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STUDIES OF ATMOSPHERIC PREDICTABILITY

by

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STUDIES OF ATMOSPHERIC PREDICTABILITY

ABSTRACT

The range at which good forecasts of the weather are possible is limited by the rate at which separate solutions of the governing dynamic equations diverge from one another. Studies aimed at determining this rate have thus far employed a dynamical approach, an empirical approach, or a dynamical-empirical approach. A comparison of these three approach points to a value of about three days as the best estimate of the average doubling time for small differences between solutions.

In separate sections of the report each approach is presented in detail.

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FOREWORD

The behavior of the atmosphere is governed by a set of physical laws. These laws may be formulated as a system of differential equations. The problem of weather prediction may be identified with the problem of discovering, by one means or another, the particular solution of these equations whose initial conditions represent the current state of the atmosphere.

In the strictest sense the laws are not deterministic; in any event they are not precisely known. Furthermore, the current state of the atmosphere cannot be measured without error. These considerations imply that perfect weather prediction is not possible. It is apparent, however, that they do not necessarily preclude the possibility of good weather forecasts at either short or long range, since similar considerations apply to the prediction of such other natural phenomena as oceanic tides and solar eclipses, which can be predicted far in advance with considerable accuracy.

The distinguishing feature of the atmosphere is its <u>instability</u>; i.e., two time-dependent solutions of the governing equations originating from slightly different initial conditions will diverge from one another and eventually become unrecognizably different. This consideration, together with the impossibility of formulating the laws and measuring the current state exactly, place a limit upon the range at which good forecasts can be made.

The "Statistical Forecasting Project" at M.I.T. has been sponsored by the Air Force Cambridge Research Laboratories continuously since 1955 under a succession of contracts. During its early years the Project was concerned primarily with methods of prediction. As the work progressed, interest was turned toward the problem of the extent to which prediction is possible. Under the present contract we have been particularly concerned with the rate at which the time-dependent solutions of the dynamic equations diverge from one another; this rate determines the time required for typical errors of observation to amplify to the point where they become intolerably large.

To date three basic approaches to the problem have been proposed. The first is dynamical, and consists of comparing separate solutions of the dynamic equations obtained by numerical means. The second is empirical, and consists of identifying states of the atmosphere which resemble one another, and comparing the atmospheric behavior subsequent to the occurrence of these states. The third is partly dynamical and partly empirical; dynamic equations governing differences between states of the atmosphere are derived, but the numerical values of the coefficients in these equations are based upon observations. We have pursued the second and third approaches.

Article I of this report, which is also to appear in the Bulletin of the American Meteorological Society in a slightly different form, compares the three approaches, and presents our principal conclusion, namely, that the average doubling time for small differences between separate solutions of the equations is about three days.

Each of the remaining articles deals with one particular approach in greater detail.

Article II does not represent work performed under this contract; it is included so that the report may contain a detailed presentation of each approach to the problem. It is an excerpt from a report entitled "The feasibility of a global observation and analysis experiment," submitted by the Panel on International Meteorological Cooperation to the Committee on Atmospheric Sciences of the National Academy of Sciences.

Articles III and IV, which are expected to appear in the Journal of the Atmospheric Sciences and Tellus respectively, represent original work performed under this contract. To our knowledge they are the only completed studies in which these approaches have been taken. Needless to say, neither study represents the final word. In particular, the last article is directed more toward the general problem of fluid predictability than the specific problem of atmospheric predictability.

We regret that this Final Report for Contract AF 19(628)-5826 is also the Final Final Report for the Statistical Forecasting Project. We have approached our objective of determining the doubling time for small differences between solutions of the equations, but much work remains to be done before our values can be accepted with confidence.

I. THREE APPROACHES TO ATMOSPHERIC PREDICTABILITY

ABSTRACT

Since errors in observing the state of the atmosphere are inevitable, the accuracy of extrapolation into the future is limited by the rate at which separate solutions of the governing equations diverge from one another. Three basically different methods for investigating the growth rate of errors have been exploited.

The dynamical method compares numerical solutions of special systems of equations. Small errors appear to double in less than a week, the growth rate decreasing when the errors become large. The empirical method examines naturally occurring analogues. Moderately large errors amplify by nearly ten per cent in one day, but extrapolation of the results suggests that small errors would double in less than three days. The dynamical-empirical method uses derived equations for the errors, with observed spectral properties of the atmosphere appearing as coefficients. Only the last method treats smaller-scale features explicityly. Small-scale errors appear to grow very rapidly, meanwhile inducing errors in the larger scales, which then double in two or three days. The dynamical method probably overestimates the doubling time because of special numerical approximations used to suppress computational instability.

An absolute maximum range of a few weeks for predicting a particular day's weather is indicated. The outlook for major improvements in short-range forecasting is favorable.

1. Introduction

The belief that man can make useful even if not perfect predictions of the weather must have first been inspired by the observation that there is some regularity in the sequence of weather events; for example, in some regions dark clouds often foreshadow a heavy shower. Today we are more inclined to base our belief in predictability upon the premise that the atmosphere is governed by a set of physical laws, which may be formulated in a manner expressing future states of the atmosphere and its environment in terms of the present.

It the laws can be so formulated, one may justifiably ask whether perfect prediction may some day be realized. Three basic reasons indicating that this is not the case may be cited. First, the system of governing laws is not strictly deterministic. Next, even if the laws were deterministic, perfect prediction would be impossible in practice because the laws are not perfectly known. Finally, even if the laws were perfectly known, perfect prediction would not be attainable because the current state of the atmosphere and its environment cannot be perfectly measured.

Although it would be an interesting task to determine what limitations are placed upon predictability by Heisenberg's Principle of Uncertainty, the problem is purely academic because of greater limitations due to other obvious uncertainties. Among these are the effects of biological activity, and, in particular, human activity, which for our purposes must be considered nondeterministic. Local cumulus convection, for example, may be affected by fires, while if one wishes to make exact predictions at very long range he must anticipate the creation of large lakes by the construction of dams.

For the present, however, these intrinsic uncertainties in the governing laws may be overlooked, because they are of minor importance compared to the uncertainties arising from our incomplete knowledge of the laws. We do not completely understand, for example, what determines when a cloud consisting entirely of minute water droplets will become converted into a cloud containing many larger drops, which will then fall out as rain. On a world-wide basis, such lack of knowledge places far greater limitations upon predictability than any uncertainty as to the locations of fires or lakes or other man-made features.

Perhaps one can visualize the day when all of the relevant physical principles will be perfectly known. It may then still not be possible to express these principles as mathematical equations which can be solved by digital computers. We may believe, for example, that the motion of the unsaturated portion of the atmosphere is governed by the Navier-Stokes equations, but to use these equations properly we should have to describe each individual turbulent eddy — a task far beyond the capacity of the largest computer. We must therefore express the pertinent statistical properties of turbulent eddies as functions of the larger-scale motions. We do not yet know how to do this, nor have we proven that the desired functions even exist.

Supposing, however, that we some day master the problem of formulating exact equations, we still cannot make perfect forecasts from imperfectly observed initial conditions. We cannot even make good forecasts at extended range, unless our equations possess the property that separate solutions, differing only slightly at some initial time, will continue to differ only slightly as time progresses. Empirical evidence indicates that this is not the case.

If we suppose instead that we can some day learn to observe the atmosphere without error, but if we acknowledge that our equations must forever contain some imperfections, we find that shortly after the initial moment the state of the atmosphere is imperfectly known, just as surely as the <u>initial</u> state is imperfectly known if our observation system is imperfect. Again, the divergence of separate solutions of the equations assures us that we cannot make good forecasts at sufficiently long range.

Knowing that we cannot predict into the indefinite future, we face the question, "How accurately can we some day predict the weather at any specified range?" The answer to this question depends upon <u>how rapidly</u> separate solutions of the atmospheric equations diverge from one another.

Let us refer to the difference between two states of the atmosphere, or between two solutions of the governing equations, as an <u>error</u>. The case of most obvious interest occurs when, at some initial time, one state is the true state of the atmosphere, and the other is the state of the atmosphere as it has been observed. There is no necessity, however, to restrict our attention to errors resembling those errors of observation which would be likely to be made in practice.

2. The dynamical approach

How, now, are we to determine the typical growth rate of small errors? The best known line of approach is a dynamical one, and it is based upon special systems of differential equations designed to resemble those which govern the atmosphere. In short, two or more solutions of the equations, originating from slightly different initial conditions, are obtained by numerical integration, whereupon the rate of amplification of the differences between the solutions is readily evaluated.

During the early stages of planning for the Global Atmospheric Research Program, it was recognized that the tendency for small errors to amplify might place a limit upon the range of practical predictability, and that too rapid a growth rate could conceivably render some of the objectives of the program unattainable. It thus became essential to establish a reasonable estimate of the growth rate. At that time there were in existence three rather extensive working mathematical models of the general atmospheric circulation, namely those developed by Smagorinsky (1963), Mintz (1964) and Leith (1965). Each model possessed its own distinctive features, but the models were alike in representing the state of the atmosphere by several thousand numbers.

