodynamics to compute an efficiency of the atmosphere, which they define as the ratio of the work done by friction to its maximum possible value in view of the heating and temperature. This maximum value appears to the present author to be more nearly equivalent to the existing rate of generation of available potential energy than to the maximum possible rate, for it is based upon the observed distribution of temperature and the resulting distribution of heating. There is no claim that this rate is the maximum which might have resulted had an entirely different temperature distribution prevailed.

This feature of the study of Wulf and Davis does not detract from its real value. Indeed, it is noteworthy that the efficiency of 60% which the authors obtain from thermodynamic considerations agrees so well with results which the present writer [7] subsequently obtained by considering available potential energy. That the efficiency given in [11] fails to equal 100% may result partly from an omitted feature - a possible negative correlation between temperature and heating within latitude circles.

## 6. The effect of the albedo

The average albedo of the earth determines the total amount of solar energy which the earth effectively receives, and thus affects the intensity of the energy cycle. Equally important, however, may be the latitudinal distribution of the albedo.

In the one-dimensional model, the maximum intensity of the energy cycle was found to be proportional to $\overline{F'^2}/\overline{F}$, the ratio of the variance of $F$ to the mean of $F$. Although the preliminary numerical results obtained from the two-dimensional model are not in complete agreement, we shall consider the effect of variations of the albedo upon $\overline{F'^2}/\overline{F}$.

In fig. 3, the solid curve represents the function

$$F = (1-\bar{\alpha})\sqrt{1-s^2}$$

discussed previously. The dashed curve represents the function

$$F = (1-\alpha)\sqrt{1-s^2}.$$

The latitudinal distribution of the albedo $\alpha$ has been taken from a table of average values of the planetary albedo determined by Houghton [ 3 ]. For the dashed curve, the value of $\overline{F'^2}/\bar{F}$ is 50% greater than for the solid curve, because $F$ has been increased where it is already large, and decreased where it is already small.

Since the values of $\alpha$ may v ry with time, the distribution of shows irregular fluctuations, in addition to the normal seasonal variations. These fluctuations may lead to different values of $\overline{F'^2}/\bar{F}$. In general, if $\delta F$ represents a small variation of the function $F$,

$$\delta\left(\overline{F'^2}/\bar{F}\right) = \overline{\left(2\bar{F}F' - F'^2\right)\delta F}/\bar{F}^2. \qquad (36)$$

Therefore $\overline{F'^2}/\bar{F}$ will be increased by increasing $F$ in the region where

$$F' > \tfrac{1}{2}\,\overline{F'^2}/\bar{F} \qquad (37)$$

or decreasing $F$ in the remaining region. For the dashed curve in fig. 4, $\overline{F'^2}/\bar{F}^2 = 0.12$, so that an increase of $F$ where $F'/\bar{F} > .06$, i.e., south of $36°N$, or a decrease of $F$ north of $36°N$, will increase the maximum intensity of the energy cycle.

If any conclusion at all can be drawn from the two cases so far completed in the two-dimensional model, it is that the one-dimensional model underestimates the effect of variations of the albedo.

