MOIST CONVECTIVE VELOCITY AND BUOYANCY SCALES

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ABSTRACT

Velocity, buoyancy, and fractional area scales for moist convection in statistical equilibrium with large-scale forcing are derived using the twin constraints of global energy conservation and subcloud-layer thermodynamic equilibrium. The magnitudes of the velocity and buoyancy in moist convective clouds are shown to depend on cloud microphysical properties and on the vertical distribution , but not the magnitude, of the large-scale forcing. Comparisons of the theoretically derived scales with velocity and buoyancy values observed in a numerical ensemble cloud model and in a one-column radiative-convective model show very good agreement. While the theory evidently provides the correct scaling for quasi-equilibrium moist convection, it does not resolve the controversy about whether the well-mixed portion of the tropical subcloud layer is conditionally unstable.

1. Introduction

The net upward mass flux by convective clouds in statistical equilibrium with large-scale forcing is strongly constrained by the thermodynamic balance between subsidence and radiative cooling in the clear air between clouds. In the absence of large-scale vertical motion, this constraint may be written

$$c_p \frac{T}{\theta} \frac{\partial \theta}{\partial z} \sum_i M_i = -\dot{Q}_{\mathrm{rad}}, \tag{1}$$

where c_p is the heat capacity of air, T and θ are the actual and potential temperatures, M_i represents the ith vertical draft in an ensemble of convective updrafts and downdrafts, and $\dot{Q}_{\mathrm{rad}}$ is the radiative heating rate in the clear air. When (1) is coupled with a cloud model that determines the partitioning among up- and downdrafts and the effect of entrainment and detrainment on mass flux, the mass fluxes M_i are determined, though perhaps not uniquely. But while the net upward mass flux is strongly constrained by (1), the partitioning of the mass flux between the fractional area covered by the drafts and their vertical velocity remains to be determined. We can write

$$M_i = \sigma_i (\rho w)_i, \tag{2}$$

where $(\rho w)_i$ is the mass flux inside the draft and σ_i is the fractional area covered by the draft. Knowledge of M_i does not yield individual scales for σ_i and $(\rho w)_i$.

Velocity and buoyancy scales for dry convection in statistical equilibrium with an imposed atmospheric cooling and a fixed surface temperature were derived long ago by Prandtl (1925; see also the review by Emanuel, Neelin, and Bretherton, 1994). This scaling shows that both the velocity and buoyancy scales increase with the magnitude of the forcing.

The lack of knowledge of the correct scaling for velocity and buoyancy in moist convective clouds is a major impediment to progress in understanding the response of convective ensembles to changes in large-scale forcing and in formulating advanced representations of convection in large-scale models. Rennó and Ingersoll (1996) recently proposed an energy method for deducing these scales. Their method predicts that velocity and buoyancy scales increase with the magnitude of the imposed forcing. Robe and Emanuel (1996) performed numerical integrations using a nonhydrostatic cloud model to simulate a field of convective clouds in statistical equilibrium with imposed radiative cooling and surface fluxes. They showed that the buoyancy and velocity scales in the simulated clouds actually remained constant or slightly *decrease* with increasing radiative cooling.

To explain this behavior, we have developed a theory of velocity and buoyancy scales that is similar to those of Rennó and Ingersoll, but which departs from it in several respects.

The theory is presented in section 2 of this report. In section 3 we describe a series of experiments with a one-dimensional radiative-convective equilibrium model and compare the results of these and the numerical experiments of Robe and Emanuel (1996) to the theoretical predictions. These results are discussed and summarized in section 4, which also provides a physical interpretation of the theoretical scales. We also address implications of the theory for the idea of convective neutrality.

2. Convective velocity scales from global energy and subcloud-layer thermodynamic balances

Consider a horizontally homogeneous system in a state of radiative-convective equilibrium. The net radiative cooling of the troposphere is balanced by surface sensible and latent heat fluxes, which are redistributed aloft by dry convection in the subcloud layer and by moist convection in the remainder of the troposphere. We define $\dot{Q}_s$ as the net absorption of radiation by the surface:

$$\dot{Q}_s \equiv SW^+_{\mathrm{net}} \downarrow - \Delta SW - LW^- \uparrow, \tag{3}$$

where $SW^+_{\mathrm{net}} \downarrow$ is the *net* incoming shortwave radiation at the top of the atmosphere (i.e., the solar flux minus shortwave radiation reflected by the earth, atmosphere, and clouds), ΔSW is the reduction of solar radiation owing to absorption by the atmosphere and by clouds, and $LW^- \uparrow$ is the net upward longwave flux at the surface.