Following a special conference, each of these investigators decided to use his model to study the growth rate of small errors. The results obtained from the separate models did not agree. Mintz found that after an initial period of adjustment, small errors tended to double, in the root-mean-square sense, in about five days. Smagorinsky deduced a considerably slower growth rate, while Leith obtained no systematic growth at all. It appeared, however, that Leith's atmosphere was varying nearly periodically, so that little growth was to be expected. In Smagorinsky's and Mintz's models, the growth rate subsided as the errors became larger.

In their report concerning the feasibility of a global observational system (see Article II of this report), Charney et al. (1966) conclude that a reasonable estimate of the doubling time for small errors is five days. If this conclusion is accepted, it is not unreasonable to entertain the possibility that good day-to-day forecasts up to two weeks in advance may eventually be produced. Such an achievement would of course demand a better observational system than the one currently existing.

Subsequent numerical experiments performed with more and more elaborate numerical models seem to confirm a doubling time of somewhat less than a week. However, even the most recent models share certain shortcomings with the earlier ones. Specifically, the equations of a model can never be the exact equations of the atmosphere. If thus becomes important to seek other means of estimating the growth rate.

3. The empirical approach

Such means are afforded by a second line of approach, which is empirical, and is based upon the natural occurrence of analogues, i.e., similar weather situations. We certainly cannot repeat the procedure of the numerical experiments, using the real atmosphere, for even though we might succeed in introducing a disturbance, and study the behavior of the disturbed state, we should then not know how the undisturbed state would have behaved. However, in principle, if we wait long enough, we may expect to encounter a state which rather closely resembles some state which has previously occurred. Either state is then equivalent to the other state, plus a small error, and the growth of the error may be studied by observing the behavior of the atmosphere subsequent to the two states.

In practice this procedure may be expected to fail, because of the high probability that no truly good analogues will be found within the recorded history of the atmosphere. Accordingly, we note that moderately large errors may in general be expected to amplify at a slower proportional rate than small errors (cf. Lorenz, 1968). By studying mediocre analogues, i.e., states bearing only a moderate resemblance to one another, we may hope at least to obtain a maximum estimate for the doubling time for small errors.

We have now completed a study of this sort (see Article III). Our basic data have been the heights of the 200-, 500-, and 850-millibar surfaces at a grid of 1000 points over the northern hemisphere, for the years 1963-1967. We have compared each state of the atmosphere with each other state occurring within one month of the same time of year, but in a different year, thereby comparing altogether about 400,000 pairs of states. As a measure of the difference between two states, or the error, we have taken the ratio of a weighted root-mean-square height difference to an estimate of the normal value of this weighted difference for the time of the year.

There are indeed no truly good analogues. In fact, the smallest error encountered is more than half as great as the average error. The smaller errors do indeed grow more rapidly, with larger-than-average errors tending to decrease rather than increase. The smallest errors amplify by nearly ten per cent in one day; thus it may be inferred that truly small errors would double in not more than eight days — a result which, incidentally is in agreement with the numerical experiments.

Presumably, however, the doubling time of small errors is considerably less than that of the smallest encountered in the study. If we introduce the postulate that the principal nonlinear processes are represented by quadratic terms in the dynamic equations, we can extrapolate the results of the study to obtain a doubling time for truly small errors. This turns out to be between two and three days.

4. The dynamical-empirical approach

In both the dynamical and the empirical procedures the state of the atmosphere is represented or described by numerical values of the weather elements at points separated by several hundred kilometers. The errors which are indicated as doubling in several days are therefore errors in representing the larger-scale features of the atmosphere. It seems likely that errors in smaller-scale features will double much more quickly. An error in estimating the intensity of a thunderstorm, for example, should amplify at least as rapidly as the thunderstorm itself, doubling in perhaps twenty minutes. At the same time, this error may be instrumental in producing errors in the larger scales. A third line of approach explicitly takes this possibility into account.

The new approach is partly dynamical and partly empirical. A system of equations whose dependent variables describe the spectral distribution of the errors is first derived from the original atmospheric equations. Numerical values of the coefficients appearing in the new equations are based upon the observed spectral distribution of atmospheric energy.

In the only study of this sort so far completed (see Article IV), we have used as dependent variables the contributions of twenty different scales of motion to the mean-square error. Each scale covers an octave of the spectrum, so that wave lengths from 40,000 kilometers down to 40 meters are included. In place of the actual atmospheric equations, we have used the equations for two-dimensional incompressible flow. The coefficients are based upon an estimated spectrum of atmospheric kinetic energy.

When the initial error is confined to the smallest scale of motion, it is found to grow very rapidly, at the same time inducing errors in slightly larger scales. These in turn grow slightly less rapidly, and induce errors in still larger scales. In the course of half an hour, errors in the cumulus-sized scales have become appreciable, while after two days the errors have invaded the synoptic scales. Large errors in all scales are present after two weeks.

If small-amplitude initial errors are instead contained in the medium or larger scales, they quickly induce errors in the smallest scales, which then proceed to behave as if they had been present from the beginning. Thus, regardless of the initial spectral distribution of the errors, the errors in the most rapidly amplifying scales, i.e., the smallest, will soon dominate the field, and only somewhat later will they succeed in inducing further errors in the larger scales. It follows that if the initial-error amplitude is small enough, a further reduction in the amplitude by a factor of two will increase the range of predictability of all scales, and hence of the atmosphere as a whole, only by the doubling time for the smallest scale present, perhaps a minute or two.

Indeed, we may extrapolate our results to the case where still smaller scales are admitted. We then conclude that the atmosphere possesses an intrinsic range of predictability, of perhaps three weeks. Presently we are far short of our goal of making the best possible forecasts, and our observation system requires major improvements. However, if the hoped-for improvements are some day realized, no further improvements will ever appreciably increase the range of predictability.

We must be quick to note that our conclusion is based upon a number of assumptions which cannot be rigorously defended. We are a long way from incorporating the true atmospheric equations into our procedure. Nevertheless, we believe that the evidence favoring our conclusion is substantial.

We must also observe that our conclusion applies only to prediction of the conditions on a specific date. Nothing is stated, for example, about the possibility of saying whether next summer will be a warm one or a cool one. We maintain that it is not possible to say which days during the coming summer will be the warmer ones or the cooler ones.

5. Further considerations and conclusions

One result of our computations to be noted is that once the errors in the synoptic scales have become noticeable but not large, further doubling, in the root-mean-square sense, requires somewhat more than two days. This doubling rate is consistent with the one deduced by the empirical procedure, but it is appreciably more rapid than that indicated by the dynamical studies. We must therefore note a particular shortcoming of the dynamical approach.

In the earlier days of numerical simulation of the atmosphere, it was found that the numerical solutions, after behaving in a reasonable fashion for perhaps several weeks, would suddenly go into wild oscillations. This phenomenon was eventually recognized as a type of nonlinear computational instability by Phillips (1959), who also accounted satisfactorily for its presence. Various computational schemes, which by no means duplicated the manner in which the real atmosphere is prevented from blowing up, were eventually devised to overcome the instability.

It seems likely that these schemes, which prevent certain computational errors from becoming unduly large, may also have a damping effect upon real errors, and thereby raise the doubling time above its proper value. We have tested one scheme for this effect. The scheme was devised by Arakawa (1966), and was used by Mintz in his study of error growth.

In short, we have repeated the dynamical-empirical procedure, using new values of the coefficients in the equations, which are the values which the coefficients would assume if the Arakawa computation scheme were a part of the equations governing the real atmosphere. We have assumed in addition that scales of motion too small to be resolved by the computational grid used in the numerical experiments are completely absent.

Using the coefficients compatible with the Arakawa computation scheme, we find that small-amplitude errors should double in five days. This is almost exactly the doubling time actually obtained by Mintz. With the more appropriate coefficients, small-amplitude errors are indicated as doubling in about 2.5 days.

The effect of the Arakawa computation scheme is to deemphasize the smallest scales actually retained, while treating the larger scales in an essentially correct manner. We venture the guess, then, that if Mintz's computations were to be repeated with a considerably closer grid-point spacing, so that the smaller synoptic scales would no longer be the smallest scales retained, a doubling time of three days or less would be found. Of course, substantially decreasing the grid-point spacing would enormously increase the required amount of computation.

It thus appears that all three approaches lead to nearly the same result, namely, that small errors in scales large enough to be resolved by conventional grids should double in somewhat less than three days, in the root-mean-square sense. Once the errors have attained a moderate size, they will grow less rapidly. In the scales too small to appear in conventional analyses, the errors may grow very rapidly indeed.

Are these results encouraging or discouraging? Certainly they must be discouraging to those who may have hoped that the often mentioned figure of two weeks could actually be pushed closer to a month. They do not even offer encouraging prospects for predicting the positions of migratory cyclones and anticyclones two weeks ahead. In another respect, they are rather promising.

