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ON THE POSSIBLE REASONS FOR LONG-PERIOD FLUCTUATIONS OF THE GENERAL CIRCULATION

by

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Among the many characteristic features of the general circulation of our atmosphere there are some which have so far defied sound theoretical explanation. There are others for which reasonably satisfactory, although not necessarily rigorous, explanations have been given. We shall begin by enumerating three features of the latter type.

The first of these is the existence of a normal, or a seasonal normal, field of motion. If the motion at each point were replaced by the long-period time-averaged motion at the same point, or by the average motion at that point for the particular time of the year, a circulation of appreciable magnitude would still remain.

The second is the existence of instantaneous departures from the normal. If the normal, or the seasonal normal, field of motion were subtracted from the field existing at any particular time, a circulation of appreciable magnitude would also remain.

The third is the aperiodic behaviour of the motion. If the annual and diurnal components and all other real or suspected periodic components were removed from the total motion, a circulation of appreciable magnitude would again remain. Were it not for this lack of periodicity, there would of course be no forecast problem.

In seeking to account for these features, we have been guided by the results of certain related studies. We first turn to the well-known laboratory experiments (Fultz et al. 1959, Hide 1958) in which a fluid contained in a rotating annular vessel is heated at the outer radius and cooled at the inner radius. If the intensities of the heating and cooling, or, more precisely, the temperatures to which the fluid is exposed, are held fixed, and if the rate of rotation is made sufficiently slow, a symmetric steady-state circulation develops. This circulation is unlike that of the atmosphere in that it lacks departures from its long-period average state. If the rate of rotation is made somewhat higher, travelling waves develop, and proceed at a uniform rate without changing their form. This circulation is again unlike that of the atmosphere in that it lacks any departure from periodicity. If, however, the rate of rotation is made still higher, the circulation resembles that of the atmosphere to the extent that travelling waves become superposed upon the time-averaged flow, and proceed in an irregular aperiodic manner.

If the last experiment is regarded as an analogue of the behaviour of the atmosphere, we are led to conclude that the proper explanation for the qualitative features of the atmosphere which we have enumerated may require quantitative considerations. If the earth were

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rotating at a slower rate, it is conceivable that the motion of the atmosphere might vary periodically, or perhaps might not vary at all.

In the case of the various numerical experiments, in which the behaviour of the atmosphere has been represented by solutions of systems of differential equations, the appropriate equations must inevitably be simplified before they can be handled by even the largest existing electronic computers. In some instances, however, they have been simplified to the point where they may equally well be regarded as numerical simulations of the atmosphere or numerical simulations of the laboratory experiments.

In one of the simplest systems of equations (Lorenz, 1963b), when a sufficiently slow rate of rotation is chosen, all solutions are asymptotic to a single steady-state solution. With somewhat higher rotation rates, the steady-state solution is still found, but it is now unstable; nearby solutions recede from it instead of approaching it, and almost all solutions are asymptotic to special solutions which are periodic but not steady-state. Again, with certain still higher rotation rates, the steady-state and the periodic solutions are unstable with respect to still further modes of oscillation, and almost all solutions are aperiodic.

In the numerical model, then, one may say that steady circulation is always a mathematical possibility. Unsteady periodic circulation occurs, when it does, because the steady-state circulation is unstable. Aperiodic circulation occurs, when it does, because all steady-state and periodic circulations are unstable.

The laboratory experiments lead to the conclusion that, even for the higher rates of rotation, a steady-state circulation exists in the mathematical sense, although, because of its instability, it cannot be observed experimentally. If a sufficiently good approximation to the steady-state solution could somehow be introduced into the apparatus, it should persist as a good approximation for some time before finally breaking down. Likewise, we are led to conclude that, for the highest rates of rotation, periodic circulations exist in the mathematical sense, but are unstable. Indeed, an aperiodic circulation, because of its very aperiodicity, may by chance temporarily assume a state nearly compatible with an unstable periodic solution. The system will then undergo nearly periodic oscillations for a while, before the periodicity finally breaks down.