The net radiative heating of the atmosphere is given by

$$\dot{Q}_A = LW^- \uparrow - LW^+ \uparrow + \Delta SW, \tag{4}$$

where $LW^+ \uparrow$ is the net upward longwave flux at the top of the atmosphere. In general, $\dot{Q}_A$ is negative. Summing (3) and (4) gives

$$\dot{Q}_s + \dot{Q}_A = SW^+_{\text{net}} \downarrow - LW^+ \uparrow = 0. \tag{5}$$

In equilibrium there is no net radiative flux at the top of the atmosphere, and thus $\dot{Q}_s = -\dot{Q}_A$.

We shall *assume* that the absorption and emission of radiation are thermodynamically reversible processes. Then the net change of entropy in the system owing to reversible processes is given by

$$\frac{1}{g}\int\left(\frac{\partial s}{\partial t}\right)_{\text{rev}} dp = \frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_A}{\overline{\overline{T}}} = \dot{Q}_A\left(\frac{1}{\overline{\overline{T}}} - \frac{1}{T_s}\right), \tag{6}$$

where s is the specific entropy of the system (the entropy of soil or ocean water beneath the surface and the entropy of moist air and cloud in the atmosphere), T_s is the surface temperature, $1/\overline{T}$ is the mean inverse temperature at which radiation is emitted by the atmosphere, the integral is taken over the mass (pressure) of the atmosphere, and g is the acceleration of gravity. Clearly, if $T_s < \overline{T}$, the net production of entropy by absorption and emission of radiation is negative, since $\dot{Q}_s > 0$, $Q_A < 0$. In equilibrium, however, the *net* entropy tendency must be zero:

$$\frac{1}{g}\int\left(\frac{\partial s}{\partial t}\right)_{\text{net}} dp = 0, \tag{7}$$

implying that there exists and irreversible entropy source:

$$\frac{1}{g}\int\left(\frac{\partial s}{\partial t}\right)_{\text{irr}} dp = \dot{Q}_A\left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}}\right). \tag{8}$$

Since $\dot{Q}_A < 0$ in the atmosphere and $T_s > \overline{T}$, this will be a positive quantity. This is simply a statement of the second law of thermodynamics.

Many irreversible processes contribute to the entropy source given by (8). These include convective turbulence in the atmospheric and oceanic boundary layers, moist convective turbulence, dissipation owing to falling precipitation, and irreversible sensible and latent heat fluxes from the surface. For the purposes of this scale analysis, *we shall assume that the majority of dissipation in the system is associated with dry and moist atmospheric convective turbulence.* We shall further assume that the convective turbulence is all in the form of dry convection in the subcloud layer, and moist convection above the subcloud layer.

We next associate irreversible entropy production with dissipation and with buoyancy fluxes. The kinetic energy equation for the atmosphere may be written

$$\frac{\partial}{\partial t}\frac{1}{2}\rho|\mathbf{V}|^2 + \nabla \cdot \frac{1}{2}\rho|\mathbf{V}|^2 = -\mathbf{V} \cdot \nabla p + \rho\mathbf{F} \cdot \mathbf{V}, \tag{9}$$

where ρ is the air density, $\mathbf{V}$ the velocity, p the pressure, and $\mathbf{F}$ is the net frictional force. Integrating over the radiative-convective system in statistical equilibrium gives

$$\int_V [-\mathbf{V} \cdot \nabla p + \rho\mathbf{F} \cdot \mathbf{V}] = 0, \tag{10}$$

where the notation $\int_V$ indicates an integral over the volume of the system. This states simply that in equilibrium, pressure work balances dissipation.

