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A PROPOSED EXPLANATION FOR THE EXISTENCE OF TWO REGIMES OF FLOW IN A ROTATING SYMMETRICALLY-HEATED CYLINDRICAL VESSEL

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The forced flow of a fluid in a rotating cylindrical vessel has been the subject of several recent experimental investigations. In some of these experiments (1, 2) the fluid (water) is heated near the rim of the cylinder and cooled near the center, the distribution of heating being symmetric about the axis. The heating establishes horizontal density differences, and hence horizontal pressure differences, which force the fluid to move relative to the cylinder. Details of the flow at the upper surface, and general features in the interior, are observed by means of tracers.

The experiments reveal the existence of at least two different types of flow patterns. In the "low rotation regime," which occurs with sufficiently low rotation or sufficiently strong heating, the observed flow is symmetric about the axis, and is primarily zonal (horizontal and perpendicular to the axis), with a weak meridional (in planes containing the axis) circulation superposed. In the "high rotation regime," which occurs with sufficiently high rotation or sufficiently weak heating, the flow loses its symmetry, and disturbances resembling those found on upper-level weather maps are superposed upon the zonal flow. The high rotation regime seems to prevail when the Rossby number, which is essentially the ratio of the relative speed at a representative point in the fluid to the absolute speed of a representative point of the cylinder, lies below a critical value.

The purpose of this discussion is to propose a physical explanation for the existence of two regimes of flow, and for the dependence of the regime upon the Rossby number.

In this discussion we shall propose several hypotheses, each dealing with a specific feature of the problem. Along with these hypotheses we shall present pertinent pieces of information. Finally, in summary, we shall assemble these hypotheses to form our proposed physical explanation.

Our fundamental hypothesis, upon which all of our following hypotheses are based, is that symmetric flow is mathematically possible at subcritical as well as supercritical Rossby numbers, in the sense that it satisfies the governing equations and boundary conditions. However, at subcritical Rossby numbers it is unstable, in the sense that small superposed disturbances can grow until they become important parts of the total flow. Therefore, symmetric flow is not observed experimentally at subcritical Rossby numbers.

The disturbances existing at low Rossby numbers usually appear to be so shaped as to transfer angular momentum into the regions where the zonal angular velocity is already greatest, at the expense of the other regions, and hence to increase the kinetic energy of the zonal flow. Computations made by Starr,¹ based upon experiments of Long (cf. Starr and Long (3)), support this idea. A similar situation usually prevails in the atmosphere, as shown observationally by Starr (4).

We therefore propose the hypothesis that at subcritical Rossby numbers the growing disturbances gain their kinetic energy from existing potential and internal energy, rather than

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from existing kinetic energy. They are thus a manifestation of a baroclinically unstable flow, rather than a barotropically unstable flow.

Among the possible procedures which we may use to test these hypotheses, one in particular naturally suggests itself. It consists first of determining analytic expressions for the symmetric flow, using suitable equations and boundary conditions, and then testing the flow for stability. The success of this procedure may be limited by our ability to solve the equations, or by our knowledge of stability of baroclinic flow.

At various times investigators (Arakawa (5), Davies (6), and Oberbeck (7)) have sought simultaneous solutions of the equations of motion, the equation of continuity, and the thermal equation, satisfying boundary conditions appropriate to a rotating heated fluid. Davies' recent investigation applies specifically to Fultz's experiments. In most of these investigations, the nonlinear terms in the equations have been neglected. Davies has considered certain nonlinear terms in the equations of motion, using the method of small parameters, and has shown that these terms are necessary to explain the presence of "easterlies" and "west-erlies" near the bottom of the fluid.

In view of the widespread application of linearized equations to problems of this sort, it is important to consider the exact meaning of their solutions. The motion under consideration is forced by the heating; i.e., with no heating there would be no motion relative to the cylinder. Hence if the solutions of the nonlinear equations are expanded in power series in some parameter representing the heating, the terms independent of the parameter describe the state of no motion just mentioned. The coefficients of the first power of the parameter are precisely the solutions of the linearized equations. The effect of the nonlinear terms is represented by the coefficients of the second and higher powers.

It follows that the solutions of the linearized equations resemble those of the nonlinear equations most closely when the heating is weak, and less closely when the heating is strong. But, for a given rotation, weak heating is precisely the condition under which the symmetric state is not observed experimentally. Thus, perhaps paradoxically, the most easily obtainable symmetric solutions are the ones which are not observed. The observed symmetric solutions are more difficult to obtain, since they involve to a greater extent the higher order terms in the power series, if indeed the series converge at all.

