Experiments in Barotropic Hurricane Track Forecasting

FREDERICK SANDERS1

National Hurricane Research Laboratory, ESSA, Coral Gables, Fla.

AND ROBERT W. BURPEE

Massachusetts Institute of Technology, Cambridge (Manuscript received 15 December 1967)

ABSTRACT

The barotropic prediction model, applied in a way appropriate to the character of tropical circulations and of the information available to describe them, is found to be capable of providing a basis for a significant advance of the state of the art of hurricane track forecasting in the range from 24–72 hr in regions of relatively dense rawinsonde data coverage. The distinctive features of the technique are application of the barotropic equation to tropospheric mean data computed from information at 10 constant-pressure levels, prognostic use of a stream function derived from direct analysis of the mean wind field, and numerical calculation over a grid with relatively small mesh length, without separation of the tropical vortex from the residual flow.

A series of test forecasts for hurricanes Donna 1960, Carla 1961, and Flora 1963 illustrates the performance of the model in dealing with storms characterized by a wide variety of behavior. The most prominent failure, a premature forecast of the recurvature of Donna, is found to be attributable to the baroclinic filling of a trough off the east coast of the United States.

1. Introduction

In view of the success achieved in the application of dynamic methods to the prediction of daily flow patterns in middle and high latitudes, it is surprising that the current standard for objective hurricane track forecasting, and indeed the only operationally viable method, is statistical (Miller and Chase, 1966). The limited success of barotropic and baroclinic models in the tropics has left the impression that the physics of motion of tropical vortices is at best extremely complex and at worst quite mysterious. We wish to reexamine this question under the guidance of the working hypothesis that the simplest conceivable physical process is appropriate, provided that the data are handled with sufficient care; and we wish to propose an objective methodology suitable for operational application which will produce results at least competitive with statistical forecasts.

The analysis of synoptic data in the tropics presents formidable problems. The errors in radiosonde pressure-heights are comparable in magnitude with the true variability of the pressure field, a circumstance which suggests that the analysis of the flow pattern must be based almost exclusively upon the wind observations.

Yet tropical wind data present particular problems. Absent is the strong and systematic vertical wind shear characteristic of the higher latitudes as illustrated in Fig. 1. The erratic variability in the tropical wind sound-

ing is not small compared with the wind itself. Now the success of the equivalent barotropic model in 500-mb prediction poleward of the tropics is probably due in fair measure to the tendency of the vertical shear to parallel the tropospheric mean wind and to the tendency for the vertical shear profile in individual air columns to resemble the climatological average. These tendencies seem to be present to a much lesser degree in tropical data, with the results that only slight skill has been shown in barotropic forecasting for a particular pressure level, and that this skill is about the same for a number of tropospheric levels (Vederman et al., 1966). Consequently, we are led to consider that forecasting should be carried out for the tropospheric mean flow itself and that the mean flow should be determined by a suitable averaging of the synoptic tropospheric data rather than by selection of a single, presumably representative, level.

We will regard the flow as governed by the simple form of vorticity equation,

$$\frac{\partial \zeta}{\partial t} = -\mathbf{V} \cdot \mathbf{\nabla} \eta + f \frac{\partial \omega}{\partial p},\tag{1}$$

where η is the absolute vorticity. Upon averaging this equation over the depth of the troposphere, say from 1000 to 100 mb, we find for the mean flow

$$\frac{\partial \bar{\zeta}}{\partial t} = - \, \overline{\mathbf{V}} \cdot \mathbf{\nabla} \bar{\eta} - \overline{\mathbf{V}' \cdot \mathbf{\nabla} \eta'} + f(\omega_{1000} - \omega_{100}), \tag{2}$$

¹ Currently on leave from Massachusetts Institute of Technology.