Integrating the first law of thermodynamics over the same volume gives

$$\int_V \rho\dot{Q} = \int_V c_p \left[\frac{\partial \rho T}{\partial t} + \nabla\cdot\rho\mathbf{V}T\right] - \int_V \left[\frac{\partial p}{\partial t} + \mathbf{V}\cdot\nabla p\right]. \tag{11}$$

Again assuming a closed system and statistical equilibrium, this reduces to

$$\int_V \rho\dot{Q} = -\int_V \mathbf{V}\cdot\nabla p = -\int_V \rho\mathbf{F}\cdot\mathbf{V}, \tag{12}$$

in which we have made use of (10). Since, by definition, the dissipation is related to irreversible processes, (11) may be written

$$\overline{T}_{\mathrm{irr}} \int_V \rho \left(\frac{\partial s}{\partial t}\right)_{\mathrm{irr}} = -\int_V \mathbf{V}\cdot\nabla p = -\int_V \rho\mathbf{F}\cdot\mathbf{V}, \tag{13}$$

where $\overline{T}_{irr}$ is the mean temperature at which entropy is produced by irreversible processes. This just states that in a closed system in equilibrium, the irreversible heating equals both the dissipation and the pressure work, which is the rate at which potential energy is converted to kinetic energy.

The pressure work integral may be approximated by an integral of the turbulent buoyancy flux as follows. Divide p into hydrostatic and nonhydrostatic parts:

$$p = \overline{p}_{\mathrm{hy}} + p'_{\mathrm{nh}}, \tag{14}$$

where by definition

$$\frac{d\overline{p}_{\mathrm{hy}}}{dz} = -\overline{\rho}g, \tag{15}$$

and $\overline{\rho}$ is the time- and horizontal-average density. The pressure work term in (13) can then be expanded:

$$-\int_V \mathbf{V}\cdot\nabla p = -\int_V \mathbf{V}\cdot\nabla\overline{p}_{\mathrm{hy}} - \int_V \mathbf{V}\cdot\nabla p'_{\mathrm{nh}}. \tag{16}$$

Using (15), this becomes

$$
\begin{aligned}
-\int_V \mathbf{V}\cdot\nabla p &= +\int_V w\overline{\rho}g - \int_V \mathbf{V}\cdot\nabla p'_{\mathrm{nh}} \\
&= \int_V w\frac{(\overline{\rho}-\rho)}{\rho}\rho g dz + \int_V w\rho g dz - \int_V \mathbf{V}\cdot\nabla p'_{\mathrm{nh}}
\end{aligned}
\tag{17}
$$

The second integral on the right vanishes because, in equilibrium, there is no net vertical mass flux. Thus the pressure work can be expressed

$$
-\int_V \mathbf{V}\cdot\nabla p = \int_V MB - \int_V \mathbf{V}\cdot\nabla p'_{\mathrm{nh}}, \tag{18}
$$

where $M \equiv \rho w$ and

$$
B \equiv g\frac{\overline{\rho}-\rho}{\rho}
$$

is the buoyancy.

Using integration by parts and recognizing that the system is closed, the last term in (18) can be re-expressed so that (18) may be written

$$
-\int_V \mathbf{V}\cdot\nabla p = \int_V MB + \int_V p'_{\mathrm{nh}}\nabla\cdot\mathbf{V}. \tag{19}
$$

The last term on the right of (19) is difficult to evaluate. It represents a correlation between nonhydrostatic pressure fluctuations and three-dimensional velocity divergence, both of which are usually small. For the purposes of deriving appropriate scales, we neglect this contribution and using (19) write the first equality in (13) as

$$
\overline{T}_{\mathrm{irr}}\int_V \rho\left(\frac{\partial s}{\partial t}\right)_{\mathrm{irr}} \simeq \int_V MB. \tag{20}
$$

Finally, making use of (8), we get

$$\int_V MB \simeq \dot{Q}_A \overline{T}_{\mathrm{irr}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}} \right). \tag{21}$$

Note that *three* temperatures appear in (21): the mean temperatures at which radiation is absorbed and emitted, and the mean temperature at which kinetic energy is dissipated into heat. The expression (21) departs from Rennó and Ingersoll (1996) in two important respects: First, the integral on the left is the sum over all the turbulent elements of the buoyancy flux, rather than the flux of undilute buoyancy by an undilute updraft; second, the quantity $\overline{T}_{\mathrm{irr}}$ appearing in (21) was approximated by T_s in Rennó and Ingersoll (1996). This second approximation produces an error of the order of 10%, and we show presently that the first approximation is satisfactory from a scaling standpoint, justifying the approximations made by Rennó and Ingersoll (1996).