Returning now to the atmosphere, we are led to conclude that, if the solar radiation reaching each geographical location did not vary with time, there would exist a hypothetical steady-state circulation - one which, if approximately established, would remain nearly steady for some time. Because of the annual and diurnal variations of the solar heating, however, we should expect instead a hypothetical circulation possessing only an annual and a diurnal variation. This circulation is of course unstable, and so is not observed in nature. Likewise, we may expect additional periodic circulations, whose periods need not be related to the annual and diurnal periods. These circulations are also unstable, and are not observed. As a result of the instability of all periodic circulations, the observed circulation of the atmosphere is aperiodic.

One might object that we have not really accounted for the qualitative features which we have enumerated, since we have offered no explanation as to why the various hypothetical periodic circulations should be unstable. Certainly we have not shown rigorously that the atmosphere must vary aperiodically, when we have merely used the term "instability," without applying the known theory of instability. Nevertheless, we have succeeded in classifying the phenomena which we are studying into a category with other well-studied instability phenomena, and we have presented an outline which any rigorous explanation will have to follow if it is to succeed.

Having accounted, in a sense, for fluctuations of the general circulation, let us now seek specifically the cause of the long-period fluctuations. We must immediately acknowledge the possibility that there is no additional cause. When a system is varying aperiodically, the mean values and other statistics of successive samples of a given length will themselves vary from one sample to another, and the variations of these statistics will be interpreted as long-period fluctuations. The variance of a sequence of sample means will be further increased, relative to the variance of the unaveraged sequence, by any persistence in the unaveraged sequence, and any system which is varying continuously must have some persistence.

To investigate the possibility that the long-period fluctuations result from chance, let us consider the variations of the index of the zonal westerlies (or simply the zonal index), defined as the space-averaged geostrophic westerly wind speed at sea-level, in the belt between 35°N and 55°N (Rossby, 1939). Among the various proposed indices of the general circulation, the zonal index has probably received the most study, and many years of data are available. Figure 1 shows the daily values for a particular five-month period.

A statistical study of the zonal index by Enger (1957), based upon 25 years of winter data, revealed a serial correlation coefficient of 0.834, at a lag of one day. Moreover, the correlation coefficient fell off nearly exponentially with the lag, i.e., the coefficient at a lag of n days was very close to $(0.834)^n$, for values of n up to twelve. The zonal index was thus revealed as closely approximating a first-order linear Markov process, which, by analogy with the now familiar term "white noise", has sometimes been called "red noise" (Gilman, Fuglister, and Mitchell, 1962).

The "noise" or aperiodic nature of the process is evident in Figure 1, but there are also some apparent regularities, especially the intervals of a month or more when the departure of the index from its long-term mean is almost entirely of one sign. One may reasonably ask whether such regularities would be expected in a typical first-order Markov process, or whether they imply some additional properties not revealed by the linear correlation coefficients.

Accordingly, we have generated a first-order linear Markov process by choosing an initial value X_o and successively applying the generating equation

$$X_{n+1} = 0.834\, X_n + E_{n+1} \tag{1}$$

where the series E_1, E_2... consists of randomly chosen numbers from a normally distributed population, i.e., Gaussian white noise. A "five-month" section of the series X_o, X_1 ... is presented in Figure 2.

The similarities in the qualitative features of Figures 1 and 2, and particularly the tendency for extended durations of one sign, are evident. While it is true that because of the randomness of the series E_1, E_2, ..., virtually any desired curve could have been obtained for Figure 2 by continuing the process for a long enough time, and then selecting a suitable segment, such means were not resorted to; Figure 2 is typical of all the red noise which was actually generated by equation (1).

It thus appears that extended-period fluctuations of the type appearing in Figure 1 and Figure 2 are to be expected in any aperiodic process in which high persistence, as measured by a high serial correlation coefficient at a single lag, is present. The only required physical cause, other than the cause of the aperiodicity itself, is the cause of the persistence.