Variations of the albedo therefore offer a possible mechanism for

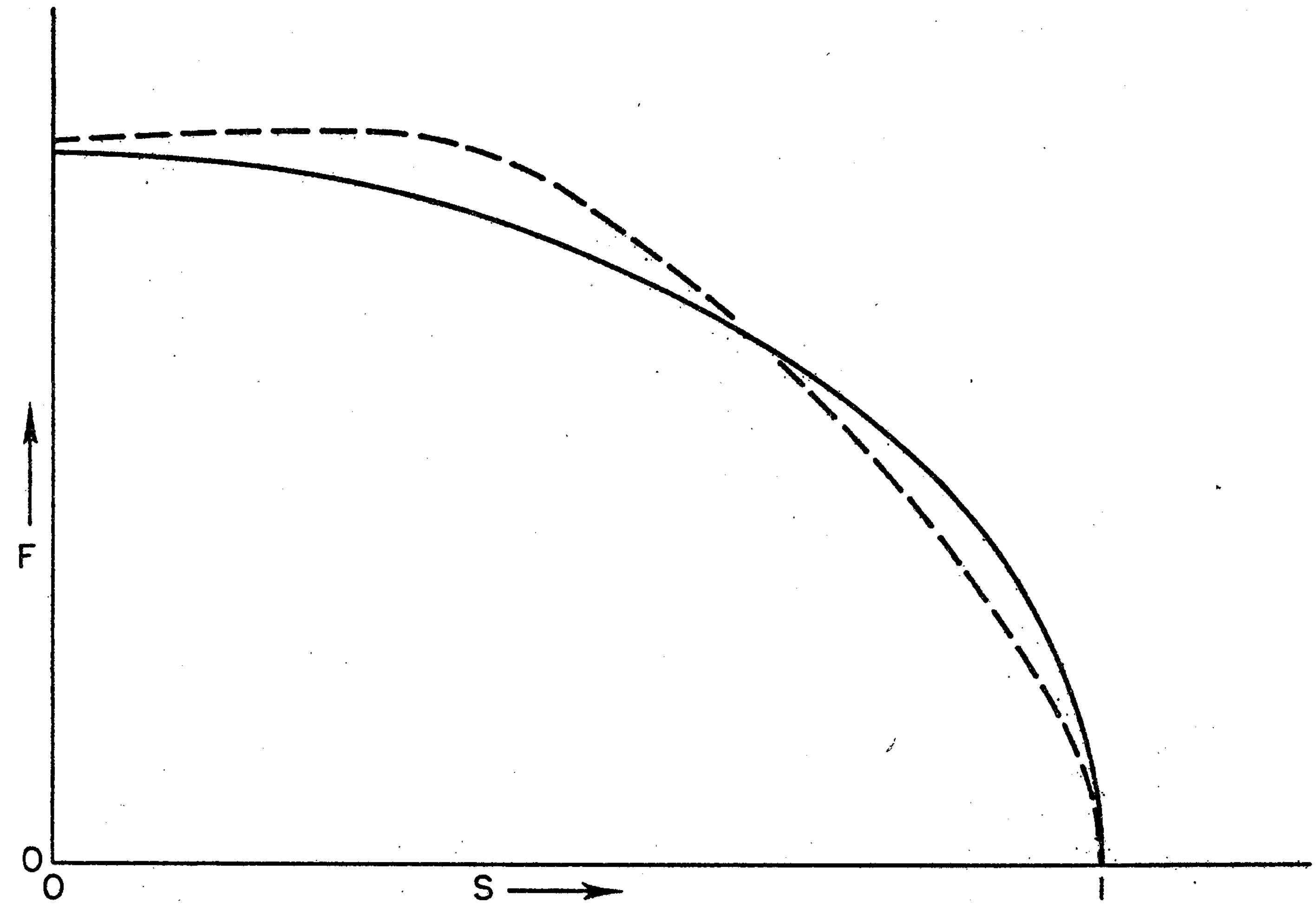


Fig. 3 Curves of $F = (1-\bar{\alpha})\sqrt{1-s^2}$ (solid) and $F = (1-\alpha)\sqrt{1-s^2}$ (dashed).

irregular fluctuations in the intensity of the general circulation. For suppose that the energy cycle is taking place at nearly its maximum possible intensity. Suppose then that for some reason, perhaps because of an equatorward shift of the belt of maximum cloudiness, the albedo increases in low latitudes and decreases in middle latitudes. The maximum intensity of the energy cycle then falls, and available potential energy can no longer be generated at the rate at which kinetic energy is being dissipated. Eventually the amount of kinetic energy must decrease. Next suppose that the albedo returns to its original distribution. The maximum intensity of the energy cycle then increases. Eventually the existing rate of generation of available potential energy may also increase, after which the amount of kinetic energy will also increase.

## 7. Maintenance of the energy cycle at nearly maximum intensity

We have considered some of the properties of the maximum possible intensity of the energy cycle, and have concluded that the prevailing intensity may be fairly near the maximum. It is now logical to ask whether there is any reason why the energy cycle must proceed at nearly its maximum intensity. If we can answer this question in the affirmative, we shall have confined the theoretical intensity of the general circulation to within rather narrow limits, instead of merely placing an upper bound upon it.

Such a reason, if it exists, must be rather involved, since it depends upon the large-scale exchange processes. There is, however, one special case where a plausible explanation is possible. This case specifically concerns not the actual atmosphere, but certain experimental models which have been constructed and examined by Fultz [ 2 ] at the University of Chicago.

In the experiments in question, sometimes called the "dishpan" experiments, a cylindrical vessel containing water is rotated at a uniform rate about its vertical axis, and is subjected to heating at the rim ("equator") and cooling at the center ("north pole"). With a sufficiently low rotation and a sufficiently great heating contrast, the observed flow consists of a symmetric circular vortex, with a weak meridional circulation superposed. With a more rapid rotation or a smaller heating contrast, the observed flow loses its symmetry, and disturbances resembling those found on upper-level weather maps are superposed upon the zonal flow.