In applying (21) to moist convective velocity scales, it must be recognized that the integral on the left side of (21) includes both moist convective fluxes and dry convective turbulence in the subcloud layer. Symbolically, we write

$$\int_V MB = \int_{\mathrm{sc}} MB + \int_{\mathrm{cl}} MB, \tag{22}$$

where "sc" and "cl" denote "subcloud layer" and "cloud layer," respectively. We first estimate the first integral on the right side of (22). For this purpose, we suppose that turbulence in the subcloud layer is generated by radiative cooling of the subcloud layer and by surface fluxes of sensible heat. We also suppose that the turbulent sensible heat flux vanishes at the top of the subcloud layer; that is, we neglect the moist convective flux of sensible heat at cloud base. Observations suggest that this will introduce an error of about 20% in this term. Following through arguments similar to the proceeding, we get

$$\int_{\mathrm{sc}} MB \simeq \dot{Q}_{A_{\mathrm{sc}}} \overline{T}_{\mathrm{irr}_{\mathrm{sc}}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}_{\mathrm{sc}}} \right). \tag{23}$$

Thus the irreversible entropy production in the subcloud layer is approximately balanced by the turbulent buoyancy flux by dry convection. Using this in (22) and (21) gives

$$\int_{\mathrm{cl}} MB \simeq \dot{Q}_A \overline{T}_{\mathrm{irr}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}} \right) - \dot{Q}_{A_{\mathrm{sc}}} \overline{T}_{\mathrm{irr}_{\mathrm{sc}}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}_{\mathrm{sc}}} \right). \tag{24}$$

We now show that in spite of large variations of the moist convective mass flux and buoyancy with height, the integral on the left side of (24) is not too different from the product of the cloud base mass flux and an undilute buoyancy. Start with a hypothetical undilute updraft of mass flux M_u (by definition, constant with height) and undilute buoyancy B_u. Now consider the effect of turbulent entrainment on the buoyancy flux:

$$\delta(MB) = M\delta B + B\delta M. \tag{25}$$

In a *dry* convective plume, buoyancy is linearly mixing, so

$$\frac{\delta B}{B} = -\frac{\delta M}{M}. \tag{26}$$

Thus, in this case, entrainment does not affect the buoyancy flux, and at every altitude $MB = M_u B_u$.

But in a moist convective plume, buoyancy is *not* linearly mixing and can even change sign owing to entrainment. Moreover, entrainment can actually cause M to change sign locally. In addition to this problem, evaporation and melting of precipitation and condensate loading can cause downdrafts with negative mass flux and negative buoyancy. Thus, in general,

$$\sum_i M_i B_i \neq M_u B_u,$$

where the sum is taken over each individual turbulent element.

It is helpful to consider a few special cases. First, consider a profile of radiative cooling that demands a mass flux that increases with height. In this case, much of the entrainment must result in dilute updrafts for which, owing to the nonlinearity of buoyancy mixing from evaporation of cloud water, the buoyancy is always less than that resulting from linear buoyancy mixing. Thus in this case,

$$\sum_i M_i B_i < M_u B_u \qquad \text{Case I.} \tag{27}$$

Now consider a case in which the radiative cooling profile is conducive to a large population of nonprecipitating clouds. These will contain many negatively buoyant, penetrative downdrafts, for which

$$\sum_i M_i B_i > M_u B_u \qquad \text{Case II.} \tag{28}$$

Substituting these into (24) and taking into account the fact that M_u does not vary with height, we get

$$\dot{Q}_A \overline{T}_{\text{irr}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}} \right) - \dot{Q}_{A_{\text{sc}}} \overline{T}_{\text{irr}_{\text{sc}}} \left(\frac{1}{T_s} - \frac{1}{\overline{\overline{T}}_{\text{sc}}} \right) \begin{cases} < M_u \text{ CAPE} & \text{Case I} \\ > M_u \text{ CAPE} & \text{Case II,} \end{cases} \tag{29}$$

where

$$\mathrm{CAPE} \equiv \int_{\mathrm{cl}} B_u \, dz. \tag{30}$$

Thus approximating the total buoyancy flux by the flux of undilute buoyancy by a single updraft will lead to either an underestimate or an overestimate of CAPE, depending on the circumstances.