One might explain persistence by noting that certain features of the atmosphere seem to be rather sluggish, or quasi-conservative, in their behaviour; the extent to which they will vary during a short interval is small compared to the extent to which they can vary when given sufficient time. That is, sluggishness is sufficient to explain persistence, and hence to explain extended-period fluctuations. Whether sluggishness is indeed the cause of the persistence of the zonal index and other atmospheric parameters is another matter. It should be remembered that persistence is not simply a property of short-period changes; it is a comparison of the variability during short periods to the total variability, i.e., the variability during an infinite period. Any additional physical process tending to bring about large changes in the circulation at infrequent intervals will also lead to persistence.

In short, simple observations of the long-period fluctuations are not enough to tell us that anything other than the sluggishness of an aperiodically varying system is at work. At the same time, they do not bar the presence of other identifiable physical causes.

If other physical causes are indeed present, they evidently must fall into one or more of three categories : (1) long-period variations in the extraterrestrial influences upon the atmosphere; (2) long-period variations in the terrestrial environment, which may in turn be influenced by feedback from the atmosphere; (3) physical processes entirely internal to the atmosphere. The long-period variations of the atmosphere may "look the same" regardless of which type of process is acting, but the appropriate procedure for forecasting these variations will very definitely depend upon the type of process.

Consider, for example, the extended numerical integration of elaborate systems of dynamic equations, which has been used to reproduce various features of the general circulation (Leith 1964, Mintz 1964, Smagorinsky 1964), and is now being proposed as an operational method for extended forecasting (Adem 1964). If it is true that aperiodic solar variations are largely responsible for long-period fluctuations of the atmosphere, any system of dynamic equations which regards the "solar constant" as a true constant will fail to predict these fluctuations properly. Likewise, if the fluctuations of the general circulation are largely due to fluctuations of the ocean-surface temperature field, any system of equations using a pre-assigned field of ocean-surface temperature will fail to predict the general circulation properly. It thus becomes important to discover the true nature of the physical causes. We cannot do this simply by observing that long-period fluctuations are present. The ultimate test of the importance of a physical process may well be the ability of a system of equations incorporating this process to predict the long-period fluctuations of the atmosphere.

Among the possible internal mechanisms for the production of long-period fluctuations, one of the most interesting to consider is due to the non-linearity of the governing physical laws. It seems possible that there may exist a number of distinct régimes, such that the general circulation, upon finding itself in any particular régime, tends to become stuck there. That is, the atmosphere may readily progress from one state to another within a given régime, but only occasionally acquire just such a state as to allow it to pass from one régime to another. The changes of régime will then appear as long-period fluctuations.

As a closing remark, a description is given of a procedure for extended-range prediction which may well be an optimum prediction procedure, if the existence of régimes is indeed a fact. The aperiodicity of the variations indicates that if the initial conditions are not precisely known, prediction at a sufficiently long range is impossible, no matter how well the governing laws may be formulated (Lorenz 1963a). Indeed, the aperiodicity may be looked upon as a result of instability with respect to small modifications of the initial conditions. If distinct régimes are present, however, it may be possible to predict the régime, with a reasonable probability of success, at a considerably longer range than that at which one can hope to predict the state within the régime.

The proposed procedure chooses a finite ensemble of initial states, rather than the single observed initial state. Each state within the ensemble resembles the observed state closely enough so that the differences might be ascribed to errors or inadequacies in observation. A system of dynamic equations previously deemed to be suitable for forecasting is then applied to each member of the ensemble, leading to an ensemble of states at any future time. From an ensemble of future states, the probability of occurrence of any event, or such statistics as the ensemble mean and ensemble standard deviation of any quantity, may be evaluated. Between the near future, when all states within an ensemble will look about alike, and the very distant future, when two states within an ensemble will show no more resemblance than two atmospheric states chosen at random, it is hoped that there will be an extended range when most of the states in an ensemble, while not constituting good pin-point forecasts, will possess certain important features in common. It is for this extended range that the procedure may prove useful.