In the numerical solutions obtained in the dynamical-empirical procedure, we have found that the spectra of the errors at intermediate stages of the computation tend to resemble the initial spectra of errors which would be present if the observational data were confined to a regular network of points. The larger scales are almost free of error and the smaller scales are completely dominated by errors, while there is rather narrow band of intermediate scales where the errors are medium sized. We are thus led to postulate a sort of additive law for predictability.

Specifically, if the largest scale of motion not resolved by the observational network has an intrinsic range of predictability of, say, three days, introducing a fine enough network to resolve all scales of motion (an impossible task, of course), would increase the realizable range of predictability of all the larger scales by just three days. Likewise, improving the network so that the largest remaining unresolved scale has an intrinsic range of predictability of one day, instead of three days, would increase the realizable range of predictability of the larger scales by two days. Systems unresolved by the present network are probably intrinsically predictable at least three days ahead; reducing this figure to one day by improving our network does not seem to be beyond our capabilities.

To be able to forecast 16 days in advance as well as we now forecast 14 days in advance would not be a particularly spectacular achievement. To be able to predict three days ahead as well as we now predict one day ahead would be a major accomplishment. Indeed, it is altogether possible that one of the practical outcomes of current efforts to improve our observational network will be a new level of excellence in short-range forecasting.

II. ATMOSPHERIC PREDICTABILITY AS INDICATED BY NUMERICAL EXPERIMENTS

NOTE

The following article does not represent work performed under Contract AF 19(628)-5826. It is included to provide a description of the numerical experiments upon which some of our conclusions are based. The article is an excerpt from a report (Publication 1290, National Academy of Sciences) entitled "The feasibility of a global observation and analysis experiment", prepared by the Panel on International Meteorological Cooperation for the Committee on Atmospheric Sciences of the National Academy of Sciences. The report also appears in the *Bulletin of the American Meteorological Society*, Vol. 47, pp. 200-220 (March 1966). The members of the Panel on International Meteorological Cooperation were Jule G. Charney (Chairman), Robert G. Fleagle, Vincent E. Lally, Herbert Riehl, and David Q. Wark.

3. *Theoretical limits of predictability*

A basic problem in long-range forecasting is concerned with the theoretical limits of predictability of the atmosphere as a determinate system. If one knew the laws of motion with perfect accuracy, had accurate initial grid data for all the atmosphere, and could reduce truncation error to negligible proportions, it would still be impossible to make determinate forecasts for arbitrarily long time intervals. This limitation is not a matter of quantum indeterminacy or of thermodynamic fluctuation; it derives fundamentally from the continuous character of the turbulence spectrum and the limitations of any observational net. As long as some turbulent energy remains at the mesh scale, then, no matter how much the observational net is refined, part of this energy will inevitably appear under an alias as energy of the large-scale flow, i.e., as an error of observation. This will remain true until the mesh size is reduced to the smallest eddy size permitted by viscosity. It will then obviously have become impossibly small. Thus it appears that errors of observation are inevitable even when they are not strictly instrumental. In practice, instrumental errors will be appreciable and will have to be reckoned with (they, too, are related to the existence of turbulence).

The question of predictability may now be asked in a more realistic manner. Given the inevitable small errors, will these remain small or will they grow, and, if they grow, how fast will they grow? How long will it be before the error has grown beyond acceptable bounds? These questions were asked by Lorenz[1] who related them to the question of the stability of a dynamical system with a finite number of degrees of freedom. If the system is represented by a point in a phase space and occupies a bounded volume in this space, it must pass arbitrarily close to the same point on more than one occasion. If the system is stable, in the sense that a small perturbation of the system in the phase space will remain small, it is easily seen that it must be periodic or almost periodic. Conversely, if it is not periodic it must be unstable. Figure 1, taken from Lorenz's article, illustrates this

1. (1963b)

situation. Lorenz also showed that if a solution is nonperiodic, it is not only unstable but the neighboring solutions must eventually become as far separated as two randomly chosen solutions. He carried out numerical experiments with a very simple atmospheric analogue containing only 28 degrees of freedom and was able to obtain steady, periodic, and nonperiodic solutions.

There is every indication that the actual atmosphere is aperiodic and therefore unstable, although it contains periodic or quasi-periodic components such as the lunar and solar tides. The question is, then, how fast will a given error, interpreted as a perturbation of the atmospheric flow, grow before the perturbed motion differs from the unperturbed motion by as much as two randomly chosen flows. To estimate the limit of predictability in this sense, a series of numerical experiments were performed by Drs. C. Leith, Y. Mintz, and J. Smagorinsky, at the Livermore Laboratory of the University of California, the University of California at Los Angeles, and the Geophysical Fluid Dynamics Laboratory of the U.S. Weather Bureau, respectively. Numerical predictions were performed with each of

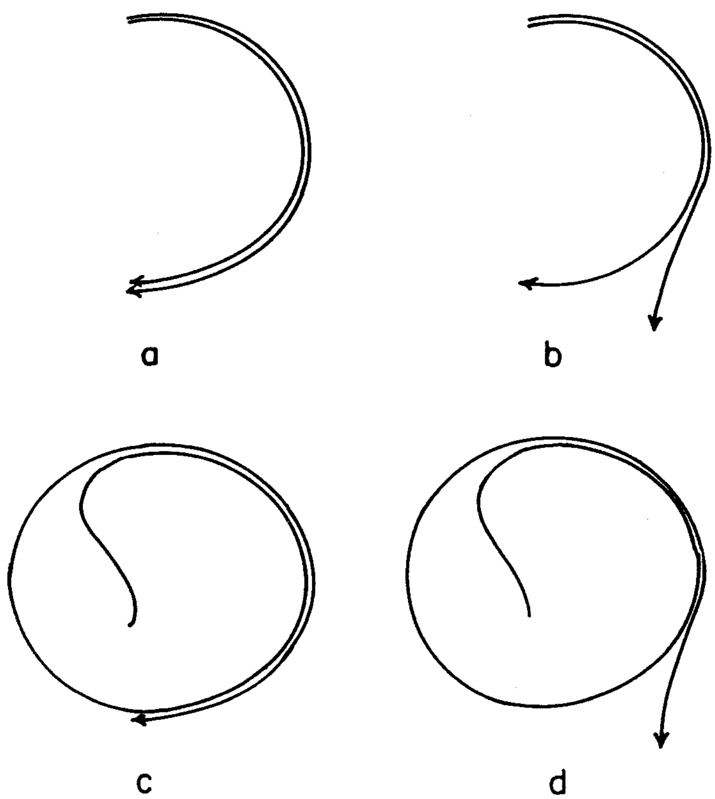


FIGURE 1. Schematic diagrams illustrating predictability.

their models for so long a period that the starting conditions had ceased to have any discernible effect. At this time a sinusoidal "error" perturbation in the temperature field was introduced and a prediction was made for at least 30 additional days. This prediction was then compared with the evolution of the unperturbed flow for the same period. The individual models will now be described.

(a) *The Leith model.*[2] This model is global in extent. The grid divides the atmosphere into six pressure levels in the vertical and covers the globe with a horizontal grid spacing of about 500 km. Heating is due to incoming solar radiation, the release of heat of condensation, and small-scale convection. Infrared radiative transfer is included by specifying a cooling rate as a function of pressure alone. The surface temperature is a fixed function of latitude and longitude, and the ground is flat. Vertical and horizontal diffusion of heat and momentum by turbulent eddies are provided for by linear diffusion laws with constant eddy coefficients. We note especially that the horizontal diffusion coefficient, D, is given the unusually large value 10^{10} cm^2 sec^{-1} in order to ensure computational stability. This results in a dissipation half-life of $L^2 \log_e 2/8\, \pi^2 D = 5.2$ days for a sinusoidal disturbance whose wavelength, L, is 6,000 km in both the zonal and meridional directions.

(b) *The Mintz-Arakawa model.*[3] This model is also global in extent. The grid divides the atmosphere into two pressure levels in the vertical and covers the globe in 9° intervals of longitude and 7° intervals of latitude, except in small regions near the poles. The heating due to incoming solar radiation is fixed in space and time; the heat of condensation is brought in parametrically and rather unrealistically only to keep the lapse-rate of temperature from exceeding the moist-adiabatic; heating by small-scale convection is allowed at the lower level; and the infrared cooling rate at both levels is given as an empirically determined function of the temperature at the lower level. The ocean temperatures are fixed, but the land temperature is permitted to vary in accordance with the temperature at the lower grid level. These features do not differ significantly from those in Leith's model except for the variable land temperature and the absence of large-scale precipitation. There are two additional features, however, in which the model does differ significantly: first, owing to the employment of a completely stable difference scheme devised by Arakawa, it was not necessary to assume so large a hori-

2. (1965)

3. (1964)

zontal diffusion coefficient; and second, orographical variations are taken into account.

