A Test of the Application of Vorticity Charts

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ABSTRACT

A brief description is given of the preparation of charts depicting vorticity and space-mean flow patterns, in a manner similar to that proposed by Fjørtoft (1952). A comparison of the velocity of pronounced vorticity centers and the velocity of the wind over the centers indicated by the space-mean flow pattern is made from a series of daily charts for the autumn and early winter of 1953. It is found that the directions of motion of the centers and of the winds do not differ by more than ten degrees in slightly more than half the cases. A correlation coefficient of +0.69 is found between the speed of the centers and the speed of the winds. Use of the space-mean wind as a method of forecasting the displacement of the vorticity centers appears to be superior to the use of extrapolation. Application of the Rossby wave formula to all ridges and troughs in the space-mean flow patterns yields forecasts of the displacement of these features which are slightly better than extrapolation forecasts.

1. INTRODUCTION

FJØRTOFT (1952) has proposed a graphical method for solution of the simplified vorticity equation

$$\frac{\partial \zeta_g}{\partial t} = - V_g \cdot \nabla(\zeta_g + f), \qquad (1)$$

where $\mathbf{V}_g$ is the horizontal geostrophic wind, ζ_g is the vertical component of relative geostrophic vorticity (hereafter referred to as "vorticity"), and f is the Coriolis parameter. The method is applicable near the 600-mb or 500-mb level, where the assumptions inherent in equation (1) are perhaps least questionable. The ease of application of the method makes possible the routine evaluation in weather stations of the important effects of horizontal vorticity advection. During the autumn and early winter of 1953 vorticity charts based on Fjørtoft's exposition were prepared at M.I.T., from the 0300−GMT 500-mb analyses, and some of the properties of these charts were investigated.

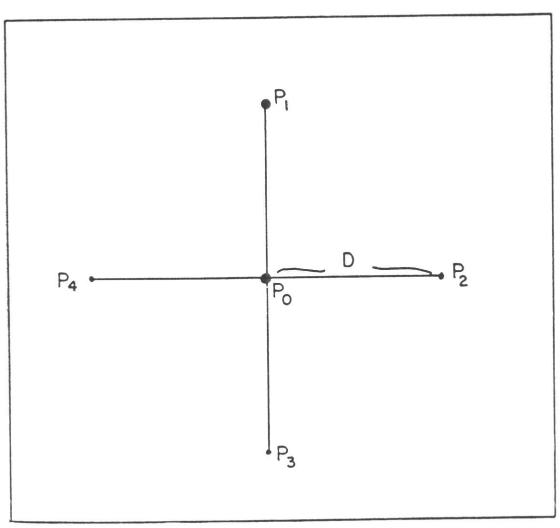

FIG. 1. Grid of points used in the computation of vorticity for the point P_0.

Figure 1 illustrates how a grid of points may be used to evaluate the vorticity ζ_g, given by $\partial v_g/\partial x - \partial u_g/\partial y$. For point P_0 this quantity may be approximated by $\zeta_g = (4g/fD^2)(\bar{Z} - Z_0)$, where Z refers to the height of a constant-pressure surface and $\bar{Z}$ is the average of the heights Z_1, Z_2, Z_3 and Z_4. Fjørtoft recommends that a grid distance D equal to 6 latitude degrees is most appropriate for application to disturbances of synoptic scale, and this distance was employed in the present study. In Fjørtoft's paper determination of the field of $\bar{Z}$ is accomplished by graphical addition of copies of the original 500-mb analysis, but the present in-investigators chose to compute the quantity $\bar{Z}$ at a suitable number of points. A celluloid square, 12 latitude degrees along the diagonals, was laid on the 500-mb chart in such a way that two corners of the square fell on the same contour line. The value $\bar{Z}$ was then determined mentally and plotted on a blank map at the location of the center of the square. These computations were made along adjoining segments of alternate 200-foot contours. Additional computations were then made at points where the $\bar{Z}$ field was not clearly defined. Finally the $\bar{Z}$