It is hardly necessary to point out that application of this procedure will place a far heavier demand upon the computer than any procedure currently in use. At present its application to even one weather situation appears prohibitive. However, the properties of the method are readily illustrated by applying it to a very simple equation, which need have nothing in common with the system of atmospheric equations except its non-linearity and its ability to generate a statistically stationary series.

Accordingly, a simple cubic difference equation in one variable is chosen, namely

$$X_{n+1} = 0.875 \ (3X_n - 4X_n^3) \qquad (2)$$

Given an initial value X_0, with $|X_0| < 0.875$, Equation (2) will generate a series X_0, X_1, X_2, ..., with $|X_n| < 0.875$. The value of X_n is to be identified with the state of the atmosphere on the n^{th} day. Equation (2) has many features in common with the quadratic difference equation recently studied by the writer (1964).

Figure 3 presents a graph of X_{n+1} against X_n. It is obvious that most values of X_n lead to values of X_{n+1} having the same sign. However, because of the choice of the coefficient 0.875, there is a narrow range of positive values, $0.866 < X_n \leqq 0.875$, for which X_{n+1} is negative, and a similar range of negative values of X_n for which X_{n+1} is positive. Likewise, there is a narrow range, $0.487 < X_n < 0.513$, for which X_{n+1} is positive but X_{n+2} is negative, and a similar range of negative values of X_n for which X_{n+2} is positive. There are still narrower ranges of positive values of X_n, for which X_{n+3}, or X_{n+4}, or some more remote member of the series, is negative. Thus there are two régimes, one consisting of all the positive values of X_n, and one of all the negative values, such that the state easily varies within a régime but only occasionally switches from one régime to the other.

Figure 4 shows four series generated by Equation (2), all beginning with nearly the same initial value. For a while, but not forever, the series are nearly identical. Each series shows the tendency for a régime to persist.

For application of the prediction procedure, the "observed" value $X_0 = 0.800$ has been chosen, and the series X_0, X_1, ... has been generated. An ensemble of initial values, consisting of the 401 numbers ranging from 0.799 to 0.801 at intervals of 0.000005, has then been chosen, and a series of ensembles has been generated.

Table 1 shows the original series, and the series of ensemble means and ensemble standard deviations, up to n = 20. For n = 0, ---, 6, the ensemble means closely resemble the original series, and the standard deviations are small. Here pin-point forecasting is possible. For n = 14, ---, 20, the ensemble means are near zero, and the standard deviations are close to the population standard deviation, 0.54. Here virtually no forecasting is possible. For n = 7, ---, 13, however, the ensemble means, although not agreeing with the original series, still differ significantly from zero, and the standard deviations are definitely less than 0.54. (The cases n = 7 and n = 13 are borderline).

Table 1. Particular solution X of Equation (2), and mean $\bar{X}$ and standard deviation σ of ensemble of solutions of Equation (2), for twenty iterations.

n	X	$\bar{X}$	σ
00	.800	.800	.000
01	.308	.308	.001
02	.706	.706	.004
03	.621	.621	.010
04	.792	.791	.014
05	.340	.341	.057
06	.756	.745	.079
07	.474	.461	.243
08	.871	.586	.231
09	-.028	.520	.305
10	-.075	.418	.345
11	-.194	.333	.364
12	-.484	.276	.435
13	-.874	.194	.498
14	.041	.137	.527
15	.108	.100	.532
16	.278	.054	.519
17	.655	.053	.535
18	.737	.016	.544
19	.534	-.010	.538
20	.869	-.018	.528

Here, then, is the extended range, where useful probability forecasts can be made, even though pin-point forecasting will fail. The occurrence of this range for n = 7, ---, 13, rather than for some other values of n, of course depends upon the standard deviation of the chosen ensemble of initial states.