(c) *The Smagorinsky model.*[4] The Geophysical Fluid Dynamics Laboratory's current nine-level model could not be used in this test because of the prohibitive amount of computation that would have been required. An earlier two-level model was used instead. In this model the flow is bounded by vertical walls at the equator and at latitude 64.4°. The grid is a square net on a Mercator projection, with a mesh size varying from 555 km at the equator to 240 km at the north boundary. The earth's surface is flat and homogeneous. The thermal structure of the atmosphere is characterized by a single variable temperature at an intermediate level and a constant static stability. Diabatic heating is given as a linear function of the temperature. Surface friction is treated as a boundary layer phenomenon, and lateral eddy diffusion of momentum is assumed to take place with a coefficient of viscosity dependent on the deformation tensor, so that strong momentum diffusion occurs only when the deformation field is large.

In each of the models a temperature perturbation of the form $\Delta T \sim \sin 6\lambda \cos 11\phi$ was introduced. Here λ is the longitude and ϕ the latitude, giving six waves zonally and six from pole to pole.

Graphs showing the time variation of the root-mean-square (r.m.s.) temperature deviation,

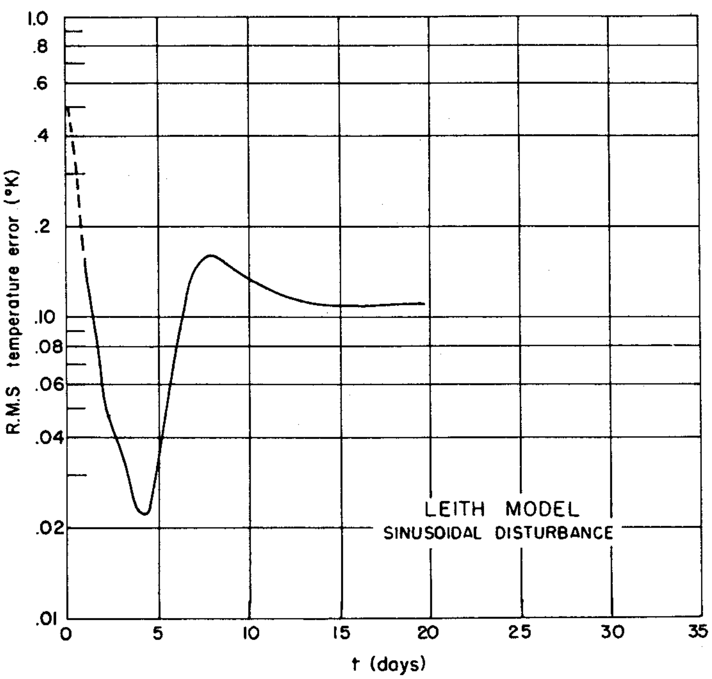
$$\left[\frac{1}{N}\sum_{1}^{N}(\Delta T_n)^2\right]^{1/2},$$

averaged over all the grid points ($n = 1, 2, \ldots N$), are presented in Figures 2, 3, and 4 for the Leith, Mintz-Arakawa, and Smagorinsky models, respectively. The perturbation temperature amplitudes were taken to be 2.0°, 0.5°, 0.1°, and 0.02°K in the Smagorinsky model and 1°K in the others. The r.m.s. errors were calculated separately for each hemisphere and each level in the Mintz-Arakawa model, and for the whole atmosphere in the others.

The initial temperature errors were chosen so small that it was thought that they could be regarded initially as linear perturbations on the finite-amplitude time-variable flow. It was expected that they would at first grow exponentially until they reached a finite-amplitude nonlinear stage and would then grow at a decreasing rate. It

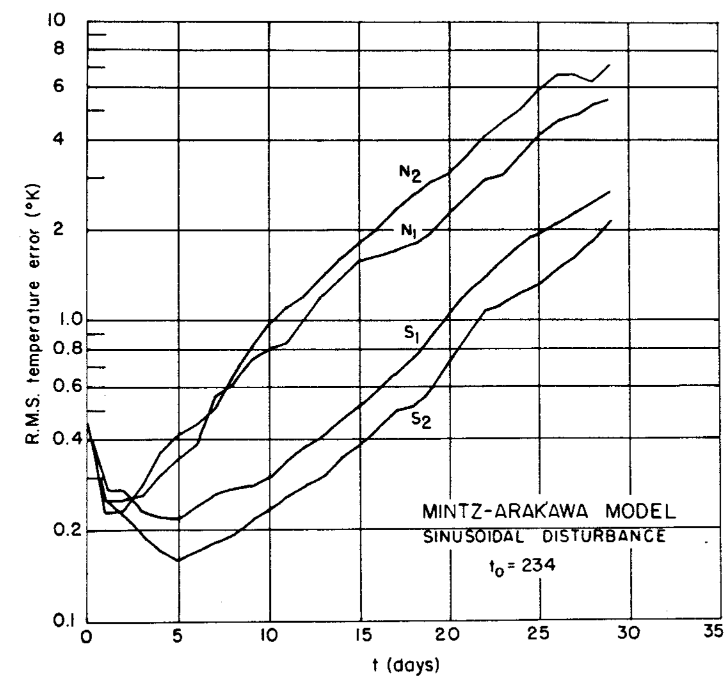
4. (1963)

FIGURE 2. Root-mean-square temperature error in Leith model.



will be seen from the figures that this expectation was borne out only in the Mintz-Arakawa model. In the Leith model the r.m.s. error started at 0.50°K and, after undergoing a transient oscillation, leveled off at 0.11°K. The calculation was terminated after 20 days because of a computational instability associated with the condensation process. The flow at the upper levels became nearly a constantly

FIGURE 3. Root-mean-square temperature error in Mintz-Arakawa model.



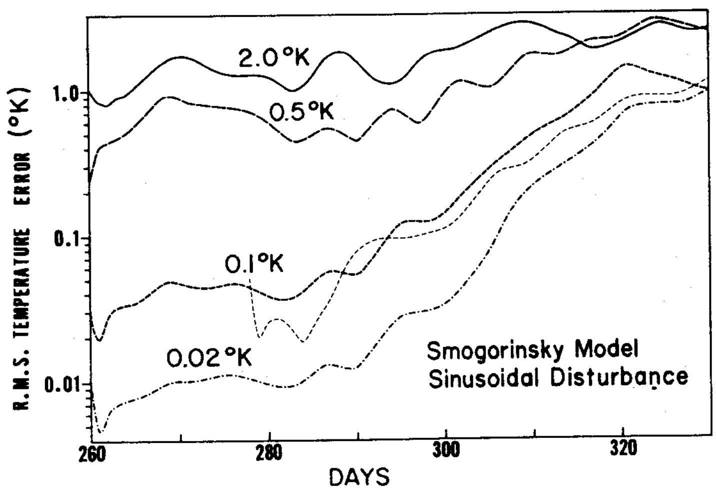
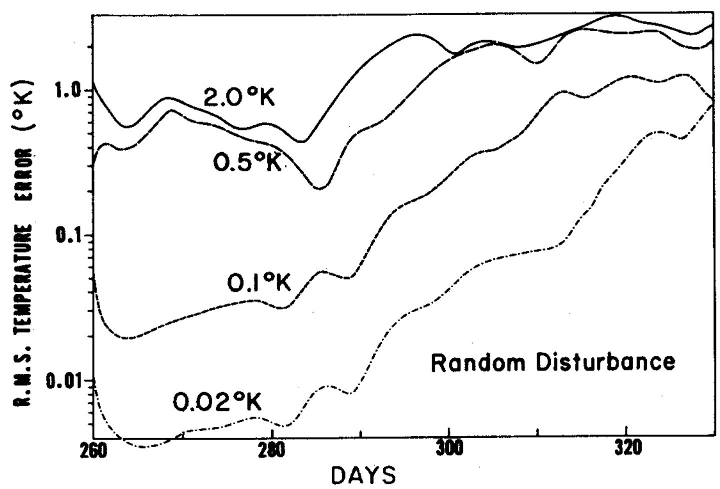
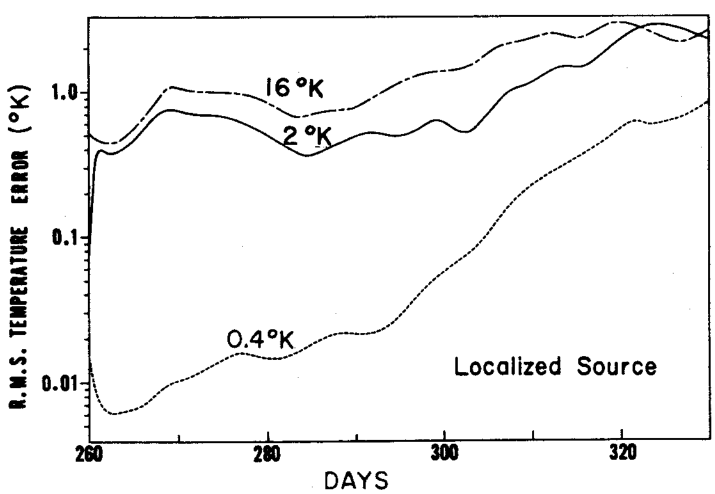


FIGURE 4. Root-mean-square temperature error in Smagorinsky model.





translating wave pattern of small amplitude. The cause of this upper-level stability is probably the excessive eddy viscosity and the weak coupling with the surface.