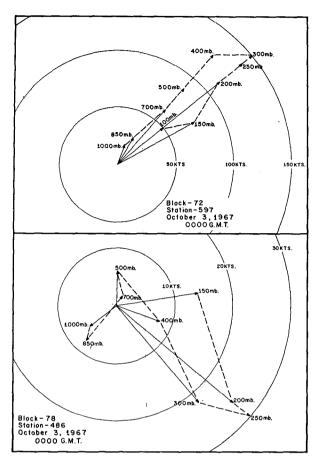


Fig. 1. Hodographs for typical wind soundings: upper, in the middle-latitude westerlies; lower, in the tropics.

where the overbar denotes the vertical average and the prime a deviation from this average. The first two terms on the right side of (2) represent the vertical average of the vorticity advection. In the equivalent barotropic model these terms are perfectly correlated, and middle latitude climatological wind profiles indicate that the second term is about one-quarter the magnitude of the first. The erratic character of vertical wind profiles in the tropics makes this modeling approximation unappealing and encourages us to hope that the second term is not large compared to the uncertainty in measuring the first. In this light, and in the interest of physical simplicity of the model, we shall neglect it. The effect of divergence upon the averaged flow, as represented by the third term on the right side of (2) is perhaps an order of magnitude smaller than the comparable effect upon the flow at a particular level because of the relative smallness of the mean divergence. Perhaps it, too, can be neglected in the short-range prediction of features of synoptic scale, whether in the tropics or in higher latitudes. We have then a rationale of sorts for attempting to predict vertically averaged tropical flow patterns on

the basis of the simple barotropic equation,

$$\frac{\partial \bar{\zeta}}{\partial t} = -\bar{\mathbf{V}} \cdot \nabla \bar{\eta}. \tag{3}$$

Hurricane track forecasting poses an additional problem due to the relatively small size of the phenomenon. Kasahara (1957), among others, has chosen to deal with this problem by separating the hurricane vortex from a residual basic current and calculating explicitly the interactions between the two. This approach, while yielding useful qualitative insights, appears to leave some ambiguity of interpretation and has not led to satisfactory quantitative prediction. We have chosen instead to follow Birchfield (1960) by selecting a relatively small grid distance and, in effect, calculating implicitly the interaction between the vortex and its environment. While our choice of a 165-km mesh length is still not sufficiently small to permit accurate representation of the intensity of the hurricane, we are encouraged by Kasahara's (1957) finding that the motion of the vortex is influenced by its size but not by its intensity.

2. Preparation of data and diagnostic calculations

The first step is the calculation of an appropriate vertical average of wind and pressure-height data for all synoptic observations in the area bounded approximately by 7N and 55N latitude and by 40W and 120W longitude. In a series of tests for hurricanes Carla 1961 and Flora 1963, King (1966) compared forecasts prepared from 500-mb data alone, from a linear combination of data at 1000,700, 500 and 200 mb which employed Birchfield's (1960) weighting coefficients, and from data at all 10 mandatory levels between 1000 and 100 mb combined on the assumption of linear variation between pressure levels. The last combination yielded the best forecasts, while the 500-mb data alone yielded the worst. Ahn (1967) added to the data at the mandatory pressure levels wind information from an additional 18 elevations between 1 and 16 km, but found in a series of forecasts for Flora no significant difference from the results obtained from use of the data at mandatory levels alone. Thus, it appears that the 10 mandatory levels represent in this context an optimum vertical sample.

These vertically averaged data provide the basis for a manual analysis of isogons and isotachs. A sample of the averaged wind data is shown in Fig. 2 along with the corresponding 500-mb data. The former display less station-to-station variability than the latter; note, for example, the strong northeasterly wind at Jacksonville in the 500-mb data, in this instance the highest speed in the entire tropospheric sounding, and the light southwesterly wind at 500 mb at Swan Island, representative of a shallow layer between from about 18,000 to 25,000 ft in an otherwise generally northeasterly sounding. Aside from these evidences of unrepresentativeness of single-level data due to aberrations of presumably small