We therefore propose the hypothesis that the symmetric solution of the linearized equations is unstable. The nonlinear terms constitute a stabilizing influence, and stability can exist only when their effect is sufficiently great.

If the flow is zonally symmetric, the principal nonlinear terms in the equations governing the flow represent the transport of momentum and heat by the meridional circulation. Our last hypothesis therefore implies that the stability of the observed symmetric flow results from the meridional transport of some quantity.

The stability of baroclinic flow has recently received much attention, although no completely general criterion for stability has been established. Very recently, several investigators (Eady, (8), Eliassen, (9), Sawyer and Bushby, (10), and Thompson, (11)) have independently derived simplified sets of equations for quasi-geostrophic baroclinic flow. These sets of equations, which are essentially equivalent to each other, were derived for the purpose of numerical weather prediction, but are applicable to the present problem with suitable modifications for lack of compressibility. In addition to prediction, they are suitable for testing the stability of a given flow.

The simplified equations contain, in addition to two dependent variables, a "static stability parameter," which is customarily, but not necessarily, regarded as constant. Besides varying inversely as the static stability (essentially the decrease in density with elevation, for water), this parameter varies directly as the square of the rate of rotation.

There appears to be a simple mechanism through which the static stability can be affected by the meridional transport of heat. The meridional circulation must carry warm fluid from

the rim of the cylinder toward the center in the top layers, and cold fluid from the center toward the rim in the bottom layers. It thus brings about a field in which the temperature increases with height, superposed upon whatever temperature field would exist otherwise.

We therefore propose the hypothesis that heating and rotation both affect the regime of flow principally by affecting the static stability parameter. An increase in heating increases the strength of the meridional circulation, thereby increasing the static stability and decreasing the static stability parameter. The rate of rotation, on the other hand, enters directly into the static stability parameter.

The proposed hypotheses can hardly be convincing unless it can be demonstrated that the values of rotation and heating actually present in the experiments lead to appropriate values of the static stability parameter. In order to investigate this question, we shall attempt to obtain approximate solutions to the appropriate equations, taking into account the nonlinear terms in the thermal equation. Following Davies (6), we shall use the method of small parameters. Since a few terms of a power series do not constitute an exact solution, and since we do not know precisely the boundary conditions in the experiment, we can hardly hope to do more than determine the order of magnitude of the static stability parameter. It therefore appears both convenient and permissible to make numerous approximations during the solution. Our result should in no way be considered an alternative result to that of Davies; it will contain mere approximations to the wind field and the static stability parameter. It will also contain an approximation to the Rossby number, so that the significance of that number may be examined.

We shall use the same primitive equations used by Davies, namely, the Navier-Stokes equations of motion, the equation of continuity, and an equation of heat transfer. These equations appear in standard textbooks on fluid dynamics (12).

Since the flow is assumed to be steady and zonally symmetric, there remain two independent variables, namely

$$\begin{aligned} r &= \text{radial distance from axis of cylinder} \\ z &= \text{vertical distance from bottom of fluid.} \end{aligned}$$

The dependent variables may be taken as

$$\begin{aligned} \rho &= \text{density} \\ p &= \text{pressure} \\ u &= \text{tangential (counterclockwise) velocity component.} \\ v &= \text{radial (inward) velocity component} \\ w &= \text{vertical (upward) velocity component} \end{aligned}$$

The constants entering the problem are

$$\begin{aligned} a &= \text{radius of cylinder} \\ h &= \text{depth of fluid} \\ g &= \text{acceleration of gravity} \\ f &= \text{Coriolis parameter (twice the angular speed of rotation)} \\ \nu &= \text{kinematic viscosity} \\ \kappa &= \text{thermometric conductivity} \\ \rho_0 &= \text{mean density of fluid} \\ \mathcal{E} &= \text{"heating parameter"}. \end{aligned}$$

The parameter \mathcal{E} will soon be more precisely defined.

The number of quantities appearing in the computations may be reduced by introducing dimensionless independent variables, dependent variables, and constants. These quantities, all denoted by capital letters, are defined by the relations

$$\begin{aligned} R &= k_1 a^{-1} r \\ Z &= h^{-1} z \\ \rho &= \rho_0 Q \\ p &= \rho_0 g h P \\ u &= g^{1/2} h^{1/2} U \\ \rho v &= g^{1/2} h^{1/2} \rho_0 V \\ \rho w &= g^{1/2} h^{3/2} k_1 a^{-1} \rho_0 W \\ F &= g^{-1/2} h^{-1/2} k_1^{-1} a f \\ N &= g^{-1/2} h^{-5/2} k_1^{-1} a \nu \\ K &= g^{-1/2} h^{-5/2} k_1^{-1} a \kappa \\ H &= k_1 a^{-1} h. \end{aligned}$$

Here k_1 is the first positive root of J_1 , the Bessel function of order one. The dimensionless quantity

$$T^2 = \frac{1}{2} N^{-1} F = \frac{1}{2} \nu^{-1} f h^2$$

plays an important part. It has been called the rotation Reynolds number, while T^4 has been called the Taylor number.