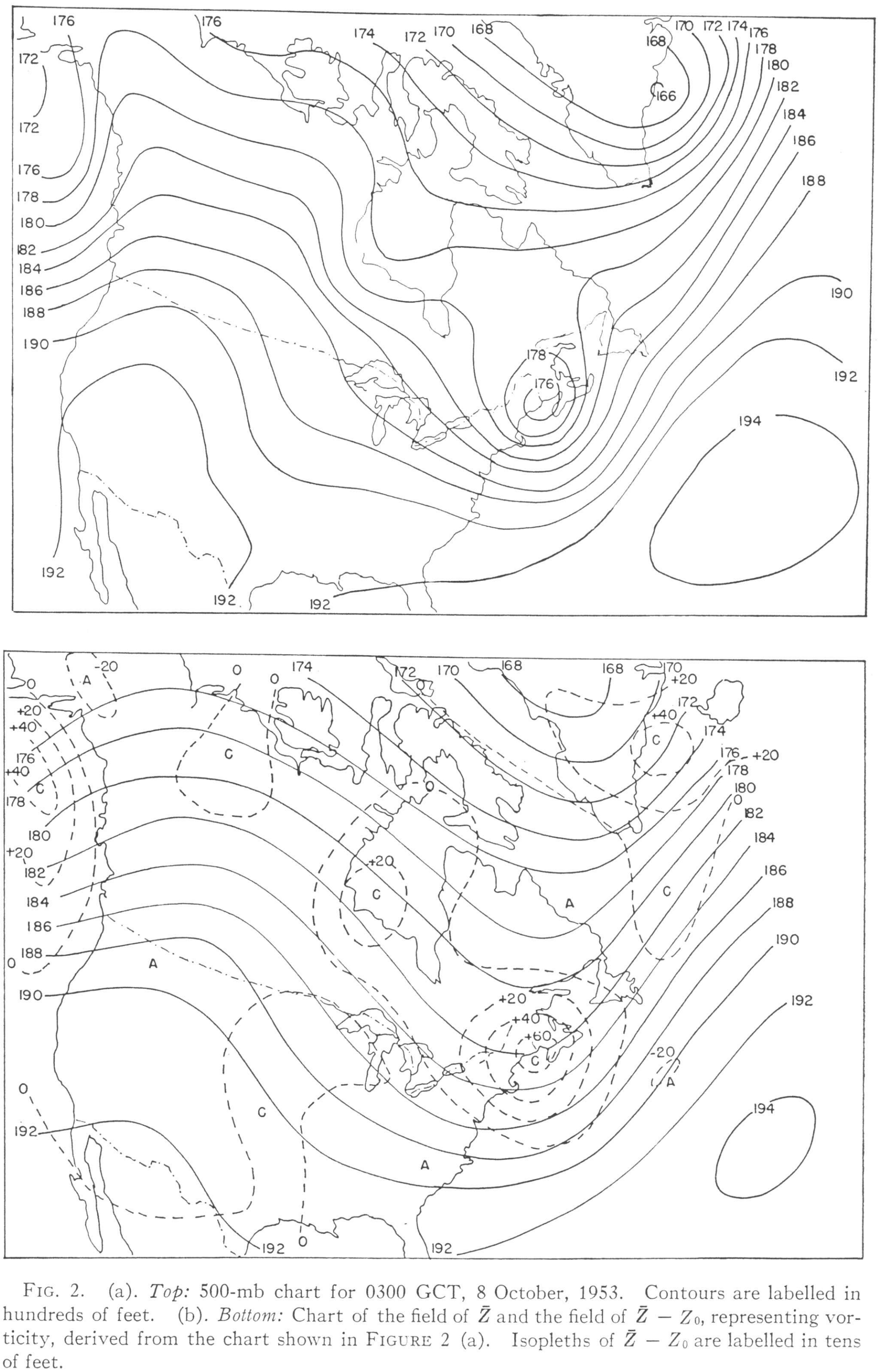

FIG. 2. (a). *Top:* 500-mb chart for 0300 GCT, 8 October, 1953. Contours are labelled in hundreds of feet. (b). *Bottom:* Chart of the field of $\bar{Z}$ and the field of $\bar{Z} - Z_0$, representing vorticity, derived from the chart shown in FIGURE 2 (a). Isopleths of $\bar{Z} - Z_0$ are labelled in tens of feet.