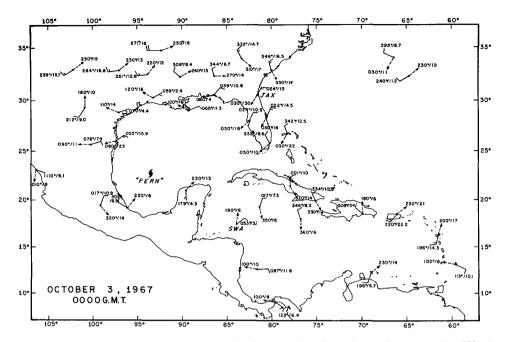


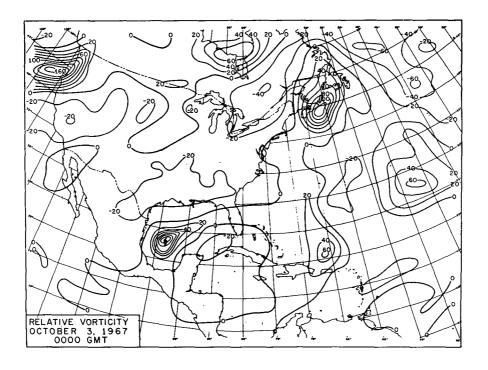
Fig. 2. A sample of vertically averaged wind data, in conventional notation, with corresponding 500-mb data, indicated by the dashed arrow head. The direction and speed of both winds at each station is given in degrees and knots. The positions of hurricane Fern and of the stations at Jacksonville, Fla. (JAX) and at Swan Island (SWA) are shown.

scale, there are differences between the two sets of data on a somewhat larger scale. At almost all stations bordering the Gulf of Mexico there is a stronger southerly or a weaker northerly component in the 500-mb winds than in the averaged ones. (It seems likely that forecasts for the track of hurricane Fern, whose initial position is shown in Fig. 2, prepared from the two sets of data would differ significantly.) Throughout higher latitudes, on the other hand, the averaged data strongly resemble the 500-mb data.

The subjective analysis depends strongly on the averaged pressure-height data over the Atlantic Ocean poleward of about 30N, less strongly over Canada where wind observations are relatively sparse, and almost not at all elsewhere. Over the United States the wind data themselves are quite adequate, while over the subtropical oceans we know of no better guidance than a climatological estimate, except where reconnaissance and other aircraft provide some information. In the vicinity of the hurricane, subjective modeling of the wind field is a necessity when reconnaissance or conventional data offer no better estimate.

Input for the computations is wind direction and speed taken from the analysis at a 35×49 array of grid points with a mesh length of 165 km. In the initial diagnostic phase of the calculations, fields of relative vorticity ζ and divergence, div V, are obtained by use of simple centered finite differences. Fig. 3 illustrates a sample result, in which hurricane Fern appears as a

small intense maximum of vorticity and an equally prominent center of convergence. The latter characteristic, as will be seen, is spurious and arises from finite differencing across the small region of strong winds associated with the storm. In general, the vorticity pattern displays three to four times the peak magnitudes and a less choppy pattern than we see in the divergence field. The latter, in fact, affords an opportunity to assess the reliability of the wind analysis, since the true divergence of the averaged wind should be extremely small. If we generously estimate, for example, that the difference in the values of ω in an air column between the 1000- and 100-mb levels is 9 mb (3hr)-1, then the true value of the mean divergence is 1×10^{-6} sec⁻¹. Yet, numerous values an order of magnitude larger appear in the computed divergence field shown in Fig. 3 and must represent error. Since there is no reason to suppose that comparable errors do not also exist in the vorticity calculations, we can regard these latter values as subject to an uncertainty approaching 10×10^{-6} sec⁻¹, representing at least 10% of the computed vorticity value. It is reasonable to suppose further that calculated vorticity advections, which form the basis of the prognosis, are subject to a comparable uncertainty. Now the second and third terms on the right side of Eq. (2), effects which could in principle be computed in a baroclinic model, are probably no larger than the uncertainty in measuring the first, barotropic, term. While this argument may be simplistic, it seems doubtful that the data are good



