Radiative Equilibrium

• Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
• Radiative heating drives actual state toward state of radiative equilibrium
**Extended Layer Models**

\[ \sigma T_e^4 = \sigma T_2^4 \rightarrow T_2 = T_e \]

**Middle Layer:** \[ 2\sigma T_1^4 = \sigma T_2^4 + \sigma T_s^4 = \sigma T_e^4 + \sigma T_s^4 \]

**Surface:** \[ \sigma T_s^4 = \sigma T_e^4 + \sigma T_1^4 \]

\[ \rightarrow T_s = 3^{1/4} T_e \quad T_1 = 2^{1/4} T_e \]
Effects of emissivity $\varepsilon < 1$

**Surface:** \[ 2\varepsilon_A \sigma T_A^4 = \varepsilon_A \sigma T_1^4 + \varepsilon_A \sigma T_s^4 \]

\[ T_A = \left( \frac{5}{2} \right)^{\frac{1}{4}} T_e \approx 321K < T_s \]

**Stratosphere:** \[ 2\varepsilon_t \sigma T_t^4 = \varepsilon_t \sigma T_2^4 \]

\[ T_t = \left( \frac{1}{2} \right)^{\frac{1}{4}} T_e \approx 214K < T_e \]
Full calculation of radiative equilibrium
Time scale of approach to equilibrium
Contributions of various absorbers

Note: All simulations have variable clouds interacting with radiation.
Problems with radiative equilibrium solution

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)
Missing ingredient: Convection

• As important as radiation in transporting enthalpy in the vertical
• Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer
When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability
Hydrostatic equilibrium

Weight: $-g \rho \delta x \delta y \delta z$

Pressure: $p \delta x \delta y - (p + \delta p) \delta x \delta y$

$F = MA$: $\rho \delta x \delta y \delta z \frac{dw}{dt} = -g \rho \delta x \delta y \delta z - \delta p \delta x \delta y$

$\frac{dw}{dt} = -g - \alpha \frac{\partial p}{\partial z}$, \hspace{1cm} $\alpha = \frac{1}{\rho} = \text{specific volume}$
Pressure distribution in atmosphere at rest

**Ideal gas:** \[ \alpha = \frac{RT}{p}, \quad R \equiv \frac{R^*}{\bar{m}} \]

**Hydrostatic:** \[ \frac{1}{p} \frac{\partial p}{\partial z} = -\frac{g}{RT} \]

**Isothermal case:** \[ p = p_0 e^{-z/H}, \quad H \equiv \frac{RT}{g} = "scale height" \]

Earth: \( H \sim 8 \text{ Km} \)
Weight: \(-g \rho_b \delta x \delta y \delta z\)

Pressure: \(p \delta x \delta y - (p + \delta p) \delta x \delta y\)

\[
F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g \rho_b \delta x \delta y \delta z - \delta p \delta x \delta y
\]

\[
\frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \quad \text{but} \quad \frac{\partial p}{\partial z} = -g / \alpha_e
\]

\[
\rightarrow \quad \frac{dw}{dt} = g \frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B
\]
Buoyancy and Entropy

Specific Volume:
$$\alpha = \frac{1}{\rho}$$

Specific Entropy:
$$s$$

$$\alpha = \alpha(p, s)$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial\alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g \frac{\left(\delta\alpha\right)_p}{\alpha} = g \left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma \delta s$$
The adiabatic lapse rate

*First Law of Thermodynamics:*

\[
\dot{Q} = T \frac{dS_{rev}}{dt} = c_v \frac{dT}{dt} + p \frac{d\alpha}{dt}
\]

\[
= c_v \frac{dT}{dt} + \frac{d(\alpha p)}{dt} - \alpha \frac{dp}{dt}
\]

\[
= \left(c_v + R\right) \frac{dT}{dt} - \alpha \frac{dp}{dt}
\]

\[
= c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}
\]

*Adiabatic:*

\[c_p dT - \alpha dp = 0\]

*Hydrostatic:*

\[c_p dT + gdz = 0\]

\[
\rightarrow \left(\frac{dT}{dz}\right)_s = - \frac{g}{c_p} \equiv -\Gamma_d
\]
\[ \Gamma = \frac{g}{c_p} \]

Earth’s atmosphere:

\[ \Gamma = \frac{1K}{100\, m} \]
Model Aircraft Measurements
(Renno and Williams, 1995)
Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium
Better, but still too hot at surface, too cold at tropopause
Above a thin boundary layer, most atmospheric convection involves phase change of water:

**Moist Convection**
Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping
Phase Equilibria

- Ice
- Liquid
- Vapor
- Critical Point
- Triple Point
When Saturation Occurs...