In order to estimate CAPE using (29), it is necessary to find an expression for the undilute mass flux, here taken to be the net upward mass flux through cloud base. Here we depart substantially from the method used by Rennó and Ingersoll (1996). We close on M_u by using the observation by Raymond (1995) that, in equilibrium, the subcloud layer moist enthalpy budget represents a balance between surface fluxes, radiative cooling, and fluxes out the top of the subcloud layer:

$$\dot{Q}_s + \dot{Q}_{\mathrm{A_{sc}}} + M_u(h_m - h_b) = 0, \tag{31}$$

where h_m and h_b represent the moist static energy of downdraft air, and of the subcloud layer, respectively. Using (5), this can be written

$$M_u = \frac{-\dot{Q}_{\mathrm{A_{cl}}}}{h_b - h_m}, \tag{32}$$

where $\dot{Q}_{\mathrm{A_{cl}}}$ is the radiative heating of the cloud layer. Substituting (32) for M_u into (29) gives

$$\mathrm{CAPE}_p \begin{cases} < \mathrm{CAPE} & \text{Case I,} \\ > \mathrm{CAPE} & \text{Case II,} \end{cases} \tag{33}$$

where

$$\mathrm{CAPE}_p \equiv (h_b - h_m)\left[\frac{\dot{Q}_A\overline{T}_{\mathrm{irr}}\left(\frac{1}{\overline{\overline{T}}} - \frac{1}{T_s}\right) - \dot{Q}_{\mathrm{A_{sc}}}\overline{T}_{\mathrm{irr_{sc}}}\left(\frac{1}{\overline{\overline{T}}_{\mathrm{sc}}} - \frac{1}{T_s}\right)}{\dot{Q}_{\mathrm{A_{cl}}}}\right]. \qquad (34)$$

Note that if the integrated radiative cooling of the subcloud layer is a small fraction of the total integrated radiative cooling, we can approximate CAPE_p by

$$\mathrm{CAPE}_p \simeq (h_b - h_m)\overline{T}_{\mathrm{irr}}\left(\frac{1}{\overline{\overline{T}}} - \frac{1}{T_s}\right). \qquad (35)$$

Also note that we have not really closed the problem, since $h_b - h_m$ must be regarded as part of the solution of the radiative-convective equilibrium problem and cannot be specified as part of the external conditions. As we will demonstrate, the quantity $h_b - h_m$ can take on different equilibrium values for exactly the same radiative cooling distribution, depending on cloud microphysical parameters, which must be considered at least partially as external parameters (depending, for example, on cloud condensation nuclei composition and size distribution). Thus we regard $(h_b - h_m)$ as, to some extent, a surrogate for specification of cloud microphysical parameters.

In the next section, we test (33) against the results of integrations using a radiative-convective equilibrium model.

3. Experiments with a radiative-convective equilibrium model

We employ the single-column model of Rennó et al. (1994), but replace the calculated radiative cooling with a specified cooling rate. This single-column model uses the convective representation of Emanuel (1991), which is an extension of the buoyancy-sorting algorithm of Raymond and Blyth (1986). The cloud base upward mass flux in the model is calculated by a relaxation toward quasi-equilibrium, and the undilute buoyancy is calculated as part of the solution.

In the first set of experiments, we attempt to reproduce the experimental conditions used by Robe and Emanuel (1996) by specifying a constant radiative cooling rate from the surface to 175 mb, and a constant sea surface temperature of 27.2° C at 1024 mb. The model is integrated until a steady state is achieved. Nominal values of the microphysical parameters are used in these experiments.

Figure 1 shows the CAPE calculated from the steady-state thermodynamic profiles of the single-column model, as a function of the radiative cooling rate. Also shown is the corresponding result from integrating an explicit nonhydrostatic cloud model over a domain large enough to contain many clouds, as reported by Robe and Emanuel (1996). Two curves, corresponding to pseudo-adiabatic and reversible ascent, are plotted for the ensemble model integration; in the single-column model there is no ambiguity in the buoyancy calculation. The single-column model and the full ensemble model produce similar values of CAPE, and variations of CAPE over the range of imposed radiative cooling is of order 10%.