In terms of dimensionless quantities, the equations to be satisfied will be taken as

$$K \left[\frac{\partial^2 Q}{\partial Z^2} + H^2 \left(\frac{\partial^2 Q}{\partial R^2} + \frac{1}{R} \frac{\partial Q}{\partial R} \right) \right] + \left(V \frac{\partial Q}{\partial R} - W \frac{\partial Q}{\partial Z} \right) = 0 \quad (1)$$

$$\frac{\partial P}{\partial Z} + Q = 0 \quad (2)$$

$$\frac{\partial^2 U}{\partial Z^2} + 2T^2 V = 0 \quad (3)$$

$$\frac{\partial^2 V}{\partial Z^2} - 2T^2 U + N^{-1} \frac{\partial P}{\partial R} = 0 \quad (4)$$

$$\frac{\partial V}{\partial R} + \frac{V}{R} - \frac{\partial W}{\partial Z} = 0. \quad (5)$$

Direct substitution of the dimensionless quantities into the appropriate equations would yield equations considerably more complicated than Eqs. (1) through (5). The additional simplification results from the following assumptions:

1. Density varies linearly with temperature, so that density may replace temperature in the thermal equation, Eq. (1).
2. The dissipation and divergence terms in Eq. (1) are negligible.
3. The equation for vertical acceleration may be replaced by the hydrostatic equation, Eq. (2).
4. The dimensionless Laplacian operator may be replaced by the operator $\partial^2/\partial Z^2$ in Eqs. (3) and (4).
5. The nonlinear terms ² in Eqs. (3) and (4) may be neglected.

Of these assumptions, the last seems to be the most questionable. Indeed, as mentioned previously, Davies has shown the importance of the nonlinear terms in determining certain details of the wind field. It must therefore be remembered that our purpose is not to obtain a complete analytic solution, but merely to investigate the plausibility of our hypotheses. It will appear that our first approximation to the value of the static stability does not depend upon these details of the wind field.

² The decision as to which terms are nonlinear depends upon whether v and w or ρv and ρw are chosen as dependent variables. Here ρv and ρw are chosen.

The simplification of the Laplacian operator in the equations of motion is justifiable if the depth of the fluid is sufficiently small compared to the radius of the cylinder, as is the case in Fultz's experiments. For Hide's experiments, such an assumption would be more questionable.

In agreement with Davies, we shall choose the boundary conditions as

$$U=V=W=0, \quad \frac{\partial Q}{\partial Z} = \mathcal{E} Q'(R) \text{ when } Z=0,$$

$$\frac{\partial U}{\partial Z} = \frac{\partial V}{\partial Z} = 0, \quad W=0, \quad \frac{\partial Q}{\partial Z} = 0 \text{ when } Z=1.$$

Here \mathcal{E} may assume any constant value, and Q' is a specified function of R , determined by the rate at which heat enters the fluid through the lower boundary.

In solving the equations, we shall assume that each dependent variable may be expanded in a power series in \mathcal{E} , so that

$$\Phi = \sum_{i=0}^{\infty} \Phi_i \mathcal{E}^i$$

if Φ represents Q , P , U , V , or W . As mentioned before, the coefficients Φ_0 of \mathcal{E}^0 represent the solution when heating is absent. They are given by

$$Q_0 = 1$$

$$P_0 = P_{\infty} - Z$$

$$U_0 = V_0 = W_0 = 0$$

where P_{∞} is a constant of integration, representing the pressure at the bottom of the fluid.

Because U_0 , V_0 , and W_0 vanish, the coefficients Φ_1 of \mathcal{E}^1 are simply the solutions of Eqs. (1)-(5) with the nonlinear terms omitted in (1). To solve for these coefficients, we must first express Q' explicitly. The simplest choice which expresses heat inflow near the rim and heat outflow near the center is a multiple of $J_0(R)$, where J_0 is the Bessel function of order zero. The particular multiple may be chosen at will, since \mathcal{E} is arbitrary. If

$$Q' = H \sinh H J_0(R),$$

Eqs. (1) and (2) yield the solution

$$Q_1 = \cosh H(Z-1) J_0(R),$$

$$P_1 = [P_{11} - H^{-1} \sinh H - H^{-1} \sinh H(Z-1)] J_0(R),$$

where P_{11} is a constant of integration which expresses the value of P_1 along the bottom of the fluid, and which will be determined by the boundary conditions for U_1 , V_1 , and W_1 .