Like the atmosphere, the dishpan possesses an energy cycle. As with the atmosphere, the cycle may be described in terms of total potential energy and kinetic energy, or available potential energy and kinetic energy. Available potential energy is defined as in the atmosphere, although the analytic expression is different, since adiabatic processes tend to conserve temperature instead of potential temperature.

Let us consider the intensity of the energy cycle in the dishpan, when it is in a symmetric steady state. If, at one extreme, there were no horizontal temperature contrast, or, at the other extreme, the contrast were great enough for thermal equilibrimm, there would be no generation of available potential energy, and the steady state could not be maintained. The temperature contrast must therefore lie between these extremes. The rate $G$ of generation of available potential energy as a function of horizontal temperature contrast $\Delta T$ is shown schematically by the solid curve in each diagram in fig. 4.

If the contrast $\Delta T$ is less than the value $E$ corresponding to thermal equilibrium, there must be a large-scale exchange process. In the symmetric case the exchange process consists of a poleward transport of warm water at high levels and an equatorward transport of cool water at low levels by a direct meridional circulation. The intentity of the heat exchange depends upon the intensity of the meridional circulation, and the vertical temperature contrast. But the vertical temperature contrast itself is created or intensified by the action of the meridional circulation. Thus a certain minimum meridional cell is necessary to maintain any value of $\Delta T$ less than $E$, and this minimum cell is more intense when $\Delta T$ is farther removed from $E$.

The meridional circulation itself contains relatively little kinetic energy, but the action of the Coriolis force upon the meridional circulation generates zonal flow. The flow is counteracted only by friction, and friction is not very effective unless the zonal flow itself is appreciable. A given meridional circulation therefore implies a certain minimum dissipation of kinetic energy by friction, which is larger when the circulation is stronger. It follows that a value of $\Delta T$ less than $E$ implies a certain minimum rate $D$ of dissipation of kinetic energy by friction which is greater when $\Delta T$ is farther from $E$. The value of $D$ as a function of $\Delta T$ is shown schematically by the dashed curve in each diagram in fig. 4.

The existing $\Delta T$ and the corresponding $G$ must obviously be represented by a point to the right of the intersection of the solid and dashed lines in fig. 4. In the first diagram, this intersection lies to the left of the highest point on the curve of $G$, so that the energy cycle may proceed at its maximum possible rate.

If the rate of rotation is increased, the same $\Delta T$ requires the same minimum meridional circulation, but this circulation requires a more