This simple example, then, illustrates the possible advantages of the proposed procedure, if the real atmosphere indeed possesses sets of states with the properties of régimes. It is hoped that the obvious disadvantage - the consumption of an enormous amount of computer time - may some day be overcome through the continued development of computers.

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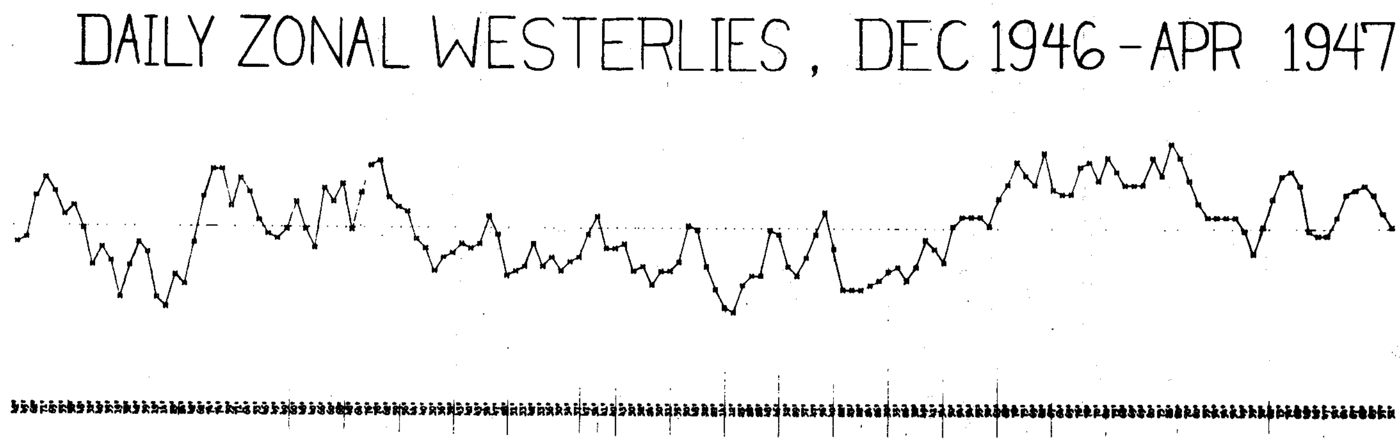

Figure 1 - Daily values of the zonal index for the period December 1946 through April 1947

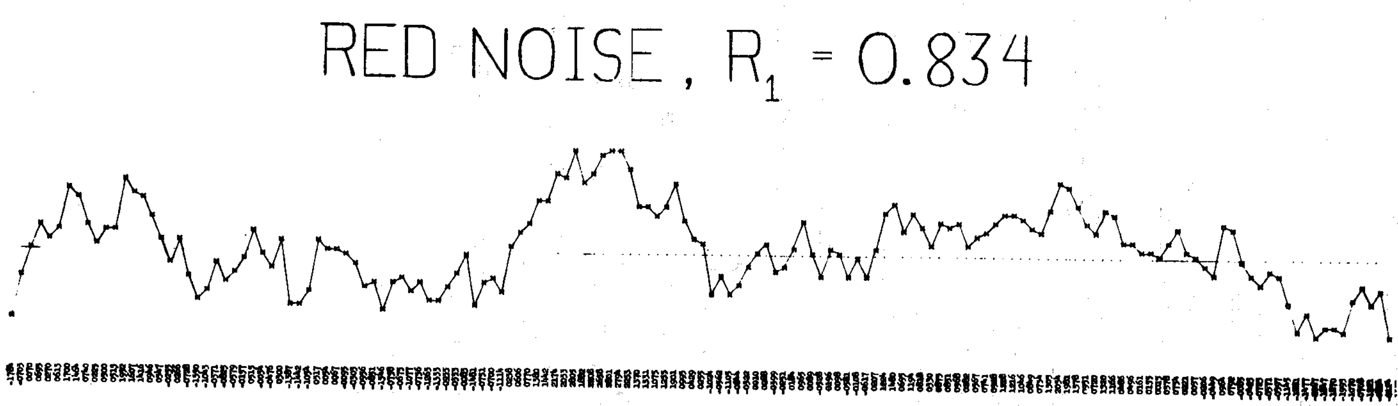

Figure 2 - A segment of an artificially generated first-order linear Markov process, possessing the same serial correlation coefficient as the zonal index shown in Figure 1