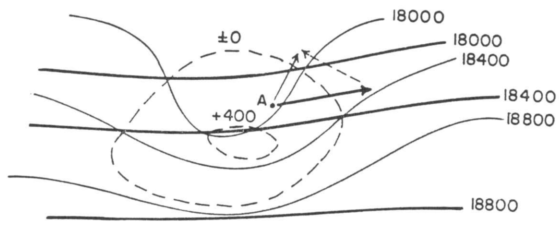

FIG. 3. Schematic representation of the fields of $\bar{Z}$ and $\bar{Z} - Z_0$, superimposed upon the original 500-mb analysis. Thin solid lines are contours of 500-mb height. Heavy solid lines are contours of $\bar{Z}$. Dashed lines are isopleths of $\bar{Z} - Z_0$, representing the vorticity. The thin solid arrow represents the geostrophic wind taken from the original 500-mb contours. The heavy solid arrow represents the wind taken from the $\bar{Z}$ contours. Note that the vector difference between the two is a component parallel to the isopleths of vorticity and hence is not effective in advecting the vorticity. Point A is the origin of the vectors.

chart was analyzed from the plotted values and the field of $\bar{Z} - Z_0$, representing the vorticity, was obtained by graphical subtraction of the original 500-mb analysis from the completed $\bar{Z}$ analysis. A map similar to that in FIGURE 2(b) can be derived from the 500-mb analysis in FIGURE 2(a) in less than one hour.

As shown by Fjørtoft and illustrated in FIGURE 3, the geostrophic wind at 500 mb may be broken down into two components, one along the isopleths of $\bar{Z}$, and one along the isopleths of $\bar{Z} - Z_0$. Since the latter component is not effective in advecting the vorticity pattern, the vorticity advection may be determined directly from the fields of $\bar{Z}$ and $\bar{Z} - Z_0$. The sign and relative intensity of the vorticity advection, moreover, may be quickly assessed by a visual inspection of the intersections of the isopleths of $\bar{Z}$ and $\bar{Z} - Z_0$.

2. A TEST OF THE ADVECTION OF VORTICITY CENTERS

If variations in the Coriolis parameter are neglected, equation (1) implies that the vorticity pattern should move with the wind at the level of non-divergence, which is taken to be near the 600-mb or 500-mb level. The concept was tested by comparing the 24-hour average velocity of centers of $\bar{Z} - Z_0$ observed on the M.I.T. charts at the 500-mb level with the average of the geostrophic winds taken from the $\bar{Z}$-field over the centers at the beginning and end of the 24-hour period. Only cyclonic vorticity centers were chosen in which the central value of $\bar{Z} - Z_0$ was +400 feet or greater at either the beginning or end of the period, in order to eliminate cases of doubtful continuity. A few anticyclonic vorticity centers were included in which the central $\bar{Z} - Z_0$ value was −250 feet or less at either the beginning or end of the period. In determining the vorticity pattern and the wind field effective in moving it some small terms in Fjørtoft's development, representing the effects of variations in the Coriolis parameter, were neglected.

FIGURE 4 gives a comparison of the observed direction of motion of the centers and the mean $\bar{Z}$ geostrophic wind over the centers. It should be noted that the relationship is quite good. In slightly more than half the cases the difference between the two directions was not more than 10 degrees. In a number of instances in which the $\bar{Z}$ flow was west-northwesterly while the motion of the vorticity center was north-northwesterly, trough development was occurring in response to upstream ridge intensification, similarly to the case described by Austin (1954). Except for these cases there seems to be little bias in the relationship.

