Axisymmetric Slab Boundary Layer Model

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The slab boundary layer is driven by an imposed horizontal pressure gradient which is constant in time and through the depth of the boundary layer, which is in this case assumed to be constant. Horizontal momentum is assumed not to vary with altitude in the boundary layer but may differ from that of air above the PBL. Where air is, in bulk, descending into the boundary layer it is assumed to carry with it the horizontal momentum of air just above the PBL. We here assume that the actual wind is equal to the gradient wind above the top of the PBL. The following differs from most of the literature in retaining horizontal diffusion terms.

For PBl depth h, drag coefficient C_D, Coriolis parameter f, and maximum gradient wind V_m, we choose the following normalization of dependent and independent variables:

$$\begin{aligned}
&v \rightarrow V_m v,\\
&m \rightarrow \frac{V_m^2}{f} m,\\
&u \rightarrow \frac{V_m^2}{f}\frac{C_D}{h} u,\\
&r \rightarrow \frac{V_m}{f} r,\\
&t \rightarrow \frac{h}{C_D V_m} t.
\end{aligned}$$

With these normalizations, the nondimensional slab boundary layer equations are:

$$\begin{aligned}
\frac{\partial u}{\partial t} = \chi\left[\frac{m^2 - m_g^2}{r^3}\right] - \left(v^2 + \frac{u^2}{\chi}\right)^{1/2} u - u\frac{\partial u}{\partial r}\\
+ 2\frac{v}{r}\left[\frac{\partial}{\partial r}\left(Sr\frac{\partial u}{\partial r}\right) - Su\right] + w_e u,
\end{aligned} \tag{1}$$

$$\frac{\partial m}{\partial t} = -\left(v^2 + \frac{u^2}{\chi}\right)^{1/2} rv + \frac{v}{r}\frac{\partial}{\partial r}\left(Sr^3\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) - u\frac{\partial m}{\partial r} - w_e\left(m_g - m\right). \tag{2}$$

Here m_g is the angular momentum associate with the gradient wind, w_e is the entrainment velocity given by mass continuity,

$$w_e = \min\left[-\frac{1}{r}\frac{\partial}{\partial r}(ru), 0\right], \quad (3)$$

the azimuthal velocity is related to angular momentum by

$$v = \frac{m}{r} - \frac{1}{2}r, \quad (4)$$

and the shearing deformation is given by

$$S^2 = \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2 + \frac{2}{\chi}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2\right]. \quad (5)$$

The two nondimensional parameters that appear in (1), (2), and (5) are defined

$$\chi \equiv \left(\frac{fh}{C_D V_m}\right)^2, \qquad \nu \equiv \frac{f^3 l_h^2 h}{C_D V_m^3},$$

in which l_h is the horizontal mixing length in the turbulence diffusion scheme.

At present, we specify the gradient wind from the Emanuel and Rotunno (2011) model:

$$m_g = 2r_m\left[\frac{\left(\frac{r}{r_m}\right)^2}{2 - \frac{C_k}{C_D} + \frac{C_k}{C_D}\left(\frac{r}{r_m}\right)^2}\right]^{\frac{1}{2-\frac{C_k}{C_D}}} \quad (6)$$

This introduces two more nondimensional parameters, the nondimensional radius of maximum gradient wind, r_m, and the ratio of exchange coefficients, $\frac{C_k}{C_D}$.

We integrate (1) and (2) to a steady state using a simple leapfrog scheme with an Asselin filter, and a combination of upstream and centered differencing for the advection terms. The model is initialized by setting the azimuthal wind equal to the gradient winds, and the radial wind to a value given by equating advection with surface drag in the angular momentum equation. The model is contained in the matlab script *pblmodel.m*.