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LIMITS OF METEOROLOGICAL PREDICTABILITY

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1. Background

During the present century we have heard frequent claims to the effect that in the foreseeable future we shall be able to make nearly perfect detailed weather forecasts many days in advance, and that all we now lack is the needed skill. Success in predicting solar eclipses and oceanic tides is sometimes cited as supporting evidence. We have also heard claims that the weather is basically unpredictable, and that forecasts will never be much better than they are today. Weather forecasting has even been compared to predicting human behavior. The truth presumably lies between these extremes; let us examine what is involved. We shall first consider the procedure currently used in numerical weather prediction, and the errors which it entails.

In numerical forecasting we begin with the observed state of the atmosphere at some initial time. Since no meteorological instruments measure perfectly, the observations possess some errors. Of greater importance, however, the observations are ordinarily made only at specific locations, and the interpolation to intermediate locations introduces further errors. The inadequacy of interpolation stems mainly from the fact that the state of the atmosphere consists of superposed features of many scales; a relatively small-scale happening, such as a thunderstorm, may go undetected if it occurs between weather stations.

We then extrapolate the observed state forward in time by means of the governing physical laws. We do not know these laws perfectly, nor do we generally use them to the best of our knowledge, whereupon we introduce additional errors during the extrapolation process.

To perform the extrapolation we must convert the state of the atmosphere into a finite collection of numbers, and we must convert the governing laws into procedures for varying these numbers. The numbers may be the interpolated values of certain weather elements at conveniently chosen locations, or some other scheme may be used, but in any event even the largest computers today cannot conveniently handle enough numbers to represent the countless small-scale features, even if such features could be captured by the observations. Therefore only the larger scales receive adequate representation. Yet the effects of the small scales upon the large scales are far from negligible, and our numerical procedures for approximating these effects in terms of the large-scale features themselves are far from perfect.

Replacing the numerical procedure by a subjective one will not eliminate these problems. The amount of relevant information is simply more than can be handled. The human brain can probably store far more information than even a large computer, but it presumably cannot absorb so much new information as a computer in the course of a few minutes. Inevitably the initial state will contain errors, and additional errors will accumulate throughout the range of the forecast.

Let us now visualize a make-believe situation in which a noticeable error has accumulated between the initial time and a later "key" time, but where errors in the extrapolation method subsequent to the key time are completely absent. The extrapolation itself (not the method) subsequent to the key time will still contain errors, since at each step the time derivative will be computed from a current state which itself contains errors. According to whether the errors in the time derivative tend to be negatively or positively correlated with the errors in the state itself, the latter errors will diminish or amplify. In the former case, the physical system being predicted (the atmosphere or something else) is said to be stable; in the latter case it is unstable.

To a first approximation, the error in the time derivative should be proportional in magnitude to the error in the state from which the derivative is computed. The decay or growth of the error should therefore be quasi-exponential. If the system is stable, the error should become unobservably small. If the system is unstable, the error should become large, but it will not grow without limit. The worst that a correct extrapolation method can do is to replace the true state by another physically realistic state, wherefore the limiting magnitude of the error should be no larger than the difference between randomly chosen states of the system.

The oceanic tides offer an example of a stable system. If one could stand away from the earth and disturb the ocean surface with a giant fan, thereby setting up oscillations bearing little resemblance to the normal tides, the tides would presumably return to their normal schedule after a few days or weeks. Equivalently, if one attempted to forecast the tides numerically, using a perfect extrapolation method but completely wrong initial conditions, the forecast a few days or weeks in advance would presumably be essentially correct. As a consequence of this stable behavior, it is not necessarily to use stepwise extrapolation at all in the forecast. One can predict the tides months or years in advance without considering their behavior in the intervening time, simply by knowing the sun-earth-moon configuration at the time for which the prediction is made.

All available evidence indicates that the atmosphere is an unstable system. First of all, theory [1] indicates that a stable system should acquire a periodic behavior. The atmosphere does exhibit some periodic oscillations, notably the normal annual and diurnal variations and their overtones, but superposed upon these there is a strong nonperiodic oscillation. Moreover, numerical extrapolations [2] applied to pairs of nearly identical initial states, using the closest approximations to the laws governing the atmosphere that we have so far been able to formulate, show that the states diverge from one another as time progresses.