FIGURE 5 illustrates the relationship between the observed speed of the vorticity centers and the speed of the $\bar{Z}$ wind. The correlation coefficient between the quantities is 0.69 and seems quite encouraging in view of the difficulty of relating the speed of weather disturbances quantitatively to the steering speed of the upper flow. The data indicate that the vorticity centers move with a speed less than the wind

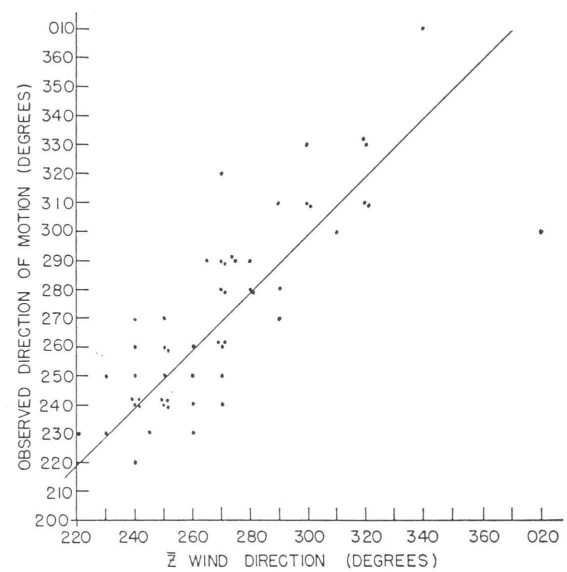

FIG. 4. Comparison of observed mean 24-hour direction of motion of vorticity centers and mean direction of $\bar{Z}$ wind over the centers.

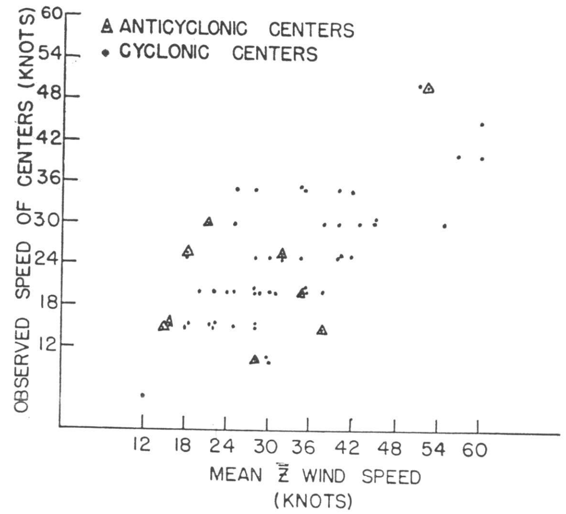

FIG. 5. Comparison of mean 24-hour speed of vorticity centers and mean speed of $\bar{Z}$ wind over the centers.

speed at 500 mb, in agreement with the findings of Cressman (1953). In order to find whether this bias depended on the direction of motion of the centers, the cyclonic centers were subdivided into three categories based upon their direction of motion. A comparison of the mean speed of the centers and the corresponding mean $\bar{Z}$ wind speed for each category is given below:

Centers moving from a direction	No. of cases	Mean $\bar{Z}$ wind speed (kts)	Mean speed of centers (kts)
<260	18	35	28
260–280	13	31	22
>280	16	31	22

It may be seen that the bias is present in all categories, and that centers moving from a southwesterly direction tend to move faster and to be associated with a stronger $\bar{Z}$ wind than other centers.

For 19 cases in which a cyclonic vorticity center and one case in which an anticyclonic vorticity center would be followed for a 48-hour period, a comparison was made of the errors incurred by using extrapolation of the preceding 24-hour observed motion as a basis of forecasting the future 24-hour motion of the centers. These errors were compared with those incurred by using as a forecast basis the wind taken from the $\bar{Z}$ contours at the position of the vorticity center at the beginning of the 24-hour forecast period. In the latter instance it was predicted that the centers would move in the direction of the $\bar{Z}$ contours and with three-quarters of the geostrophic speed measured from the contours. With respect to direction, the average error in the extrapolation forecast was 34 degrees, and in the $\bar{Z}$-wind forecast 25 degrees. The latter forecast was better on eleven occasions, while the former was better seven times.