In the Smagorinsky model the temperature amplitude of the unperturbed flow was, in the first calculation, no greater in order of magnitude than the imposed perturbation of 2°K. This placed the perturbation immediately in the nonlinear range. However, even when the amplitude was reduced to 0.5°K the error growth exhibited a similar behavior. In both cases it will be seen from Figure 4 that the r.m.s. error varies in a quasi-periodic fashion with a period of about 2 weeks and a slowly increasing mean value. The variations parallel each other for about 30 days and then begin to depart. The smaller error disturbances, of amplitude 0.1°K and 0.02°K, show a slow but continuous growth until after about 30 days, when the doubling time reaches the value of 6 or 7 days. An examination of the actual flow patterns revealed that the motion was primarily periodic, with a small aperiodic component. In accordance with what has been said about the stability of dynamical systems, it might have been expected that the instability would appear as a slow growth superimposed on a periodic fluctuation. After about 30 days the vacillating regime changed to a more aperiodic behavior, and at that time the error grew more rapidly with a doubling time of 6 or 7 days. This behavior does not resemble very well the usual condition of the atmosphere in which strong instabilities appear always to exist.

The only model exhibiting the strongly aperiodic behavior of the atmosphere was the one of Mintz and Arakawa. Integrations had been performed with this model for upwards of 284 days with the sun constantly at the Northern Hemisphere winter solstice. A sequence of sea-level pressure charts at 2-day intervals for days 229–243 are shown in Figures 5, 6, 7, and 8. It will be seen that the flows are realistic at middle and high latitudes, with the traveling disturbances and the quasi-permanent centers having typical locations and intensities. The zonally and time-averaged temperature distributions shown in Figure 9 are also seen to correspond well with observations for the winter season. We may therefore be justified in assuming that the statistical properties of the error growth will be realistic.

The primary deficiency of the model is its unrealistic treatment of the condensation process; this deficiency is indicated most markedly in the tropics by too broad a low-pressure trough and the absence

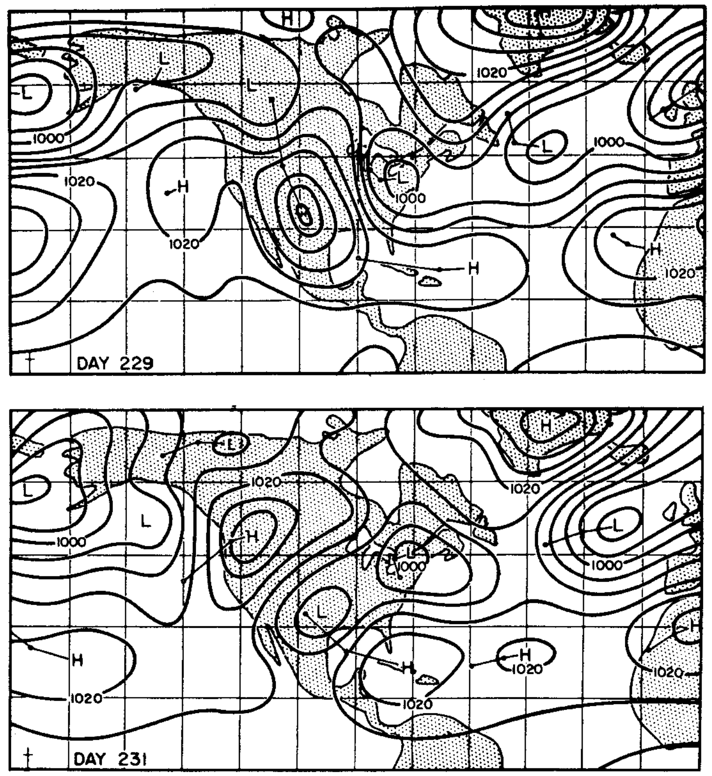


FIGURE 5. Sea-level-pressure maps as computed for days 229 and 231. The isobars are at 5-mb intervals and the small circles, connected by thin lines, show the positions of the low and high centers in the preceding two days.

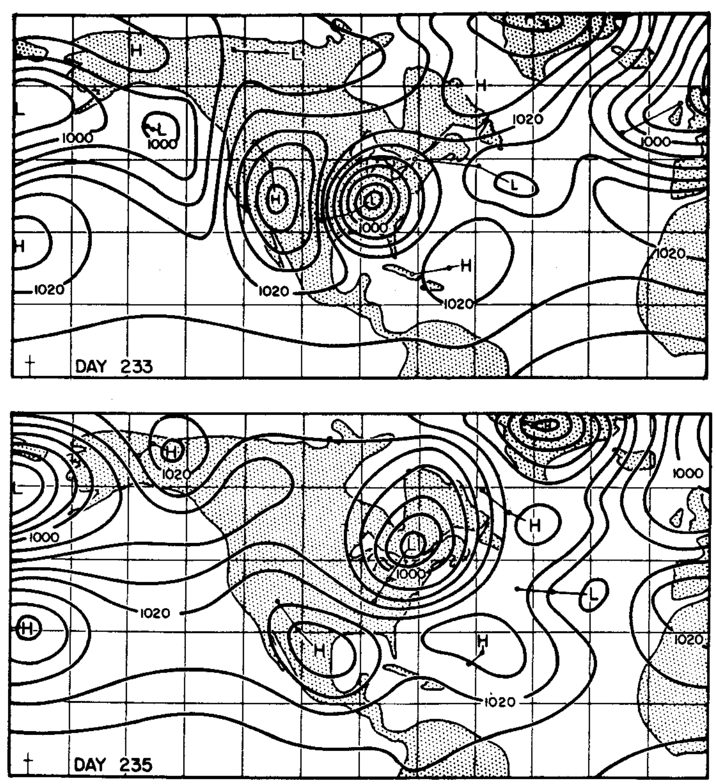


FIGURE 6. Sea-level-pressure maps as computed for days 233 and 235.

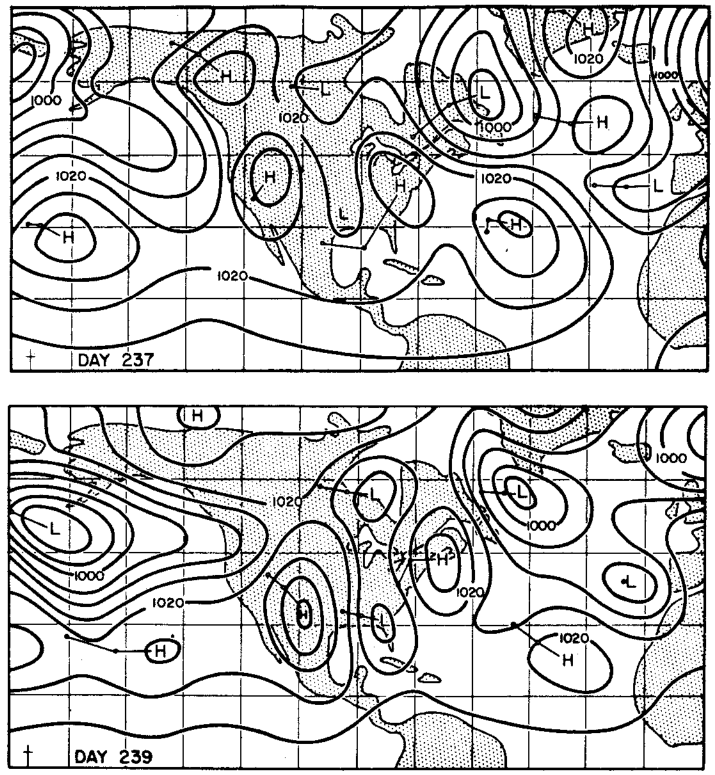


FIGURE 7. Sea-level-pressure maps as computed for days 237 and 239.

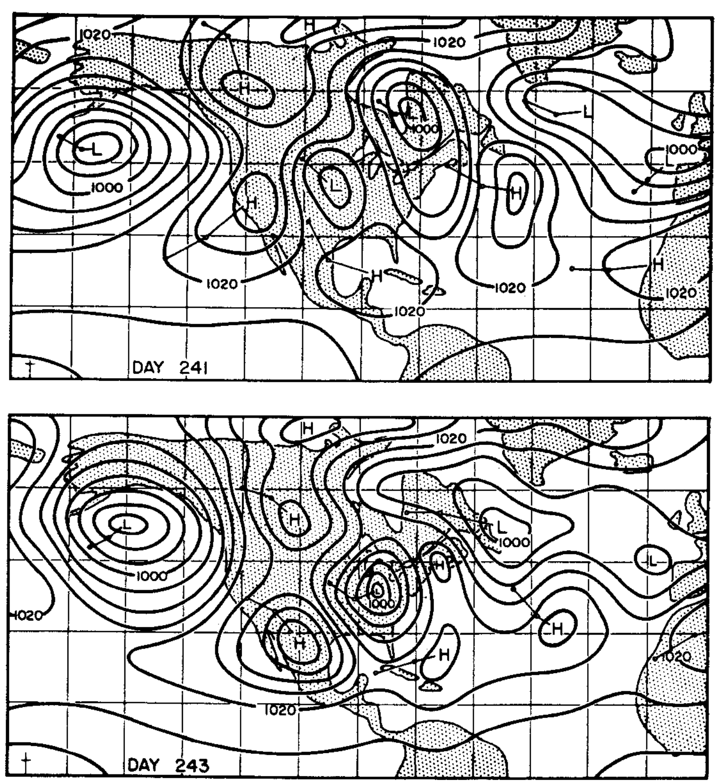


FIGURE 8. Sea-level-pressure maps as computed for days 241 and 243.