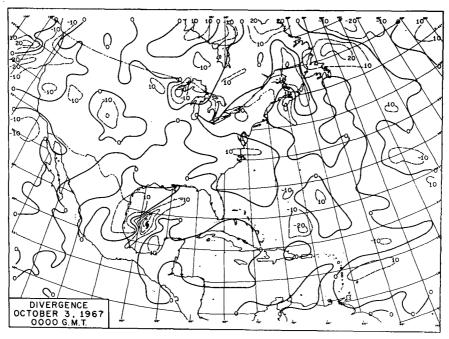
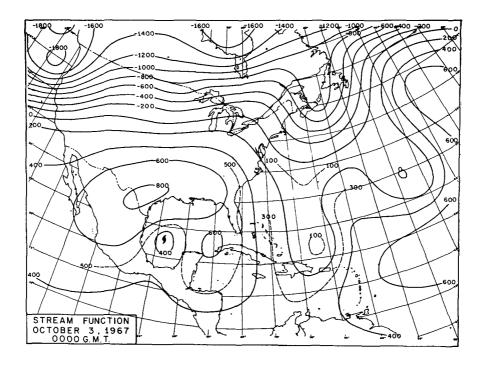


Fig. 3. Fields of relative vorticity, upper, and horizontal divergence, lower, for the same time as Fig. 2. The isopleths are labeled in units of $10^{-6}~\text{sec}^{-1}$ and the position of hurricane Fern is shown.



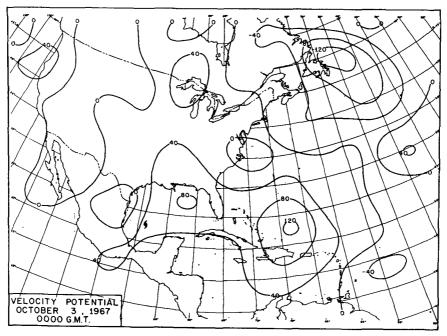


Fig. 4. Fields of stream function, upper, and velocity potential, lower, for the same time as Fig. 2. The isopleths are labeled in units of $1.65\times10^4\,\mathrm{m^2\,sec^{-1}}$. The solid line interval for stream function and for velocity potential is 200 units and 40 units, respectively. The position of hurricane Fern is shown.

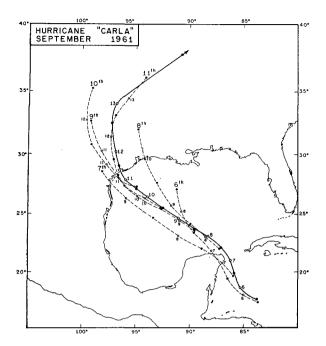


Fig. 5. Observed track of hurricane Carla 1961 (solid line) from 0000 GMT 6 September to 0000 GMT 14 September. Twelve-hourly positions are shown by solid or open circles, the latter indicating the 1200 GMT positions on the dates shown adjacent to them. The 72-hr forecast tracks appear as dashed lines, with 12-hr positions indicated in the same way. The number at the end of each forecast track represents the date of the initial time of the prediction.

enough to warrant expectation of baroclinic improvement over a barotropic forecast.

The diagnostic calculations are next completed by derivation of the fields of stream function and velocity potential from those of vorticity and divergence, respectively, after the latter pair has been smoothed so as to eliminate the two-dimensional wavelength of twice the grid distance. To ensure that maximum information from the wind analysis is retained in the stream function, which will be used for prognosis, we have applied boundary conditions which minimize the kinetic energy of the residual (and in this instance largely spurious) divergent part of the wind. Sangster (1960) has shown that this goal is achieved by assuming that the tangential component of the total wind along the boundary is associated with the stream function. Formally, since the rotational wind is given by $\mathbf{k} \times \nabla \psi$ and the divergent wind by ∇X (where **k** is the unit vertical vector, ψ the stream function, and x the velocity potential), we specify the normal derivative of the stream function at boundary and set the boundary value of velocity potential equal to a constant. Subject to these conditions we solve the Poisson equations $\nabla^2 \psi = \zeta$ and $\nabla^2 \chi = \text{div } V$ for the distributions of vorticity and divergence shown in Fig. 3 to obtain the patterns shown in Fig. 4. It is evident that more than 90% of the total kinetic energy is contained in the flow represented by the stream function; the field of velocity potential shows large areas where the divergent wind is on the order of 1 m sec⁻¹ and only a few points where it reaches 3-4m sec⁻¹.