If H is sufficiently small, these expressions may be replaced by the further approximations

$$Q_1 = J_0(R),$$

$$P_1 = (P_{11} - Z) J_0(R).$$

In determining U_1 , V_1 , and W_1 , it will be assumed that the rotation Reynolds number T^2 is so large that the quantity e^{-T} may be replaced by zero without serious error. It is convenient to introduce the quantities $\alpha = TZ$ and $\beta = T(Z-1)$, whence $e^{-\alpha}$ is negligible except near the

lower boundary, and e^β is negligible except near the upper boundary. With the indicated approximations,

$$U_1 = F^{-1} \left\{ Z + T^{-1} \left[-1 + e^{-\alpha} \cos \alpha - \frac{1}{2} e^\beta (\cos \beta + \sin \beta) \right] \right\} J_1(R),$$

$$V_1 = F^{-1} T^{-1} \left[-e^{-\alpha} \sin \alpha + \frac{1}{2} e^\beta (\cos \beta - \sin \beta) \right] J_1(R),$$

$$W_1 = F^{-1} T^{-2} \left[-\frac{1}{2} + \frac{1}{2} e^{-\alpha} (\cos \alpha + \sin \alpha) + \frac{1}{2} e^\beta \cos \beta \right] J_0(R),$$

while

$$P_1 = -(Z - T^{-1}) J_0(R).$$

In common with Davies' solution, this solution is deficient in that it fails to satisfy a side boundary condition $W=0$ when $R=k_1$, which should perhaps have been imposed. It should be noted that the horizontal portion of the meridional circulation is confined almost entirely to two thin boundary layers, one at the top and one at the bottom, while the zonal flow is quasi-geostrophic except within these boundary layers.

If the entire Eq. (1) is expanded in a power series in \mathcal{E} , the coefficient of \mathcal{E}^2 becomes

$$K \left[\frac{\partial^2 Q_2}{\partial Z^2} + H^2 \left(\frac{\partial^2 Q_2}{\partial R^2} + \frac{1}{R} \frac{\partial Q_2}{\partial R} \right) \right] + \left(V_1 \frac{\partial Q_1}{\partial R} - W_1 \frac{\partial Q_1}{\partial Z} \right) = 0,$$

which is the equation to be solved for Q_2 . If the Laplacian operator is again replaced by $\partial^2/\partial Z^2$, the solution is

$$Q_2 = NK^{-1} F^{-2} \left\{ -Z + T^{-1} \left[-e^{-\alpha} \cos \alpha + \frac{1}{2} e^\beta (\cos \beta + \sin \beta) \right] \right\} J_1^2(R).$$

Outside of the boundary layers, the dimensionless density is therefore

$$Q = 1 + \mathcal{E} J_0(R) - \mathcal{E}^2 F^{-2} NK^{-1} Z J_1^2(R) + \dots$$

The static stability resulting from the meridional circulation first appears in the \mathcal{E}^2 -term. Evidently it varies with R . If $\Delta_z Q$ is defined as the difference between the values of Q at the top and the bottom, averaged horizontally, we find, since the area average of $J_1^2(R)$ is $J_0^2(k_1) = 0.16$, that

$$\Delta_z Q = 0.16 NK^{-1} F^{-2} \mathcal{E}^2.$$

In order to examine the static stability parameter, we introduce simplified equations for quasi-geostrophic baroclinic flow. The equations derived by Thompson (11), when modified to apply to a liquid, become

$$\frac{\partial}{\partial t} \nabla^2 \psi + J(\psi, \nabla^2 \psi) + \frac{1}{3} J(\psi', \nabla^2 \psi') = 0,$$

$$\frac{\partial}{\partial t} (\nabla^2 - M^2) \psi' + J(\psi, (\nabla^2 - M^2) \psi') + J(\psi', \nabla^2 \psi) = 0,$$

where ψ is a stream function for the vertically averaged flow; ψ' is a stream function for the vertical wind shear, expressed as the difference between the flow at the top and the vertically averaged flow, ∇^2 and J are the Laplacian operator and the Jacobian with respect to dimen-

sionless horizontal variables, and M^2 is the dimensionless static stability parameter, which may be called the "stability number". For the system under study,