The quantity CAPE_p defined by (34) is calculated from the single-column model data as follows: h_b and h_m are taken to be the moist static energy in the subcloud layer (constant with height in this model) and the moist static energy at 650 mb, respectively. This level is usually near that of the minimum moist static energy. In the subcloud layer we take

$$\overline{T}_{\mathrm{irr}_{\mathrm{sc}}} = \overline{T}_{\mathrm{sc}} = (T_0 + T_{\mathrm{LCL}})/2, \tag{36}$$

where T_0 is the surface air temperature and T_{LCL} is the temperature at the lifted condensation level of subcloud-layer air.

In the cloud layer, we recognize that the mean temperature at which dissipation occurs is a function of the degree of mixing and the draft velocities. Thus we expect that the dissipation is weighted toward the upper portions of the cloud layer. For simplicity, we take

$$\overline{T}_{\mathrm{irr}} = (\tilde{T} + T_{\mathrm{LNB}})/2, \tag{37}$$

where T_{LNB} is the temperature at the level of neutral buoyancy for adiabatic displacement of subcloud-layer air. The quantity $\tilde{T}$ is the inverse of the vertically averaged inverse temperature, weighted by the radiative cooling rate.

CAPE_p calculated from (34) using the approximations just described is also plotted in Figure 1. The correspondence with CAPE from the single-column model is striking, considering the approximations used.

To test the inequalities given by (33) we perform two additional experiments with the goal of replicating the conditions of Cases I and II. In the first experiment,

Table 1. Experiments with Linearly Varying Radiative Cooling

Experiment	Cooling profile	CAPE $(\mathrm{J\ Kg})^{-1}$	CAPE $(\mathrm{J\ Kg})^{-1}$ [from (34)]
Case I	Increasing with altitude from 1 K day^{-1} to 4 K day^{-1}	3225	2923
Case II	Decreasing with altitude from 4 K day^{-1} to 1 K day^{-1}	992	1536

the radiative cooling increases linearly with height, from 1 K day^{-1} at the surface to 4 K day^{-1} at 175 mb. This necessitates an increasing upward mass flux with height and thus encourages entrainment into updrafts. In the second experiment, the radiative cooling decreases linearly from 4 K day^{-1} at the surface to 1 K day^{-1} at 175 mb, resulting in a large population of shallow clouds. The results of these experiments are summarized in Table 1.

In the Case I experiment CAPE is greater than CAPE$_p$ by 302 J Kg^{-1}, while in Case II CAPE is less than CAPE$_p$ by 544 J Kg^{-1}, supporting the inequalities (33). This clearly shows the nonlinearity of buoyancy mixing and the inaccuracy of using CAPE$_p$ to predict CAPE when mixing effects are strong. In percentage terms, the theoretical value of CAPE is especially poor when the cumulus population is dominated by shallow clouds, which have strong penetrative downdrafts.

We have argued that the appearance of the quantity $h_b - h_m$ in (34) is a consequence of the sensitivity of the equilibrium state to cloud microphysical processes. To test this idea, we perform a number of experiments in which the microphysical parameters in the convective scheme are varied. Figure 2 shows the variation of CAPE and CAPE$_p$ as well as $h_b - h_m$ with the parameter σ_s, which is the fraction of rain assumed to fall outside of cloud in the convective scheme (see Emanuel, 1991). When σ_s is small, little rain evaporates and the equilibrium state is comparatively dry. Clearly, CAPE and $h_b - h_m$ both depend on assumptions about cloud microphysics, not just the large-scale forcing. Drier atmospheres are more unstable, all other things being equal.

4. Discussion

When moist convection and large-scale processes such as radiation and surface fluxes are in a state of statistical equilibrium, the reversible entropy sink owing to absorption and emission of radiation must be balanced by an irreversible source of entropy, if the radiation is absorbed at a higher temperature than it is emitted. In equilibrium, this irreversible entropy source must equal the integrated dissipation divided by the temperature at which dissipation occurs. This must also equal the rate of conversion of potential to kinetic energy in the system; this in turn is nearly proportional to the integrated buoyancy flux. Thus we can estimate the vertically integrated buoyancy flux if we know the rate of radiative heating, the mean temperatures at which radiation is absorbed and emitted, and the mean temperature at which dissipation occurs.