In the immediate vicinity of the hurricane the stream function in this case yields a wind speed about 10% smaller than the input values at the grid points, the loss being attributable partly to truncation error and partly to diversion into the divergent wind field. The latter effect in other cases is much more pronounced when one or two of the grid points happens by chance to fall in the ring of maximum winds around the storm. Without exception, we have found that the diagnostic program replaces the actual hurricane with a much weakened vortex at very nearly the same initial location. Whether the true storm intensity affects the motion crucially can therefore be judged by the success of track forecasts, particularly the earlier portions which are influenced relatively strongly by conditions near the vortex and relatively little by conditions a great distance away.

3. Prognostic calculations

We are now ready to make a forecast from Eq. (3) and the initial fields of stream function and vorticity, after addition of the appropriate value of the Coriolis

Table 1. Position errors (n mi) in barotropic hurricane track forecasts.

Date and initial time		24 hr	48 hr	72 hr
		Carla		
9-6-61	$0000~\mathrm{GMT}$	56	140	224
9-7-61	$0000~\mathrm{GMT}$	91	210	343
9-8-61	$0000~\mathrm{GMT}$	21	126	273
9-9-61	$0000~\mathrm{GMT}$	7	91	252
9-10-61	$0000~\mathrm{GMT}$	56	161	182
9-11-61	$0000~\mathrm{GMT}$	21	42	175
		Flora		
10-4-63	0000 GMT	14	154	294
10-5-63	$0000~\mathrm{GMT}$	42	126	182
10-6-63	$0000~\mathrm{GMT}$	70	91	210
10-7-63	1200 GMT	84	182	154
10-8-63	1200 GMT	161	301	497
10-9-63	1200 GMT	175	357	728
		Donna		
9-5-60	0000 GMT	91	343	840
9-6-60	$0000~\mathrm{GMT}$	112	511	980
9-7-60	$0000~\mathrm{GMT}$	35	84	35
9-8-60	$0000~\mathrm{GMT}$	63	140	287
9-9-60	$0000~\mathrm{GMT}$	28	238	252
9-10-60	$0000~\mathrm{GMT}$	70	259	210
9-11-60	$0000~\mathrm{GMT}$	119	112	486

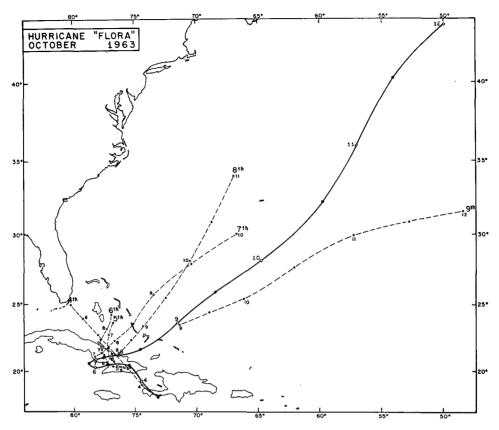


Fig. 6. Same as Fig. 5, except for hurricane Flora 1963 from 0000 GMT 4 October to 1200 GMT 12 October.