$$M^2 = 8F^2(\Delta_z Q)^{-1}.$$

Introducing the approximate value of $\Delta_z Q$, we find that

$$M^2 = 50N^{-1}K\mathcal{E}^{-2}F^4.$$

For water of room temperature $N^{-1}K$ is about 0.16, so that

$$M^2 = 8\mathcal{E}^{-2}F^4.$$

The Rossby number R_0 has been presented upon empirical evidence as a criterion for stability. If R_0 is defined as the ratio of the maximum relative speed in the fluid to the absolute speed of the rim of the cylinder, the approximate value of U_1 yields the value

$$R_0 = 0.3 \mathcal{E} F^{-2}.$$

It follows that the Rossby number and the stability number are connected by the relation

$$M^2 = 0.72 R_0^{-2}.$$

Thus a definite value of M^2 implies a definite value of R_0 . We therefore propose the hypothesis that a critical value of R_0 exists because of the relation between R_0 and M^2 .

The problem of determining the critical value of M^2 corresponding to an arbitrary wind field has not yet been solved. Investigations by the writer,³ based upon the properties of the equations for baroclinic flow, indicate that the simple wind field whose components are $\mathcal{E}U_1$ and $\mathcal{E}V_1$ should be stable if $M^2 < 2$, except that it is neutral with respect to certain disturbances of wave number one. Symmetric flows with such disturbances superposed appear as off-center vortices. According to Fultz,⁴ the center of circulation is often observed to be displaced from the center of the cylinder, when the flow otherwise appears to be symmetric.

If $M^2 = 2$ is assumed to be a criterion for stability, $R_0 = 0.6$ must also be a criterion. This value of R_0 is at least of the proper order of magnitude.

There remains the question of introducing numerical values typical of the experiments. In certain of Fultz's experiments, $a = 15$ cm, $h = 2$ cm, $g = 980$ cm sec⁻², and $\rho_0 = 1$ gm cm⁻³. If n is the number of rotations per minute, and τ is the temperature difference in °C between the center and the rim, in the fluid, then $f = 0.21 n$ sec⁻¹, and $\mathcal{E} = 1.8 \times 10^{-4} \tau$. Thus $F^{-2} = 5.8 \times 10^3 n^{-2}$, so that

$$R_0 = 0.31 \tau n^{-2}.$$

The assumed critical value $R_0 = 0.6$ is then obtained, for example, at 1 rpm if $\tau = 2^\circ$ C, or at 2 rpm if $\tau = 8^\circ$ C. Such values are certainly of the observed order of magnitude.

It must be emphasized again, for additional reasons, that we have not presented a nearly exact solution to the problem, in spite of our presentation of numerical values. Wholly aside from the approximations introduced into the equations themselves, the expressions for the wind field and the stability number are approximate in that they consist only of the first non-vanishing terms in power series in \mathcal{E} . The value of \mathcal{E} appropriate to the experiments may lie

³ Unpublished study.

⁴ Verbal communication.

in the range where the series are not well represented by their initial terms. Hence there is further reason for regarding the numerical results as no more than estimates of orders of magnitude.

With this limitation, the numerical results agree with the hypotheses which have been presented, and lead to the additional hypothesis that it is the relation between the Rossby number and the stability number which makes the Rossby number a criterion for stability. If we assemble the hypotheses, we obtain our proposed physical explanation for the experimentally observed phenomena being considered. The proposed explanation is stated in the remaining paragraphs.

"Let a fluid in a rotating cylindrical vessel be heated near the rim and cooled near the center, the heating distribution being symmetric about the axis. There then exists a mathematically possible symmetric flow, which may or may not be observed experimentally, under which the fluid is everywhere in equilibrium. This flow is characterized by a quasi-gradient zonal circulation and a weak meridional circulation. The stronger the heating, the stronger the flow, but, after a certain point, the higher the rotation, the weaker the flow.

"In such a flow the meridional circulation carries warm fluid toward the center in the upper layers, and cool fluid toward the rim in the lower layers, and thus creates static stability. The stronger the heating, the greater the static stability, but the higher the rotation, the smaller the static stability.

"The stability of a baroclinic flow is determined partly by a stability number, which varies inversely as the static stability and directly as the square of the rate of rotation. The flow under consideration is unstable for supercritical values of the stability number, and hence for sufficiently high rotation or sufficiently weak heating. Since an unstable symmetric flow is one where small superposed disturbances can grow until they become prominent, such flows are seldom observed experimentally. Hence for sufficiently high rotation or sufficiently weak heating, an unsymmetric flow is observed.

"The Rossby number depends upon the rotation and the heating in such a way that it is inversely proportional to the square root of the stability number. Hence for subcritical values of the Rossby number an unsymmetric flow is observed."

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