Knowledge of the vertically integrated buoyancy flux is not sufficient to determine velocity and buoyancy scales individually. Nor can it be assumed without some careful reasoning that the buoyancy flux is equivalent to that carried by an undilute updraft. Indeed, there can be important contributions to energy conversion in convective systems by saturated and unsaturated downdrafts, among other things. We have shown that if downdrafts can be neglected and buoyancy can be assumed to be linearly mixing, then the buoyancy flux by an undilute updraft is the same as that by an entraining plume extending to the same altitude. But in clouds, buoyancy mixing is far from linear, and downdrafts are important. We have shown that entrainment into a saturated updraft reduces the buoyancy flux from that of a nonentraining updraft, while the production of downdrafts of all kinds increases the buoyancy flux. For an undilute updraft, the integrated buoyancy flux is just the cloud base mass flux multiplied by CAPE.

A second relation is needed to find individual scales for buoyancy and mass flux. Here we determine the cloud base mass flux as that required for thermodynamic equilibrium of the subcloud layer. When this relation is combined with the energy constraint described above, the expressions (33) and (34) result. (If the subcloud layer is shallow, (34) reduces approximately to (35).) We interpret (34) as follows.

As found by Rennó and Ingersoll (1996), the greater the thermodynamic efficiency of the system, as measured by the difference between the temperatures at which radiation is absorbed and emitted, the greater the buoyancy flux must be, and this will in general be associated with higher values of CAPE. But we do not find, as they did, that the buoyancy depends on the *rate* of heat input per se. Experiments with a full nonhydrostatic model (Robe and Emanuel, 1996) and with a single-column radiation-convection model show little dependence of buoyancy on the rate of heat input.

We interpret the predicted value of CAPE given by (34) as follows. If the middle troposphere is relatively humid, then the moist static energy gradient is small and, all other things being equal, CAPE will be small. If this is the case, then the static energy of air in downdrafts (including those in clear air outside of clouds) will be relatively large, and to balance the surface entropy flux (which is constrained by the need to balance solar heating of the surface), the downdraft mass flux will have to be relatively large. This implies a relatively large convective updraft mass flux. For the same buoyancy flux, this means that the buoyancy itself must be relatively small.

We wish to point out that determination of the correct scaling for moist convection does little to resolve the issue, raised by Betts (1982) and Xu and Emanuel (1989), of whether the well-mixed portion of the tropical subcloud layer is neutral to adiabatic displacement. Neither the ensemble cloud model of Robe and Emanuel (1996) nor the radiative-convective model described here has enough resolution (or physics, in the case of the radiative-convective model) to resolve the subcloud layer. In nature, it is entirely possible that the air that determines the buoyancy of clouds originates somewhere in the surface layer, while the bulk of the well-mixed portion of the subcloud layer remains nearly neutral to vertical displacement.

To address this issue, we have examined sounding data with 5-mb resolution from the TOGA/COARE project. Figure 3 shows the buoyancy of reversibly displaced air, averaged over a sample of 344 soundings from Kapingamarongi. The small percentage of soundings that exhibited no positive buoyancy at any level have been excluded from the averaging. Problems with the sounding technique render questionable the lowest 100 m or so of data, but it is clear that the interior of the subcloud layer is on average extremely close to neutral. We have made some attempt to correct for known deficiencies of these soundings in the lowest 100 m, and the CAPE of air from the lowest sounding point is about 2000 J kg^{-1}. Above 50 m, CAPE decreases from about 500 J kg^{-1} to close to zero at the top of the subcloud layer. This confirms the findings of Xu and Emanuel (1989), who had to work with soundings with a vertical resolution of 50 mb.

Williams and Rennó (1992) point out that allowing all condensed water to freeze at 0C adds enough heat to raise CAPE to near its pseudo-adiabatic value, but it is unlikely that freezing occurs at such a high temperature in undilute cores. (This does not contradict the finding that most clouds are glaciated at temperatures not far below 0C in the tropics, since only a very small fraction of cloudy air represents undilute ascent.) Moreover, when the entire equilibrium system is considered (not just the updraft branch of the cycle), ice processes can be seen to reduce the available energy since, on average, water freezes at a lower temperature than it melts or evaporates at, so that the latent heat of fusion contributes negatively to the thermodynamic efficiency. If ice does make an important contribution to evaluations of stability, then the finding that air near the top of the subcloud layer is almost precisely neutral to reversible lifting (without freezing) must be viewed as an odd coincidence.