parameter to the latter. The predictions are carried out to 72 hr in time steps of 30 min. Among the hurricane forecasts to be presented, those for Carla 1961and Flora 1963, based on analyses prepared by King (1966), employed simple centered finite-difference approximations for the Tacobian calculations and fixed boundary values of stream function and vorticity, and take no account of variation in map scale over the forecast area (on a conformal conic projection with standard parallels at 30 and 60N.) In the forecasts for hurricane Donna, Ahn (1967) applied a scheme resulting from an extensive series of numerical trials, which comprises a higherorder differencing scheme due to Arakawa (1966): extrapolation of vorticity values from the interior to the boundary points, a variable map-scale factor, and a constraint that the mean value of the vorticity over the forecast area remain constant. Ahn's tests disclosed that this scheme did not yield a significantly different or significantly better hurricane track forecast than did the simple scheme described above; it did, however, show some success in controlling area-average kinetic energy and in mitigating truncation error. In all cases it was necessary to smooth the stream field at 12-hr intervals in order to control irregularities in the vorticity pattern, particularly near outflow boundaries. As before, the smoothing operator eliminated the two-dimensional wavelength of twice the grid distance.

4. A series of test forecasts

In the selection of hurricanes for test forecasting, relative abundance of upper-level data was regarded as a necessary requirement. Within this provision we chose storms which exhibited a variety of behavior: Carla 1961 for its long, nearly straight path, slow and relentless prior to landfall on the Texas coast; Flora 1963 for its erratic path and nearly stationary interlude over Cuba prior to acceleration toward the northeast; and Donna 1960 for its recurvature over the United States.

At the initial time the hurricane was represented by a minimum in the stream function and a nearly coincident maximum in the vorticity field, as in Figs. 3 and 4. During the forecast these two points tended to drift slowly apart; the nominal forecast position of the hurricane was taken halfway between them. On occasion the former point disappeared during the course of the forecast, specifically when the accelerating storm became superposed on an ever-stronger basic flow. In these

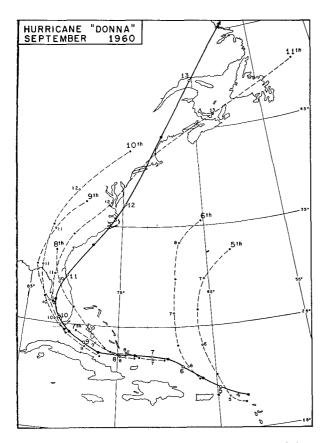


Fig. 7. Same as Fig. 5, except for hurricane Donna 1960 from 0000 GMT 5 September to 0000 GMT 14 September.

cases the vorticity maximum, though suffering from truncation error, was still clearly identifiable and served to determine the forecast position of the hurricane.

The forecast and observed tracks are displayed in Figs. 5-7, while the errors in forecast position are summarized in Table 1. We resist three temptations: to compute average errors, because of the small size of the sample and the appreciable variability within it; to compare with errors in the corresponding operational forecasts, because the forecasts were prepared in a research environment; and to proliferate the number of statistics which might be used to judge the skill of the predictions, because the usefulness of these forecasts as guidance must ultimately be judged by forecasters on bases which may be difficult to express quantitatively. We nevertheless have the impression that if this technique can be translated into operational practice without serious loss of accuracy, a significant improvement of the state of the art will be within our grasp.

Some specific comments seem appropriate. In the Carla forecasts (Fig. 5) the first three were somewhat erratic, but those beginning at 0000 GMT 9 September and thereafter drew a consistent bead on a point on the the Texas coast close to ultimate landfall on the 11th.

The excessive forecast speed, which might not be a serious detriment to operational utility, was attributable to excessively strong winds associated with spurious growth of a large anticyclone on the right flank of the storm. This synoptic-scale error is a manifestation of a decrease in area-average vorticity which occurred during most of the forecasts and which seems to result principally from advection of the Coriolis parameter by a mean southerly component in the initial wind data. An entirely satisfactory way of avoiding this difficulty has not yet been devised nor have we found a satisfactory way of controlling the area-average growth of kinetic energy.