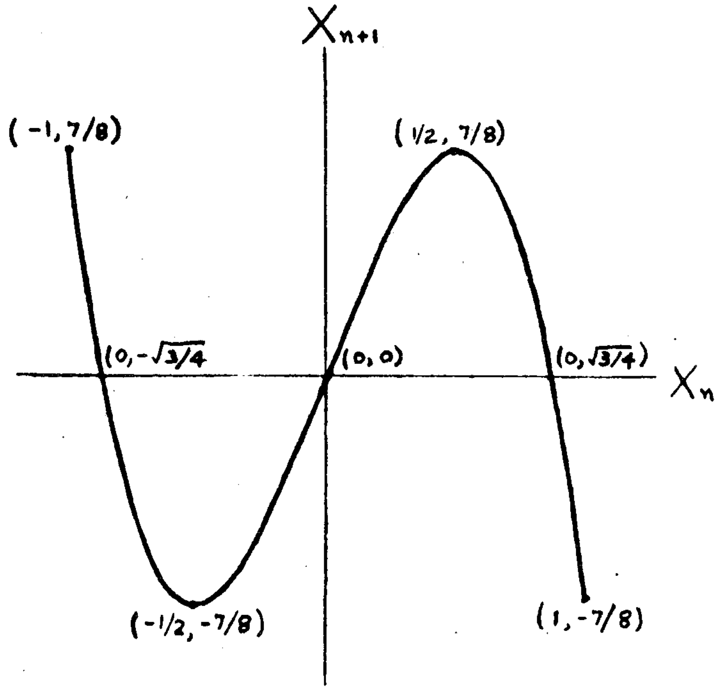

Figure 3

Graph of X_{n+1} versus X_n, as given by Equation (2), showing the co-ordinates of some of the points of principal interest

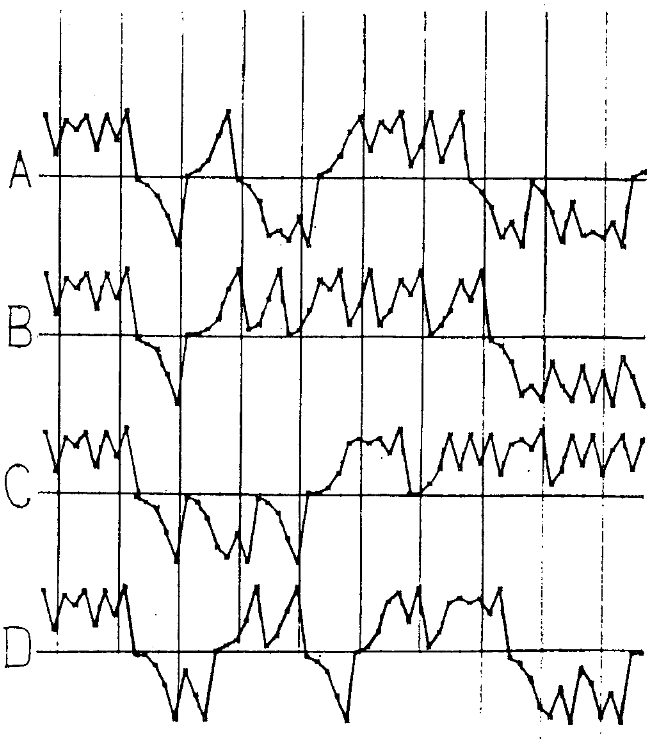

Figure 4

Four time series generated by Equation (2), originating from nearly equal initial values, namely (A) 0.800008, (B) 0.800004, (C) 0.800002, and (D) 0.800001