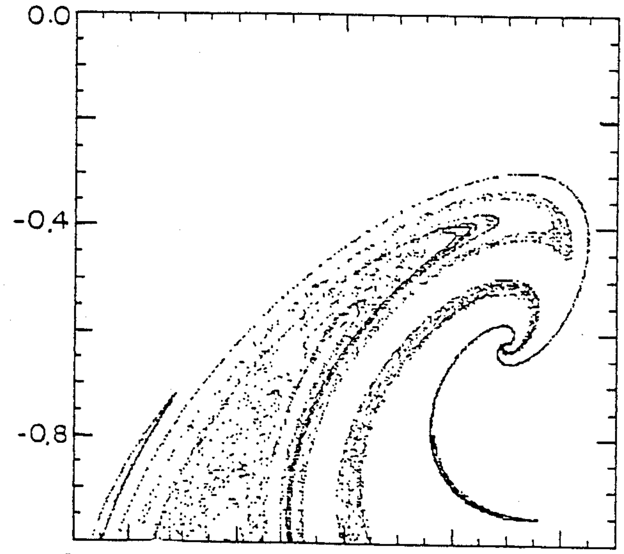

Fig. 5. An enlargement of a portion of Fig. 4, as represented by 8000 successive intersections of a single orbit with the included portion of the plane z = 0.

For our next example of chaos we proceed from one of the smallest possible "global circulation models" to one of the largest yet constructed. This is the operational forecasting model of the European Centre for Medium Range Weather Forecasts (ECMWF). The principal dependent variables of the model are horizontal wind components, temperature, and water-vapor mixing ratio; other variables are determined from these by auxiliary diagnostic formulas. The variables are independently defined at 15 elevations, and, in a recent version of the model, each horizontal field is represented by more than 10000 spherical-harmonic coefficients. The model thus consists effectively of more than 600000 ordinary differential equations in as many variables.

The model contains such physical features as orography. The effects of structures which are unresolved by the model, such as cumulus clouds, are included via parameterization. The intent is to make the model as good an approximation to the real atmosphere as is practical, in view of today's observation and computation systems. Diagnostic studies are regularly performed to determine how closely the climate produced by the model resembles the real atmosphere's climate, and significant differences generally lead to further research aimed at eliminating the discrepancies.

As the name of the Centre might imply, the principal purpose of the model is to produce weather forecasts at the "medium range" extending from a few days to a week or two. The present operational routine

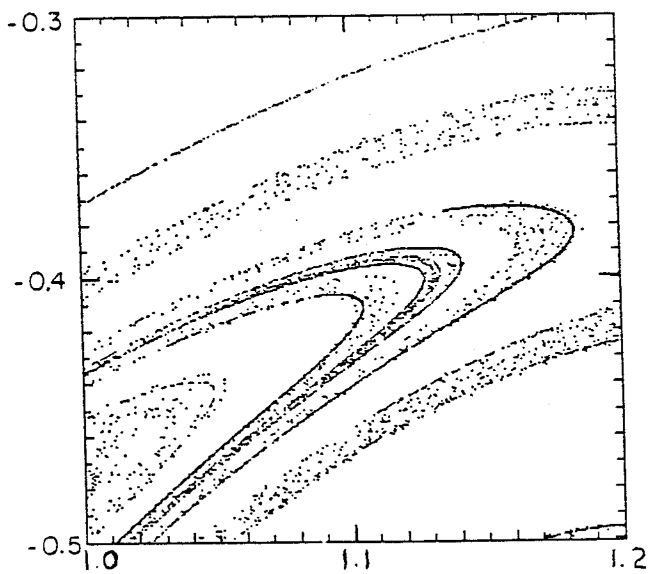

Fig. 6. An enlargement of a portion of Fig. 5, as represented by 6000 successive intersections of a single orbit with the included portion of the plane z = 0.

involves preparing, every day, a ten-day forecast of the global atmospheric state, using the present day's state as initial conditions. Since the equations are solved by stepwise integration, forecasts for intermediate ranges are automatically produced, and one-day, two-day, . . ., ten-day forecasts are routinely archived and made available for further research. However, forecasts more than ten days in advance are not generally prepared, and anything like an 18-month time series, comparable to Fig. 1 or 2, is unavailable.

Since the climate of the model differs from that of the real atmosphere, initial states determined from the real atmosphere need not lie on the model's attractor, and, since transient effects may well take more than ten days to die out, not even one point on the model's attractor set is known, let alone an entire attractor. That the model behaves chaotically rather than periodically is best determined by examining it for sensitive dependence on initial conditions.

We have performed a detailed examination of this sort. It would have been computationally expensive to perform many additional runs, in which the operationally used initial states were slightly modified. Instead we have capitalized on the fact that the model produces rather good one-day forecasts, so that the state predicted for a given day, one day in advance, may be regarded as equal to the state subsequently observed on the given day, plus a relatively small error. By comparing the one-day and two-forecasts for the following day, the two-day and three-day forecasts for the day after that, etc., we can determine how rapidly the error grows. Moreover, there are no practical barriers to averaging the results over a large sample of forecasts.

Fig. 7 presents the principal results. Points labeled i,j, where i and j are integers, indicate the globally averaged root-mean-square temperature difference at the 500-millibar level between i-day and j-day forecasts for the same day, averaged over 100 consecutive days beginning 1 December 1984. A 0-day forecast is simply an initial analysis.