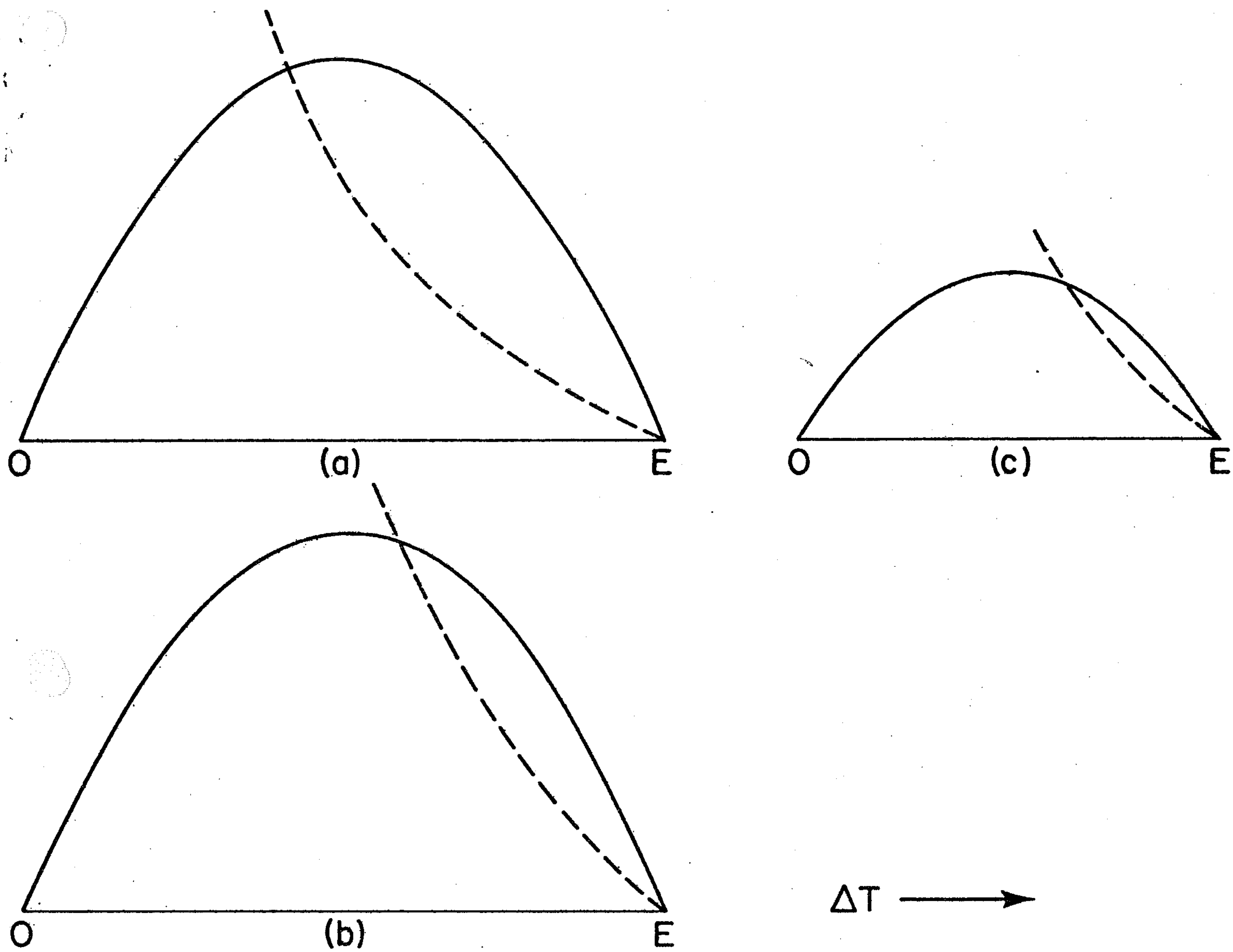


Fig. 4. Schematic curves showing generation of available potential energy (solid) and minimum dissipation of kinetic energy (dashed), corresponding to horizontal temperature contrast $\Delta T$. (a) Moderate rotation and moderate heating. (b) High rotation and moderate heating. (c) Moderate rotation and weak heating. E = value of $\Delta T$ for thermal equilibrium.

intense zonal circulation, and hence a greater $D$ . The dashed cufve in fig. 4 is therefore raised. If it is raised sufficiently, it will intersect the solid line to the right of its highest point, and the energy cycle cannot proceed at its maximum intensity. This situation is shown in the second diagram in fig. 4.

If on the other hand the heating contrast, represented by the value of $E$ , is lowered, the solid curve is lowered considerably, since the maximum value of $G$ depends upon the variance of heating, which is proportional to $E^2$. The dashed curve however is not lowered (relative to $E$ as an origin), and may be slightly raised, since with a smaller $\Delta T$ a stronger meridional circulation is required to bring about the same vertical temperature contrast. Thus the dashed curve may again intersect the solid curve to the right of its highest point. This situation is shown in the third diagram in fig. 4.

It follows that if the rotation is too rapid or the heating contrast too weak, the energy cycle in the dishpan cannot operate at its maximum intensity, if the large-scale heat exchange is accomplished by a meridional circulation. Recalling the experiment, we observe that these conditions - rapid rotation or weak heating contrast - are precisely the conditions under which symmetric flow is not observed, i.e., under which the heat exchange is not accomplished entirely by a meridional circulation. We are thus led to the hypothesis that the breakdown of symmetric flow when critical conditions are exceeded represents an attempt of the general circulation to operate at its maximum intensity, by introducing another type of exchange process when the symmetric type of exchange is insufficient.

That the general circulation should attempt to operate at nearly maximum intensity in this special case seems plausible for the following reasons: Suppose that critical conditions have been exceeded, and that the flow is still symmetric, so that the energy cycle is necessarily at less than its maximum intensity, as in the second or third diagram of fig. 4. Suppose that the flow is also subjected to small perturbations of various forms, and thus, eventually, to a perturbation in the form of horizontal eddies in which the water moving poleward is warmer than that moving equatorward. This perturbation will possess very little kinetic energy, but it will transport heat poleward, thereby decreasing the horizontal temperature contrast and so increasing the rate of generation of available potential energy. Some of this additional energy will be converted into kinetic energy. If the new kinetic energy appears in the form of similar horizontal eddies, the horizontal temperature gradient will be further decreased, and still more energy will be realized, until the circulation reaches a new regime, in which the eddies transport a significant portion of the heat, and contain a significant portion of the kinetic energy. The eddies themselves may then possess more kinetic energy than the original meridional circulation, but they need not be accompanied by a strong zonal circulation, which was the cause of most of the dissipation of kinetic energy in the symmetric case.