Abandoning now the make-believe situation, we see that during the course of a weather forecast the error is affected by two processes. One is an <u>accumulation</u> due to an inadequate interpolation method, occurring at a roughly constant rate. (It is convenient to treat the initial error as part of the accumulation error, as if it resulted from an abrupt accumulation at the initial instant.) The other is an <u>amplification</u> of the already existing error, occurring at a roughly exponential rate. It is important to distinguish between these processes when considering the improvements in weather forecasting which may be forthcoming. Such improvements can be brought about only by reducing the rate of accumulation (or the magnitude of the initial error); the rate of amplification is an intrinsic property of the atmosphere.

The theory which tells us that errors will grow in any nonperiodically varying system does not tell us how rapid the growth will be [1]. Our knowledge of this rate is based mainly upon the numerical extrapolations [2] previously mentioned. These indicate that root-mean-square errors in both the wind and temperature fields will tend to double in about three days, during the time they remain small. As they become large they approach their limiting magnitude more gradually.

We may distinguish between two extreme situations which could arise. If the extrapolation method is very poor, the error may accumulate so fast that it approaches its limiting magnitude during an interval too short for much amplification due to instability to take place. In this event it matters little that the atmosphere is unstable; the error results mainly from the forecasting scheme, as would have been supposed by those who maintain that nearly perfect forecasting is possible. Improving the scheme so as to cut the accumulation rate in half will then virtually double the range of acceptable forecasting.

If instead the extrapolation method is very good, very little error may accumulate during an interval long enough for the total amplification to be considerable. From this time onward, until the limiting magnitude is approached, further growth of the error will be due mainly to amplification. Improving the scheme so as to cut the accumulation rate in half will then not double the range of predictability; it will simply increase it by about three days.

Further consideration [3] reveals that this picture is oversimplified, and that it leads to an overoptimistic assessment of future prediction possibilities. The factor which has not been given sufficient prominence is again the presence of significant superposed features of many scales.

Consider a hypothetical situation in which the initial state of the atmosphere has been observed perfectly, except that one developing thunderstorm has been incorrectly located. The total error is then essentially the magnitude of two thunderstorms, one appearing where it should not, and one failing to appear where it should. During the forecast, the error should then amplify about as rapidly as a thunderstorm amplifies, doubling in perhaps twenty minutes rather than three days.

More generally, the total error may be resolved into scales, and the error in each scale possesses its own doubling time. The smallest scales would appear to amplify very rapidly, while the three-day doubling time is characteristic only of those scales resolved by conventional weather-map analyses.

It is equally apparent that the accumulation error (including the initial error) should be scale dependent. Certainly our present network cannot begin to resolve the structure of a thunderstorm, nor could the equations as we use them succeed in predicting the proper development, even if the structure could be observed.

It follows that our reasoning concerning the relative importance of accumulation and amplification should be applied more or less separately to each scale. In addition, we must take into account the interactions among different scales, whereby errors in one scale may induce errors in another, even if the latter are not initially present. From the practical point of view the possibility that the inevitable errors in the smallest scales will cause errors to develop in larger scales is the most important consideration. Theory indicates that the direct effect of errors in one scale upon errors in much larger scales is negligible, but that there may be a large indirect effect, in which small scales influence slightly larger scales, which in turn influence still larger scales, until ultimately the largest scales are in error.

This possibility has received considerable discussion since it was first seriously proposed, but we know of only one set of quantitative computations which explicitly considers scales too small to be resolved by conventional observational or computational networks. We quote some of the pertinent results [3] in Table 1.

In each computation the initial error is assumed to be confined to a very small scale (~ 40 meters), and the extrapolation procedure is assumed to be perfect. The first two columns of results, headed "no gap", show the times required for the mean-square errors in the indicated scales to reach 10% and then 90% of their limiting magnitudes. According to the theory which underlies the computations, the figures represent intrinsic limits which no forecasting procedure can ever transcend.