In the predictions for Flora (Fig. 6), the initial stalling of the storm over Cuba was not caught 24 hr in advance (i.e., in the forecast beginning 0000 GMT 4 October), though a slight deceleration was forecast. The position errors, however, are not excessively large, even at 72 hr (though they are indeed serious for the problem of warnings on the south coast of Florida). The forecast begining at 0000 GMT 6 October, on the other hand, was successful in predicting the begining of systematic motion toward the northeast, after a nearly stationary initial 24 hr. The evolution of the entire flow pattern for this prediction is shown in Fig. 8. At the start Flora is embedded within the subtropical ridge, but as time progresses the small flanking anticyclones drop southward and a southwesterly steering current develops over the region of the storm. At the start, Flora would almost certainly be characterized as a storm that could "move anywhere," but the forecast correctly indicated the ultimate track. The rather large errors in the initial 24 hr of the last two forecasts for Flora appear to be due to an insufficiently strong southwesterly current in the path of the storm in the initial analysis, which could have been avoided by acceptance of stronger geostrophic guidance provided by the available pressure-height data.

The first two forecasts for Donna (Fig. 7) represent the first two incontestably "busted" forecasts we have encountered. At first glance one is tempted to attribute these egregiously erroneous recurvatures to lack of data since nothing is available save reconnaissance observations for a great distance north and east of the storm. Closer examination, however, discloses another reason. The forecast starting on 7 September is extraordinarily good even at 72 hr and the data could not have been all that much better on that date than on 6 September. Therefore, it is reasonable to suppose that the source of the error is in the environmental 24-hr forecast made on the earlier date. The relevant portion of this forecast, and of the observed development, is illustrated in Fig. 9. The trough forecast to lie just off the east coast of the United States, in advance of which the spurious recurvature is predicted, fills dramatically. The forecast change in the stream function field indicates northward acceleration of the storm while the observed change accelerates

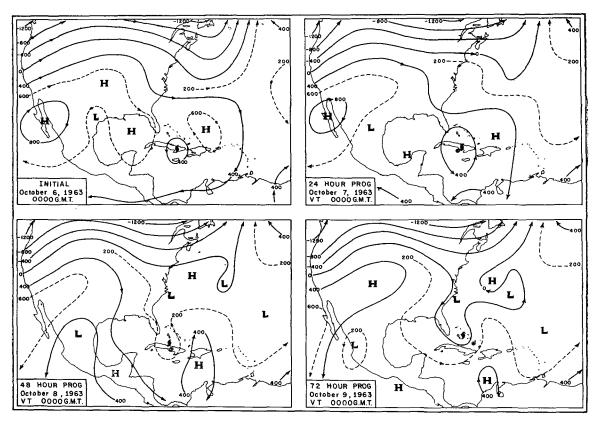


Fig. 8. Initial and forecast fields of stream function at 24-hr intervals beginning at 0000 GMT 6 October 1963. Isopleths are labeled in units of 1.65×10⁴ m² sec⁻¹. The initial and prognostic positions of hurricane Fern are shown by the conventional symbol, while corresponding observed positions are shown by the circled crosses.

it toward the southwest, in agreement with the observed track. The large error in the barotropic forecast of vorticity, shown in Fig. 9, occurs downstream from a region of plentiful data and must be taken as evidence that the failure of the hurricane forecasts was due to an unusually vigorous baroclinic process, limited in time and space, in which a crucial portion of the trough filled so as to block the northward progress of the storm.

Donna's actual recurvature and acceleration was predicted with some success. Forecasts from data at 0000 GMT 9 September (about 36 hr prior to the beginning of the distinct change in direction of the storm) and thereafter predicted recurvature and kept pace with the observed increase of forward speed. The sharpness of the recurvature across Florida was not reproduced, however, even in the short-range prognosis. This flow did not lead to excessively large position error, but it would have been operationally serious, since the forecasts did not anticipate the overwater track along the east coast of the storm, which was instrumental in preserving its intensity. The cause of this error has not been investigated.