The upper curve, connecting points labeled 0,j, for different values of j, therefore measures the model's performance, and indicates how rapidly the difference between two states, one governed by the model and one by the real atmospheric equations, will amplify. The lower curve, connecting points labeled i,j, with j - i = 1, indicates how rapidly the difference between two states, both governed by the model, will amplify.

The lower curve clearly indicates sensitive dependence on initial conditions. Extrapolation of the curve to very small differences suggests a doubling time of about 2.5 days. Detailed forecasting of weather states at sufficiently long range is therefore impractical. However, the difference between the slopes of the two curves indicates

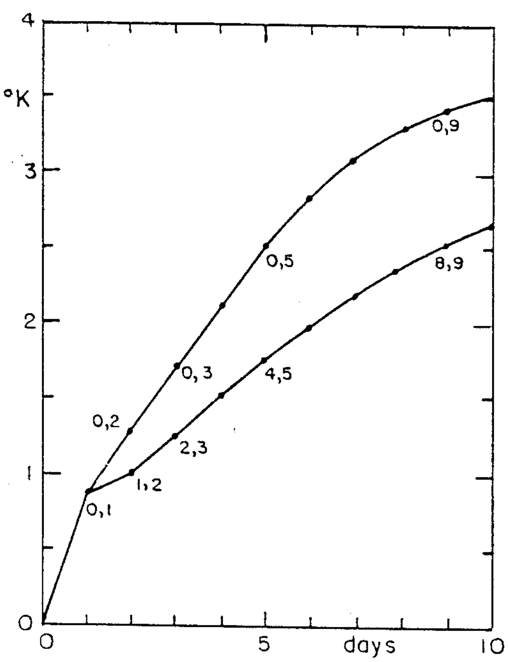

Fig. 7. Root-mean-square differences between i-day and j-day forecasts of the 500-millibar temperature for the same day, made by the ECMWF operational model, averaged over 100 days beginning 1 December 1984. Numbers i, j appear beside selected difference values, which are plotted against values of j.

that there is still considerable room for improvement in forecasting, and implies that we may, for example, some day produce one-week forecasts as good as today's three-day forecasts. Fig. 7 closely resembles a figure constructed from an earlier version of the ECMWF model [11], and both studies tend to confirm the results of earlier studies performed with less elaborate models [12].

Our final example of chaos is the weather itself. In contrast to the case of large atmospheric models, our evidence for chaotic behavior is mainly the absence of any tendency for exact repetitions, and the accompanying presence of continua in the many available variance spectra. We cannot perturb the atmosphere and observe what happens, and at the same time know what would have happened if we had not introduced the perturbation. In principle we could wait for an atmospheric state which closely resembles a previous state, and regard the new state as equal to the old state plus a small perturbation, but in practice we would have to wait too long. We recently estimated that we would have to wait 140 years to obtain one pair of states with a difference of one half of the difference between randomly chosen states [13].

Frequently we observe atmospheric states which closely resemble one another over limited regions; for example, two extratropical cyclones may look very much alike. After a few days the local resemblance will be much weaker, but it is not certain whether this is so because of local amplification or because of the influence of more distant regions where the states are quite different.

Probably our confidence in the chaotic nature of the atmosphere is fortified by the fact that the various large global models exhibit behavior resembling that of the real atmosphere fairly closely, and all of these models show sensitive dependence on initial conditions and agree fairly well as to the rate of error growth. We may also be influenced by our familiarity with baroclinic instability, where perturbed states will depart from unperturbed states.

6. CONCLUSIONS

We may now return to our question as to whether, in investigating atmospheric dynamics, we ought to treat the atmosphere as a deterministic or a chaotic system. The possibly surprising answer is that for most investigations it does not matter. The system of equations which we will be using to study the atmosphere will necessarily involve some approximations, and it may be regarded as a model. Provided that the model is realistic enough to produce a chaotic atmosphere with essentially correct gross features, its behavior will be about the same whether or not it contains some stochastic terms. Here we are assuming that the magnitude of these terms is not completely out of proportion with the actual randomness present in the laws governing the atmosphere.

Our choice between a formally deterministic and a stochastic model will therefore be one of convenience. If our reasoning can be facilitated by the knowledge that our equations contain no randomness, we should use a deterministic formulation. If explicit randomness will aid our investigation, we should introduce it.

As with most general conclusions, there are particular exceptions. If we are studying the growth of the difference between two atmospheric states, using a model in which the smaller scales have been parameterized, and if the initial difference is very small, it will grow quasi-exponentially and require a number of days to become appreciable, if the parameterization is deterministic. With a stochastic parameterization the difference, even if it is initially zero, will quickly become appreciable, possibly during the first day. The latter type of behavior seems more realistic, since it appears that if the small scales could be carried explicitly, uncertainties in these scales would rapidly spread to the larger scales [14], [15]. Once the differences in the resolved scales have become appreciable, it matters little whether the parameterization is deterministic or stochastic.

We are not maintaining that a system of equations with no random terms, and the same system with random terms added, can produce quantitatively identical results. Qualitatively the results may be nearly indistinguishable, or they may be quite different if some of the constants in the system are close to their bifurcation values. In the latter event, the addition of small random terms may still be nearly equivalent to making small alterations in the numerical values of the constants.

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