From informal discussions with numerous meteorologists we have gained the impression that these results are generally considered too pessimistic. There are a number of reasons for this attitude:

1) A "spectral gap" at scales of tens or hundreds of kilometers has not been included. The effect of such a gap would be to suppress the absolute

Table 1. Optimum predictability times estimated from a simple theoretical model [3] . Times labeled "10%" and "90%" indicate times required for mean-square errors in indicated wave lengths to reach 10% and 90% of their limiting magnitudes, assuming that extrapolation method is perfect.

wave length (km)	no gap 10%	no gap 90%	weak gap 10%	weak gap 90%	strong gap 10%	strong gap 90%
12	0.8 hr	1.3 hr	1.0 hr	1.7 hr	1.0 hr	1.5 hr
25	1.3	2.1	1.8	3.1	1.8	2.9
50	2.2	3.4	3.4	6.1	3.6	7.6
100	3.5	5.6	6.1	20.7	7.8	3.5 day
200	5.8	9.2	11.1	1.4 day	2.3 day	4.6
400	9.6	15.2	0.8 day	2.0	3.7	5.5
800	15.9	1.1 day	1.4	2.7	4.7	6.3
1600	1.1 day	1.8	2.1	3.3	5.2	6.7
3200	1.9	3.1	3.0	4.3	6.0	7.6
6400	3.3	5.4	4.5	6.6	7.6	9.8
12800	5.9	9.8	7.1	11.0	10.2	14.2
25600	9.4	16.3	10.6	17.5	13.8	20.6

errors in these scales, which in turn would reduce the absolute influence of these scales upon the larger scales. We therefore present in Table 1, in the columns headed "weak gap" and "strong gap" the results of two additional computations (not previously published), in each of which a gap spanning several octaves is assumed to be centered at a wave length of about 50 km. The exact nature of the gap in the true atmosphere is not known, and even its existence is sometimes questioned, but the often mentioned "minus-third-power spectrum" [4] in the sub-synoptic scales places the gap between the weak gap and the strong gap used in the computations.

2) A somewhat unrealistic statistical "closure assumption" has been used in the computations. This may serve to decrease the indicated predictability times; however, some closure assumption is needed to obtain any results at all, and no known closure assumption is clearly realistic.

3) For computational reasons, errors in each scale were allowed to increase abruptly to their limiting magnitudes, whereas their growth rate should have slackened sooner.

4) Viscous damping was omitted.

Nevertheless, one can also cite reasons why the results might be too optimistic. Chief among these are the omission of all vertical structure of the atmosphere, and the omission of water vapor, clouds, and precipitation.

2. Projections

In the light of these computations, we now consider the present limitations upon predictability, bearing in mind that the computations are too crude to serve as more than a guide. Over the continental United States, stations which observe upper-level winds and temperatures are somewhat irregularly located, but their density is about equal to that of a square grid of points with a 350 km grid spacing. This means that wave lengths of 350 km or less are virtually unresolved, while wave lengths of 700 km or more are captured with fair accuracy. Let us see first what would happen if a similarly dense network of stations covered the globe.

Looking at the figures under "no gap" in Table 1, we see that at about 15 hours, errors in the 800-km wave length have reached but 10% of their limiting values, while those in the 400 km length have reached 90%. That is, the errors at 15 hours with unlimited observational resolution are about the same as errors would be initially with present observational resolution. It follows that the predictability times for the larger scales would be those appearing in Table 1, reduced by 15 hours. Unlimited improvement in the observational network would add only 15 hours to the possible range of predictability.

However, this rather discouraging conclusion is obtained without regard for the spectral gap. If we apply similar reasoning to the figures under "strong gap", we find that predictability times can be increased by not 15 hours, but about 5 days, by improving the observations sufficiently.

Moreover, the assumed 350-km spacing between stations does not really exist over the oceans. Wave lengths of even 3000 km are actually not too well resolved. For forecasts more than two or three days in advance, even over populated land regions, the effective observational resolution is the hemispheric or perhaps the global resolution. The possible gain in predictability is then seen to be about 3 days if no gap is present, or 7 days with a strong gap.

Even so, the indicated optimum ranges of predictability with present-day resolution seem to be larger than current forecasting practice has attained. The discrepancies are no doubt due in part to the crudeness of the computations, but they may also be a measure of the failings of current forecasting methods. Aside from such mathematical shortcomings as truncation errors, some of the relevant physical processes are inadequately represented, or even omitted. Representation of vertical transports of moisture, heat, and momentum by cumulus and cumulonimbus convection presents a special problem. The transformation of clouds into precipitation, and the absorption and reflection of radiation by clouds, also deserve mention. In short, it is not certain whether observational inadequacies or extrapolation inadequacies contribute more to current forecasting errors.