5. Sensitivity to initial analysis

In practice it would be necessary to have an idea of the sensitivity of the forecast to the initial analysis. To this end, five teams of students in the MIT synoptic laboratory, some of whom were totally inexperienced in wind analysis, were given the plotted data for one of the Donna cases. Preparation of the analysis and determination of initial grid-point data proceeded independently, after which parallel forecasts were calculated by the simple numerical scheme employed for Carla and Flora. The resulting hurricane track predictions are shown in Fig. 10. Great similarity in the forecasts is evident for the first 36 hr, after which considerable scatter occurs (though four of the five tracks show recurvature). This result is contrary to our expectation, which was based on the assumption that the details of the analysis near the storm, about which the students would differ, would determine the earliest portion of the forecast. while the large-scale aspects of the analysis, about which the students would agree, would control the later portions. Evidently, it is difficult to upset the first part of

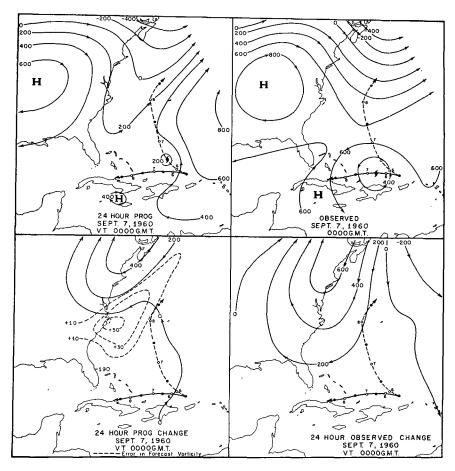


Fig. 9. Analysis of an erroneous recurvature forecast for hurricane Donna 1960: upper left, 24-hr prognostic stream-function field for 0000 GMT 7 September; upper right, observed stream-function field for the same time; lower left, prognostic stream-function change for the 24-hr period ending at 0000 GMT 7 September; and lower right, observed stream-function change for the same period. The stream function and its change are in units of 1.65×10^4 m² sec $^{-1}$ and are represented by isopleths at intervals of 200 units. The direction of the wind flow and its change is indicated by arrow heads. In the lower left, the numerical value of a minimum in the forecast stream-function change is shown near Florida, and the error in the forecast vorticity, in units of 10^{-6} sec $^{-1}$, is indicated by the thin dashed lines. The observed and predicted tracks of the hurricane during the period from 0000 GMT 6 September to 0000 GMT 9 September are shown in the same notation as in Fig. 5.

the forecast, given a reasonable data coverage. It seems, morever, that the evolution of the larger scale aspects of the pattern may be quite sensitive to the details of the initial analysis, in a nonlinear way that is not at all evident.

6. Concluding remarks

We find that the barotropic prediction model, applied in a way appropriate to the character of tropical circulations and of the data available to describe them, is capable of providing a basis for significant advance in the state of the art of hurricane track forecasting in the Carribbean Sea, the Gulf of Mexico, and the western Atlantic Ocean. By way of limitation, the technique is not now applicable in many regions of tropical cyclone activity because of lack of wind information; and even in regions of adequate data coverage large errors occasionally develop in the forecast beyond the first 24 hr bebecause of prominent baroclinic effects in the evolution of the environmental flow. The distinctive features of the technique are application of the barotropic equation to tropospheric mean data computed from information at 10 constant pressure levels, prognostic use of a stream function derived from direct analysis of the mean wind field, and numerical calculation over a grid with rela-

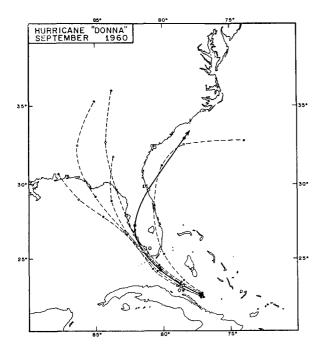


Fig. 10. Observed track of hurricane Donna 1960 from 0000 GMT 9 September to 0000 GMT 12 September, with forecast tracks from five independent analyses of the initial data. Notation the same as in Fig. 5.

tively small mesh length, without separation of the tropical vortex from the residual flow.

Regular operational use of this method requires automation of the present manual aspects, particularly the

development of an adequate wind analysis scheme. Widespread use requires wind information from currently silent areas of the tropical oceans, in which the cloud-tracking capability of the ATS satellite system offers promise.

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