- Heterogeneous Nucleation
- Supersaturations very small in atmosphere
- Drop size distribution sensitive to size distribution of cloud condensation nuclei
Precipitation Formation

• Stochastic coalescence (sensitive to drop size distributions)
• Bergeron-Findeisen Process
• Strongly nonlinear function of cloud water concentration
• Time scale of precipitation formation ~10-30 minutes
Stability

No simple criterion based on entropy. But air inside ascending cumulus turrets has roughly the same density as that of its environment. It can be shown that neutral stability corresponds to the constancy of the saturation entropy $s^*$:

$$s^* = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right) + L_v \frac{q^*(T, p)}{T}$$
Tropical Soundings

November - February

- $T_v \text{ av}$
- $T_{vp} \text{ av}$

$p (mb)$

(K)

0

100

200

300

400

500

600

700

800

900

1000

150

200

250

300

350
Average density difference between reversibly lifted parcels and their environments, deep Tropics
“Air-Mass” Showers:

- Towering Cumulus Stage
- Mature Stage
- Dissipation Stage
Fig. 15. Radar echo, plane paths, measured draft data, and cell outlines, 1310 EST 9 July 1946.

Fig. 16. Radar echo, plane paths, measured draft data, and cell outlines, 1320 EST 9 July 1946.
Precipitating Convection favors Widely Spaced Clouds (Bjerknes, 1938)
Properties of Moist Convection

• Convective updrafts widely spaced
• Surface enthalpy flux equal to vertically integrated radiative cooling
  \[ M \frac{c_p T}{\theta} \frac{\partial \theta}{\partial z} = -\dot{Q} \]
• Precipitation = Evaporation = Radiative Cooling
• Radiation and convection *highly* interactive
Simple Radiative-Convective Model

Enforce convective neutrality:

\[ T_1 = T_2 + \Delta T, \]
\[ T_s = T_2 + 2\Delta T \]
TOA: \( T_2 = T_e \rightarrow T_1 = T_e + \Delta T, \quad T_s = T_e + 2\Delta T \)

Surface: \( F_s + \sigma T_s^4 = \sigma T_e^4 + \sigma T_1^4 \)

Layer 2: \( 2\sigma T_e^4 = \sigma T_1^4 + F_c \)

Define \( x = \frac{\Delta T}{T_e} \),

\[
F_s = \sigma T_e^4 \left[ 1 + (1+x)^4 - (1+2x)^4 \right],
\]

\[
F_c = \sigma T_e^4 \left[ 2 - (1+x)^4 \right]
\]
Manabe and Strickler 1964 calculation:

[Graph showing temperature, pressure, and altitude relationships with labels for dry adiabatic adjustment and pure radiative equilibrium.]
Effect of Moist Convective Adjustment on Climate Sensitivity
Flux of water by convection makes real problem complex
Effects of Clouds on Radiative Transfer

• Responsible for much of Earth’s albedo
• Important greenhouse effect from longwave absorption and re-emission
Globally Averaged Energy Balance
Radiative-Convective Model

• Band-averaged radiation
• Radiation interacts with H₂O, CO₂, O₃, clouds
• Two-stream approximation: radiation assumed to travel vertically
• Moist convection whenever instability exists
• Convection transports H₂O, enthalpy
• Representation of layered clouds
Open MATLAB
Type “run_model”

• 1) Restart from last run? [n]
• 2) Interactive radiation? [y]
• 3) Interactive clouds? [y]
• 4) Interactive surface temp? [n]
• 5) Time-dependent radiation? [n]
• 6) Date-dependent radiation? [n]
• 7) Diurnal-average radiation? [n]
• 8) Annual-average radiation? [y]
• 9) Surface albedo [0.32]
• 10) Amount of CO2 [360 ppm]
• 11) End time of integration [100 days]
• 12) Graphics averaging time [1 days]
• 13) Frequency of radiation calls [3 hours]
• 14) Frequency of graphics output [2 hours]

• 0) Run model with current configuration.