The phenomenon which we have pictured is of course nothing more than the growth of small perturbations superposed upon an unstable baroclinic zonal current. The instability of baroclinic flow has played an important part in many recent studies of the general circulation. The breakdown of zonal flow in the dishpan has previously been discussed as an instability phenomenon by the author [ 6 ], while the impossibility of zonal flow in the dishpan when the generation of energy fails to equal the dissipation has been discussed by Kuo [ 5 ].

It is well to observe that in the unsymmetric case the intensity of the energy cycle is only <u>nearly</u> a maximum. The exchange process in this case requires temperature differences within latitude circles, which lead to heating differences, such that $T$ and $Q$ are negatively correlated within latitude circles. This negative correlation contributes negatively to the generation of available potential energy, and thus brings $G$ below its maximum value.

Returning to the general problem of the intensity of the energy cycle, we see that we have considered one case in which the energy cycle lacks its maximum intensity because the large-scale exchange process is not strong enough to bring about the optimum temperature field. In this case, a mechanism seems to be present for bringing about an alternative exchange process. Possibly this type of reasoning could be extended to more general cases in which the exchange process is too weak. In the case where the exchange process is too strong to maintain the optimum temperature field, some other line of reasoning may be necessary.

## 8. Conclusion

We have examined the problem of explaining the intensity of the general circulation, or, more precisely, of deducing the observed value of the kinetic energy contained in the atmosphere from a knowledge of the solar radiation reaching the atmosphere, using the physical and dynamic laws governing the atmosphere. We could solve this problem, at least in a rough fashion, by explaining the intensity of the energy cycle which characterizes the general circulation. For this purpose, it is convenient to express the intensity of the energy cycle as the rate of generation of available potential energy by nonadiabatic processes.

To explain the intensity of the energy cycle directly, we should have to explain the distributions of both temperature and heating. The distribution of temperature, unlike that of heating, is directly affected by the large-scale motions in the atmosphere. We have temporarily removed the necessity of considering the large-scale motions in detail by concentrating upon a less involved problem - that of determining the maximum possible rate of generation of available potential energy, and the temperature field which leads to this rate.

To make the latter problem tractable, we have assumed an atmosphere in which temperature changes result only from absorption and emission of long-wave radiation, advection by adiabatic large-scale motions, and friction. The absorption and emission are further assumed to be due to a single atmospheric constituent, which occurs with a constant mixing ratio, and has an absorption coefficient independent of wave length. Direct solar radiation is assumed bo be absorbed by the ground, which emits long-wave radiation.

The one-dimensional model, in which we further neglect variations with height and longitude, yields to analytic solution - it suggests that the observed intensity of the energy cycle may be close to the maximum possible intensity. The two-dimensional model, in which vertical variations are also considered, is best treated by numerical methods - the preliminary results appear to support the hypothesis suggested by the one-dimensional model. No three-dimensional model has been considered, but it seems probable, from considerations of symmetry, that if the solar heating is assumed to be independent of longitude, the temperature distribution leading to the maximum rate of generation of available potential energy will be independent of longitude also.

In the process of treating this problem, we have also encountered a mechanism whereby fluctuations in the latitudinal distribution of the albedo may lead to fluctuations in the intensity of the general circulation.

To complete the original problem, we must explain why the energy cycle must take place at nearly its maximum possible intensity, if indeed it does so. In one particular case, we have described a mechanism whereby, if the intensity of the energy cycle fails to equal its maximum value because the large-scale exchange process is too weak to bring about a suitable temperature distribution, an alternative exchange process will be established. A more general explanation awaits further study.

In the process of describing this mechanism, we have presented an explanation for the existence of two regimes of flow in the "dishpan", and for the dependence of the regime upon the rate of rotation and the heating contrast.

What steps must we now take before our original problem is completely solved? First of all, in determining the maximum intensity of the energy cycle, we must use a more realistic description of the atmosphere. The numerical procedure used in treating the two-dimensionsl model is designed to be readily modified for different laws of absorption as well as different distributions of solar heating. Possibly some absorption law resembling that for water vapor, or for a combination of water vapor, carbon dioxide, and ozone, could be introduced. It does not seem legitimate, however, to use the observed distribution of the mixing ratio, for this distribution is itself affected by the large-scale motions. Instead, we might find the maximum intensity of the energy cycle corresponding to each of a great many distributions of water vapor, and compare each of these maximum intensities with the existing intensity. In using this procedure, we should eliminate distributions of water vapor and temperature which imply supersaturation.

In explaining why the energy cycle must proceed at nearly its maximum intensity, we must extend our discussion to the general case where a possible exchange process is too weak to maintain an optimum temperature contrast. We must also eliminate the possibility that the exchange process may be too strong to allow the optimum temperature contrast to persist.

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