In estimating the improvements to be realized in the coming decade, we must explicitly recognize the role of such undertakings as the Global Atmospheric Research Program (GARP). One task of GARP is to determine the degree of improvement in observation and extrapolation needed to bring about a given degree of improvement in the forecasts. The findings of GARP are likely to play a part in future planning, and it therefore seems that, aside from ever present financial limitations, the observations and extrapolations will be improved just to the point where little gain is believed to be attainable from still more improvement.

Referring again to Table 1, we see that if the "strong gap" figures are to be believed, a 2-day increase in the range of predictability may be gained by reducing the largest unresolved wave length from 400 km to 100 km; the increase is at least 4 days if the reduction is to 50 km. (Only minor gains are indicated by the "no gap" figures.) For financial reasons if for no others, such an improvement in the network of conventional observing stations is utterly impossible. However, satellite observations, which indeed are already contributing to the weather forecast, offer real hope. Routine photographs of clouds in visible light already have very high resolution, but cloud observations are not easily used as a basis for numerical forecasting. Temperature measurements by infrared sensing seem to offer considerable promise, particularly if present experiments in deducing the wind from the temperature prove successful. Barring any major national or international happenings which act to give weather forecasting a low priority, we believe that before 1985 some form of satellite observations should resolve wave lengths at least down to 200 km over the whole globe.

If such high-resolution observations are to serve their intended purpose, the computational grid must be reduced accordingly, preferably to 50-km spacing. On the basis of past trends, allowing for considerable slackening, we feel that the capacity and speed of computers should increase at least 100-fold before 1985. This will allow a reduction of the grid at least to 100-km spacing, on a global basis; another factor of 10 in speed would make a 50-km spacing possible.

Neither better observations nor better computers will serve their purpose unless corresponding improvements in the extrapolation method are forthcoming. We find the extent of such improvements the most uncertain item in our projection. This is partly because we are rather uncertain as to the quality of even today's extrapolations, not knowing to what extent the operational errors result from inadequate observations rather than inadequate extrapolation. One of the aims of GARP is to learn how to parameterize sub-grid-scale influences; hoping for moderate success along these lines, we venture that the range of acceptable forecasting of synoptic systems will be increased by two to three days in the next decade.

So far we have been considering the limitations upon forecasting on a continental or global basis, whether by numerical or subjective means. Let us now consider the forecast of such specific local events as severe connective storms, at a range of less than six hours. Here we mean not the forecast of whether there will be numerous storms in a general area, which is a synoptic-scale forecast, but the forecast of whether a storm will affect a given location at a given instant.

Such storms might fit the 12-km or 25-km wave-length category in Table 1. The figures then suggest that these storms cannot be predicted on a regular basis more than one to three hours in advance; despite the crudeness of the model, this conclusion may not be too unreasonable.

To make good forecasts of short wave lengths we must observe even shorter wave lengths. We need not, however, observe over an extensive area. A network of stations a few kilometers apart, surrounding a key location such as an air base, is therefore economically feasible, provided such networks are not repeated for too many air bases. Observations would be needed only when convective storms or other critical conditions appeared likely, and some sort of automatic observation system seems to suggest itself. Satellite photographs also have the necessary horizontal resolution, and perhaps with an enhancement technique they could serve to detect virtually every thunderstorm.

It seems most unlikely that detailed fields of motion, temperature, and moisture within individual storms can be observed on an operational schedule. The best forecasting procedure will probably involve predicting the motions of the storms by dynamical techniques, but predicting the growth and decay according to the known behavior of typical storms. Projecting to 1985, we feel first that the state of the art in the subject of predictability will have advanced to the point where we can say just how predictable convective storms really are. For the present, we simply note that the existence of so much turbulence at a scale just smaller than that of the storm itself does not favor prediction. We venture that by 1985, most of the few advances in very-short-range prediction which are possible will have been achieved.

We finally consider extended-range and long-range forecasting of general weather trends or average conditions, to which Table 1 does not apply. Can we, for example, say that next month will be exceptionally stormy, even though we cannot name the days on which the storms will occur? There is some empirical and theoretical evidence that the atmosphere can become temporarily stuck in one of several "regimes"; without being able to predict the particular weather pattern a few weeks in advance, we might still be able to say with some assurance that it will be typical of one regime rather than another. Again we feel that the biggest advance by 1985 will be in learning to what extent prediction at these ranges is possible. We also anticipate some progress in actual prediction. It seems possible that the most practical long-range forecasting procedure will be demonstrated to be a relatively simple statistical technique.

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