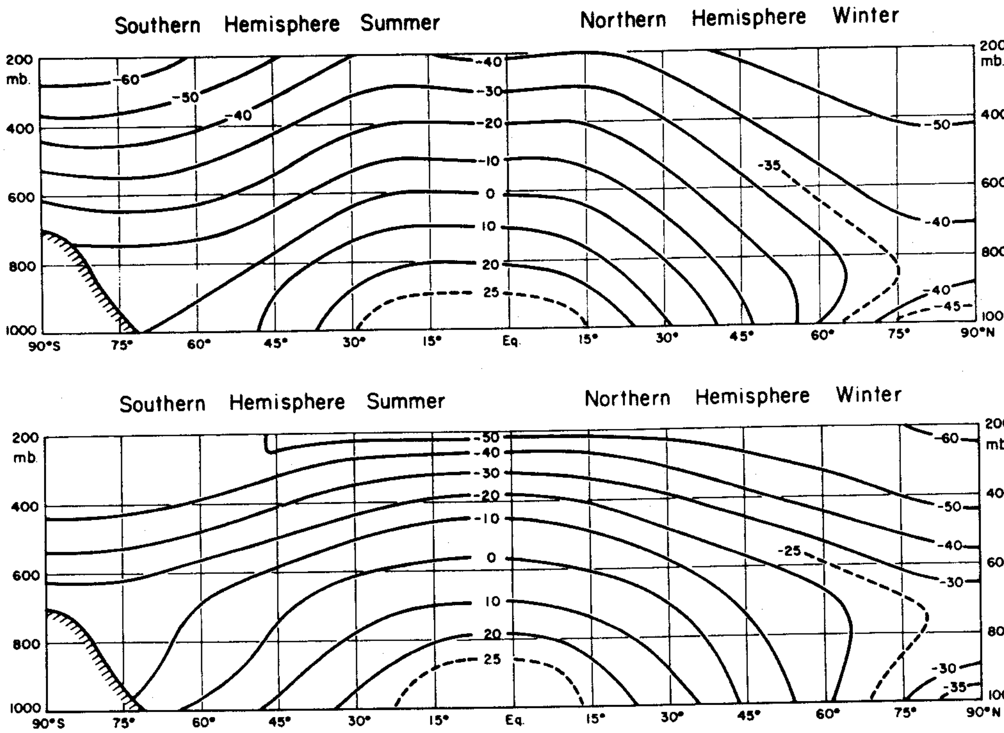


FIGURE 9. Meridional cross sections of the zonally averaged and time-averaged temperature, in degrees centigrade, for Northern Hemisphere winter and Southern Hemisphere summer. The upper figure is the 30-day mean (from day 256 to 285) computed in the numerical experiment. For representational purposes, the zonally averaged surface temperature is shown at the 1,000-mb level, except over Antarctica. The lower figure shows the observed field, according to Burdecki (1955).

of the normal tropical disturbances, as well as in a distortion of the subtropical high-pressure pattern. This is shown in Figure 10. However, the principal error growths are associated with strong middle-latitude cyclogenesis, and since cyclogenesis is far less frequent and usually less intense in the tropics, especially in the Northern Hemisphere winter, the lack of realism in the tropical motions cannot have affected the statistical results significantly. The lack of condensation in the extratropical developments is probably more serious, but since it is not the primary cause of cyclogenesis, it is not thought to have a dominating effect.

The results of the predictability calculations with the Mintz-Arakawa model are summarized in Figures 3, 11, 12, 13, 14, 15, and 16. The first of these figures shows the growth of the r.m.s. temperature error from day 234, when the sinusoidal error was inserted, to day 264 for each of the two temperature levels in the model and for each hemisphere. The temperature levels, 1 and 2, vary somewhat in elevation and have means of 400 and 800 mb, respectively.

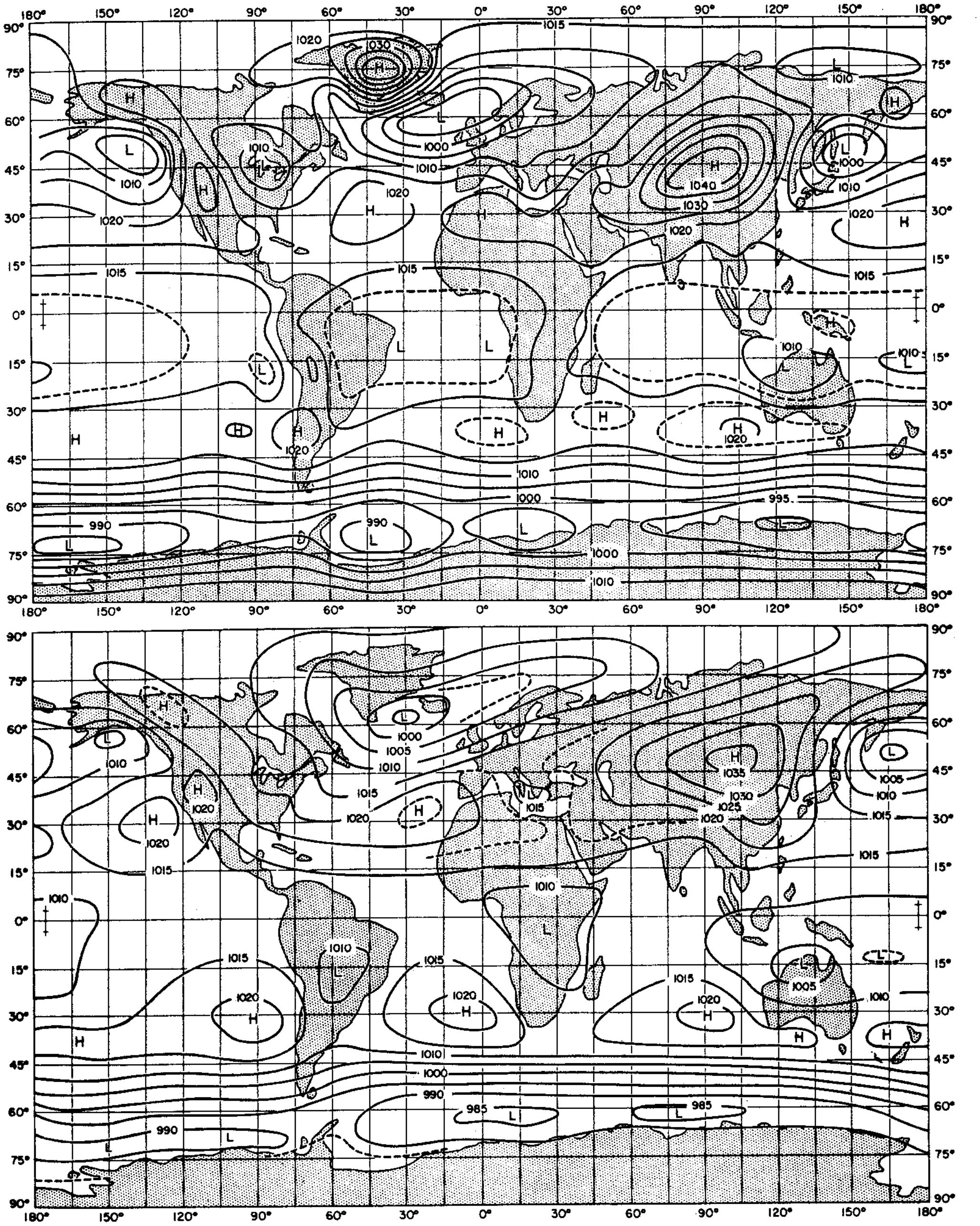
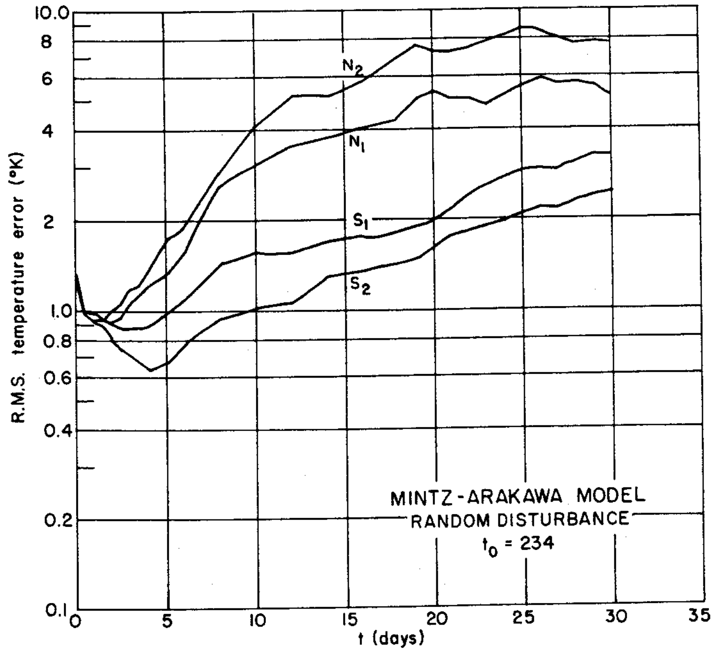