With respect to speed, the average error in the extrapolation forecast was 8.8 knots and in the $\bar{Z}$-wind forecast 6.3 knots. The latter forecast was better twelve times and the former five times. The average vector error in the extrapolation was 17 knots, and it was superior in this respect four times, while the forecast on the basis of the $\bar{Z}$ wind incurred an average vector error of 12 knots and was superior thirteen times.

It should be noted that the forecast based on the $\bar{Z}$ wind could probably be improved by making an estimate of this wind over the center at the end of the forecast interval.

3. A Test of the Applicability of the Rossby Wave Formula

The present investigators felt that Rossby's (1939) wave formula, derived on the assumption of the conservation of absolute vorticity, might be more readily applied to the relatively smooth wave patterns observed in the $\bar{Z}$ flow pattern than to the complex patterns usually observed in the 500-mb analysis itself and that it might be used to predict changes in the positions of ridges

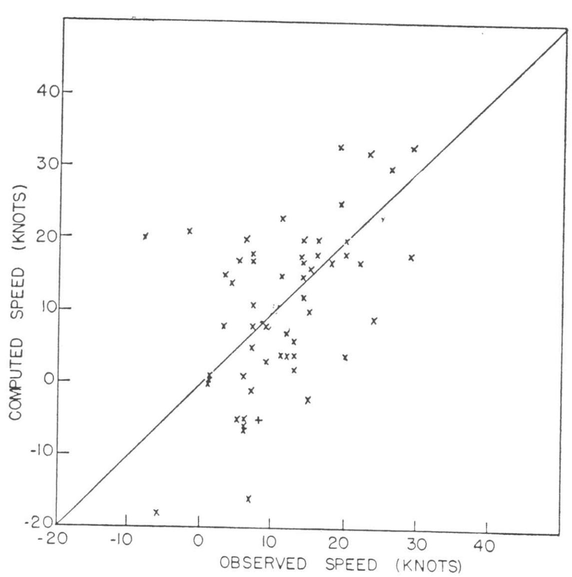

FIG. 6. Comparison of observed mean 48-hour speeds of troughs and ridges in the $\bar{Z}$ pattern and the speed at the middle time computed from the Rossby formula.

and troughs in the $\bar{Z}$ pattern, thus improving the forecast displacement of the vorticity pattern. The Rossby formula is $C = U - \beta L^2/4\pi^2$, where C is the speed of the trough or ridge, U is the speed of the zonal current, L is the wavelength, and β is the northward rate of variation of the Coriolis parameter. The formula was applied only to those troughs and ridges which could be identified on the day preceding and the day following the day for which the computation was made. No effort was made to restrict the computation to nearly sinusoidal patterns. U was taken to be the zonal wind speed measured from the $\bar{Z}$ charts, averaged longitudinally over a half wavelength or, when available, a full wavelength of the long-wave pattern and averaged meridionally over a distance of fifteen latitude degrees, centered at the inflection points of the pattern.

For this sample of cases a comparison was made between the error incurred when the speed given by the formula was used as a basis of forecasting the displacement for the following 24 hours, and the error incurred by using extrapolation, that is, by assuming that the speed for the following 24 hours would be the same as that observed in the preceding 24 hours.

A comparison between the trough or ridge speeds computed from the formula and the mean of the observed speeds for the preceding and following days is shown in Figure 6. The correlation coefficient between the two quantities is 0.53. It is noted that when the speed computation is much higher than the average of about 15 knots, the wave tends to move more slowly than the computation and vice versa. Retrogression was indicated by the formula nine times. Of these times it occurred only once. Two additional cases of retrogression occurred when the formula indicated a substantial progression.

For this sample of cases, a comparison was made between the error incurred when the formula was used to predict the speed for the following 24 hours and the error incurred by assuming that the speed for the following 24 hours would be the same as for the preceding 24 hours. The average error in the formula forecast was 8.0 knots, while the average error in the extrapolation forecast was 9.6 knots. Of the 56 cases, the Rossby formula was more accurate 29 times; extrapolation was superior 25 times, and equal errors were incurred twice. Thus there seems to be little to choose between extrapolation and the Rossby computation as a means of predicting changes in the $\bar{Z}$-chart.

Acknowledgments

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