It seems likely that air ascending in cumulus towers originates somewhere within the surface layer, rather than within the interior of the subcloud layer. The CAPE of this air, in tropical soundings in circumstances not far from radiative-convective equilibrium conditions, has values similar to those found here and in the ensemble convective simulations of Robe and Emanuel (1996). We point out that the amount of mass contained in the surface layer is so small that the CAPE of this air makes a negligible contribution to the total available potential energy of the system; were the forcing switched off, convection would deplete the surface layer in about 3 hours. In this view, the surface layer plays the same role in moist convection as it does in dry convection in the boundary layer: its buoyancy is a reflection of the magnitude of the forcing. As its mass is small, it does not represent a significant reservoir of available potential energy. The idea that air inside convective clouds originates in the surface layer receives some support in the observational analysis of Rennó and Williams (1995). They used remotely controlled aircraft to sample the detailed thermodynamic properties of thermals within dry convective boundary layers and found clear evidence that updraft air originates in the surface layer. While this does not directly address the issue of the origin level of air within moist convective updrafts, it is suggestive.

The strong tendency of tropospheric temperature to follow a virtual adiabat originating at the top of the subcloud layer is perhaps analogous to tendency of temperature to follow a dry adiabat in dry convective boundary layers.

5. Summary

We have developed a theory for the buoyancy scales of moist convection in statistical equilibrium with radiation and surface fluxes, based on global energy conservation and subcloud-layer thermodynamic equilibrium. The theory uses an energy constraint similar to that employed by Rennó and Ingersoll (1996), but constrains the mass fluxes in a very different way. The predictions of the theory compare well with the results of radiative-convective equilibrium calculations using an explicit ensemble cloud model and with a single-column model calculations. The amount of buoyancy in moist convective clouds depends on the vertical distribution of the forcing, but not on its magnitude, and is also sensitive to cloud microphysical processes. All other things being equal, relatively humid atmospheres will have relatively small buoyancy in clouds. The theory overpredicts CAPE when downdrafts are relatively important (as in nonprecipitating convection), and underpredicts it when there is much entrainment into cloudy updrafts. Mass conservation demands that for the same vertical mass flux, smaller cloud buoyancy (and thus vertical velocity) be associated with greater fractional areal coverage of updrafts.

The models used in this study are not suited to resolving the structure of the subcloud layer, but observations suggest that the source of buoyant air in convective clouds is the surface layer, leaving the bulk of the well-mixed portion of the subcloud layer nearly neutral to vertical displacement. But clearly, the relationship of the surface layer and the rest of the subcloud layer to convective clouds is a subject deserving of much greater scrutiny.

We leave to future work the dependence of moist convective buoyancy scales on other forms of large-scale forcing, such as mean ascent. The finding that CAPE can fluctuate depending on the dryness of the middle troposphere implies that strict statistical equilibrium of the kind discussed by Emanuel et al. (1994) may not imply a one-to-one relationship between surface layer entropy and tropospheric temperature. This might permit non-WISHE type instabilities of the horizontally homogeneous tropical atmosphere. This will be the subject of future work.

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Figure Captions

Figure 1: CAPE as a function of prescribed radiative cooling rate. The bold solid and dashed curves show reversibly defined CAPE (EM_REV) and pseudo-adiabatic CAPE (EM_PSA), respectively, from lifting air form the lowest model level in a time-averaged sounding from the ensemble cloud model integrations described by Robe and Emanuel (1996). The thin solid line is the CAPE of the equilibrium sounding in a single-column model with parameterized convection. The thin dashed line is the theoretical value of CAPE from eq. (34), using input from the single column model.

Figure 2: Actual (thin dashed) and predicted (thin solid) values of CAPE from the single-column model, as a function of the prescribed fraction of precipitation falling through unsaturated air, σ_s. The thick line at bottom is the 0.1 times the value of $h_b - h_m$.

Figure 3: Contours of the difference between the density temperature of a reversibly lifted parcel and that of its environment, as a function of the parcel origin level (abscissa) and level to which the parcel is displaced (ordinate). This quantity has been averaged over 344 soundings from Kapingamarongi, with soundings showing no positive buoyancy excluded. No freezing occurs in the lifted parcel.