FIGURE 10. Surface pressure reduced to sea level, for Northern Hemisphere winter and Southern Hemisphere summer, in millibars. (The broken lines are intermediate 2.5-mb isobars.) The upper figure is the 30-day mean (from day 256 to 285) computed in the numerical experiment. The surface pressure was reduced to sea level using the computed surface temperature and a subterranean temperature lapse rate of 6°C/km. The lower figure shows the normal January sea-level pressure, according to O'Conner (1961), 90°N–15°N; Riehl (1954), 45°N–45°S; and van Loon (1961), 15°S–90°S.

Since the wind field was not disturbed initially, the initial decrease in the r.m.s. temperature error most likely represents a transformation

FIGURE 11. Root-mean-square temperature error in Mintz-Arakawa model.



from perturbation potential energy, associated with the perturbation temperature fluctuations, to perturbation kinetic energy during the process of the quasi-geostrophic adjustment of the wind to the pressure field. The evidence from another calculation indicates that little gravity wave energy was generated.* The subsequent growth is seen to be approximately exponential, with a doubling time of about 5 days in each hemisphere and at each level. Since the pressure and wind fields are hydrostatically and geostrophically related to the temperature fields, the relative r.m.s. error growth in these fields should be essentially the same as that of the temperature field after the initial adjustment period.

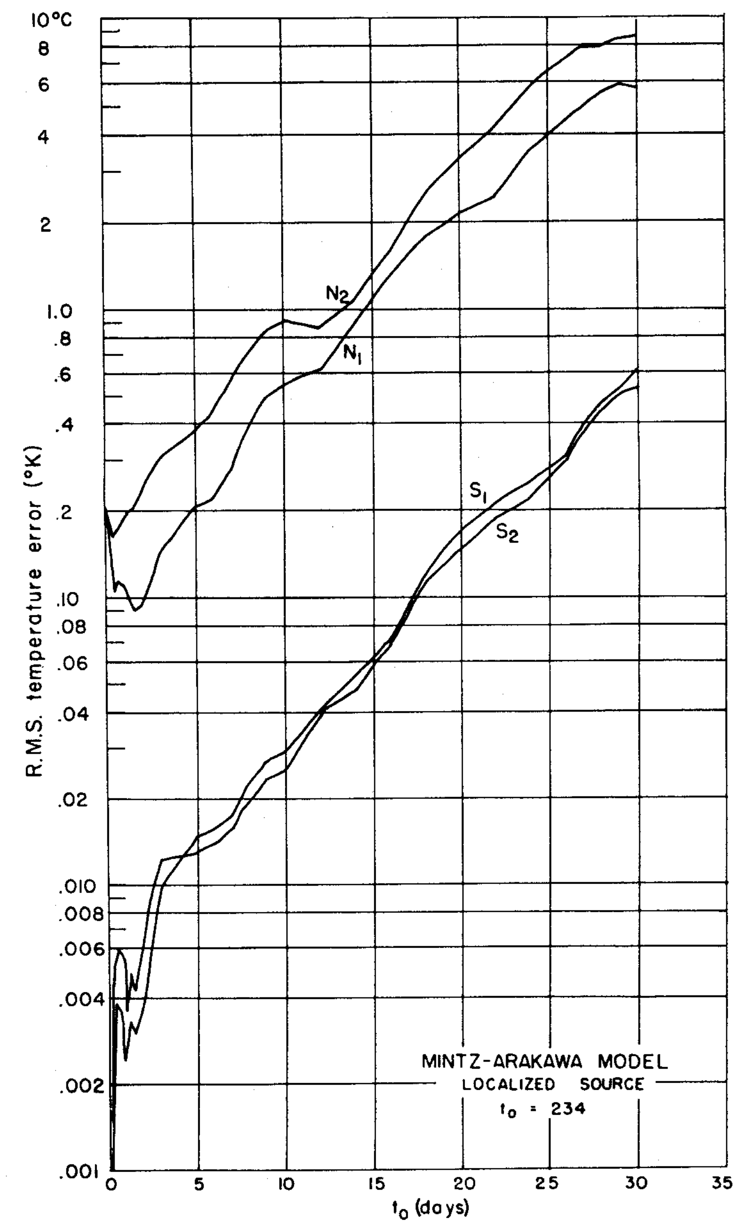
Figure 11 shows the r.m.s. error growth for a random temperature disturbance modulated by the factor cos ϕ cos 6ϕ, and Figure 12 shows the growth for an initial disturbance confined to the region 21–63°N, 157–203°W. Again, we see that after the initial adjustment period the growth becomes near-exponential with a doubling time of about 5 days.† In the former case the initial error is large, and the nonlinear range is reached sooner. In this range the growth rate first

* It may be seen from Figure 12 that the gravitational wave energy generated by a perturbation placed initially in the Northern Hemisphere invades the Southern Hemisphere within a day or two, and that at this stage it is three orders of magnitude smaller than the initial energy.

† A similar doubling time for an initially random disturbance in a two-level model was obtained theoretically by Thompson.[5]

5. (1957)

FIGURE 12. Root-mean-square temperature error in Mintz-Arakawa model.



diminishes and then levels off to an irregular fluctuation, at which time the perturbed flow ceases altogether to resemble the unperturbed flow. All deterministic predictability is lost when the r.m.s. errors become comparable to those obtained by differencing two random flows. Estimates of these magnitudes were obtained by differencing temperatures for the days 261 and 260, 262 and 259, 263 and 258, etc. The resulting r.m.s. temperature differences are shown in Figures 13 and 14 as functions of the time-difference in days. It will be seen that beyond 3 days the resemblances are so weak that the pairs may be considered essentially random. (Three days is therefore the upper

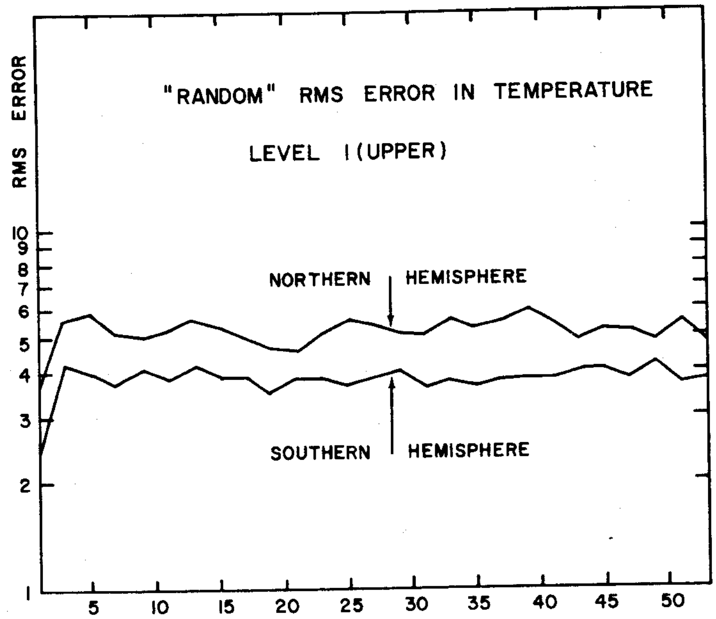


FIGURE 13. "Random" error in temperature as a function of days of separation. Upper level.

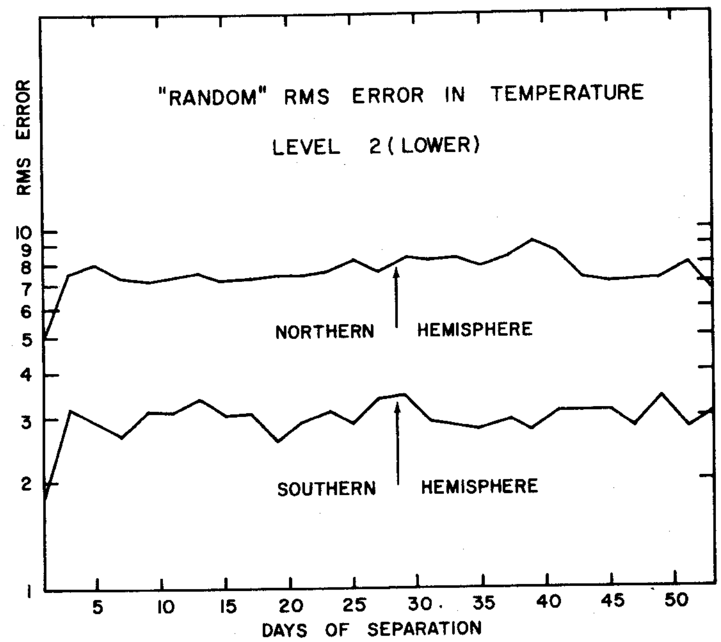


FIGURE 14. "Random" error in temperature as a function of days of separation. Lower level.

limit for a "persistence" forecast.) Referring to Figures 13 and 14 and Figures 3, 11, and 12, we note that all predictability in the Northern Hemisphere is lost at 26 days for the wave perturbation, 19 days for the random perturbation, and 29 days for the localized perturbation.

We note from Figure 4 that the behavior of the random and localized disturbances in the Smagorinsky model resembles that of the Mintz-Arakawa model after the first 25 days, but with a slightly lower growth rate.

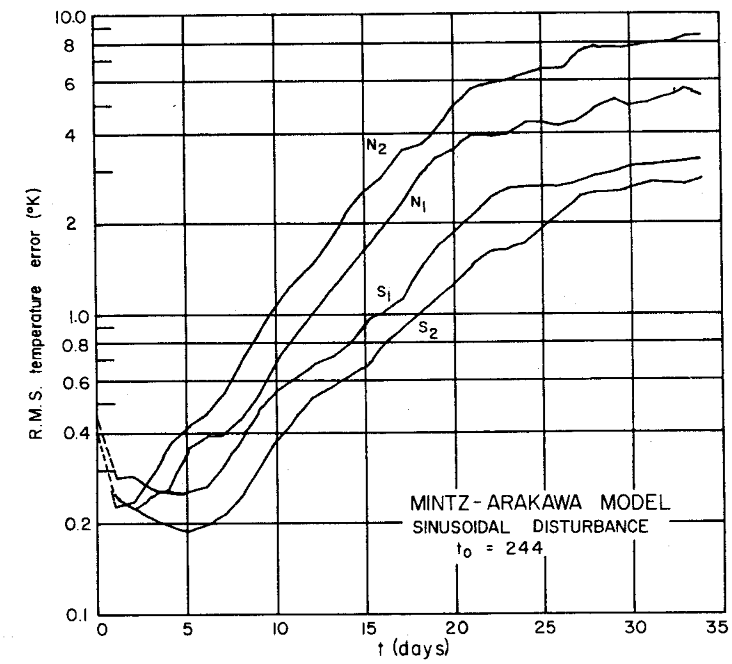


FIGURE 15. Root-mean-square temperature error in Mintz-Arakawa model.

It may seem at first surprising that the logarithmic rate of growth of the r.m.s. error is approximately independent of the nature of the initial error. An explanation of this phenomenon is suggested by Fourier analyses of the error along zones of latitude at middle latitudes. One finds for the case of the sinusoidal wave perturbation, whose wavelength corresponds roughly to disturbances of cyclone scale,

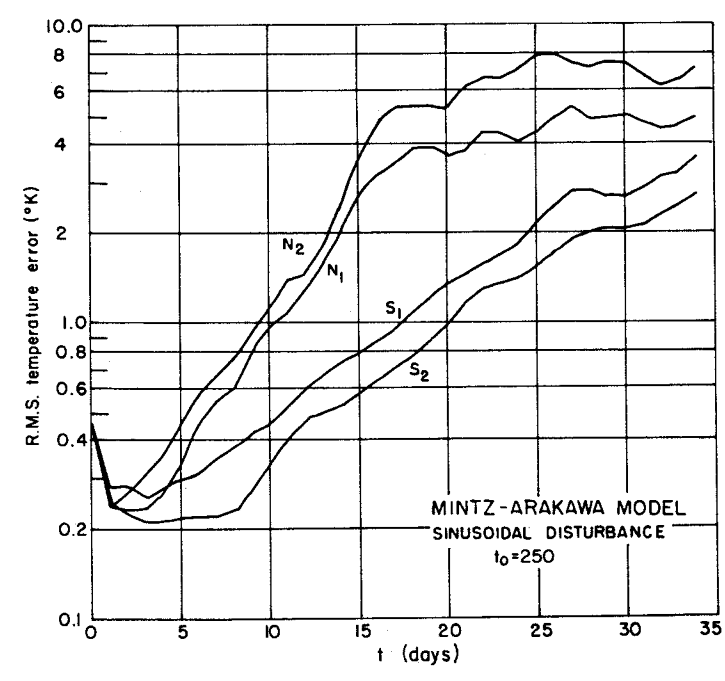


FIGURE 16. Root-mean-square temperature error in Mintz-Arakawa model.

that the spectral energy quickly spreads in such a manner that after 5 days two other peaks in the spectrum appear in the wavenumber range 1 to 4 and 8 to 12 but that the maximum energy remains near wavenumber 6. This is the spectral behavior that would characterize the interactions of the most unstable of the normal perturbation modes of the finite-amplitude time-variable flow with the finite-amplitude flow itself. It is what one would expect if the error growth were essentially a baroclinic instability appearing as cyclone development in a band centered near wavenumber 6. The first-order interactions of this *perturbation* mode with similar or larger scales of the *finite-amplitude* flow would produce the peaks of *perturbation* energy near wavenumbers 3 and 10. Since each of the error disturbances considered (and any that would be likely) has energy at either the long, intermediate, or short scales, its first-order interaction with the finite-amplitude flow will produce energy in the cyclone-scale mode, which will then dominate the development by its rapid growth and give rise to the characteristic r.m.s. growth rate.

We may infer from the correspondence of the space and time scales that the finite-amplitude irregular flow of the atmosphere is unstable in much the same way as the idealized baroclinic zonal flows which have been studied theoretically, but that, because of the irregularity of the basic flow, the unstable perturbation modes are themselves so irregular that it would require a very special initial disturbance indeed not to excite them.

An important conclusion concerning data requirements may be drawn from the above considerations. Since errors on any macroscale will ultimately grow by instability or by nonlinear interaction, one is not spared the necessity of defining the entire spectrum in the macro-range by means of the observational system. The conjecture that the need for defining the smaller scales of motion is removed by extension of the forecast range is wrong on two counts. In the first place, one cannot measure the long-wave components without measuring the shorter ones as well, for if the shorter components have appreciable energy, a large part will appear under an alias as energy of the long waves. In the second place, the errors in the neglected shorter-wave components will immediately affect the long-wave components through nonlinear interaction.

If one accepts a doubling time of 5 days as the rate of growth of the r.m.s. temperature difference and (from Figure 14) 8°K as the r.m.s. temperature difference between randomly chosen flows at 800 mb in the Northern Hemisphere winter, then a flow whose temperatures are determined within an r.m.s. error of 1°K will remain deterministically predictable for about 5 $\log_2 8$ or 15 days. If the error were random with an amplitude of 1°K its r.m.s. value would be $1/\sqrt{3}$, and the flow would remain predictable for some 4 days longer. An indication of how the rate of error growth varies with the synoptic situation was obtained by inserting the initial sinusoidal error perturbation at days 244 and 250 (see Figures 15 and 16). The integrations show doubling rates which vary from 4 to 5 days in the early stages of the prediction in the Northern Hemisphere winter and which decrease in the later stages, just as for the first case where the initial time was day 234. In the Southern Hemisphere summer the rates are somewhat less: the doubling time varies from 5 to 7 days at r.m.s. amplitudes below 1°K, and may be as long as 15 days at r.m.s. amplitudes about 1°K.

Conclusions

We may summarize our results in the statement that, based on the most realistic of the general circulation models available, the limit of deterministic predictability for the atmosphere is about 2 weeks in the winter and somewhat longer in the summer. We have assumed that the observational system defines the initial state of the atmosphere globally and that the temperature error is random and not greater than, say, $\sqrt{3}$°K, with the errors in pressure and wind having hydrostatically and geostrophically corresponding values in middle and high latitudes. Observational systems of the kind contemplated in this report can be expected to determine temperatures to within this error.

Finally, it should be stated that, in principle, prediction of certain statistical quantities might be made for longer periods. It remains to be seen whether there are predictable statistical quantities which vary significantly and yet more slowly than the individual dynamical parameters. Surface interactions due to heat storage in the mixed layers of the oceans, variable snow cover, etc., may act as governors regulating these changes. However, it would appear from the rapid growth rates due to the basic instability of the atmosphere that the slower surface interactions will not appreciably change the limit of deterministic predictability, i.e., the limit of predictability of the individual motions.