

Chaos, Organization, Self-Organized Criticality and Predictability of the Tropical Atmosphere

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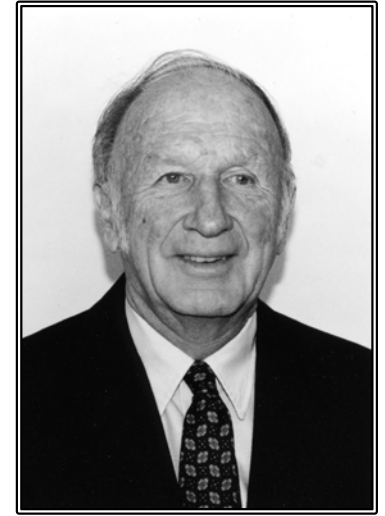
Overview

- Brief review of chaos theory
- Predictability of moist convection
- Self-organized criticality and the stability of the tropical atmosphere
- Prigogine's hypothesis of emergent organization and thermodynamic disequilibrium
- Application to the tropical atmosphere and implications for predictability
- Long-term criticality of the tropical atmosphere



Henri Poincaré
1854-1912

Chaos Theory



Edward Lorenz
1917-2008

Poincaré, H., 1890: Sur le problème des trois corps et les équations de la dynamique. *Acta Mathematica*, **13**, 1–270.

- Demonstrated that the three-body problem in orbital dynamics has no analytic solution
- Invented phase-space diagrams, now known as Poincaré sections and stated that "One is struck by the complexity of this figure, which I shall not even attempt to draw"
- Concluded that solutions were extraordinarily sensitive to initial conditions

Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, **20**(2), 130–141.

- First computational demonstration of chaos
- Reduced convection problem to three ODE's in three variables:

$$\begin{aligned}\dot{x} &= \sigma(y - x), \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= xy - bz.\end{aligned}$$

- Defined "Strange Attractor" and demonstrated extreme sensitivity to initial conditions

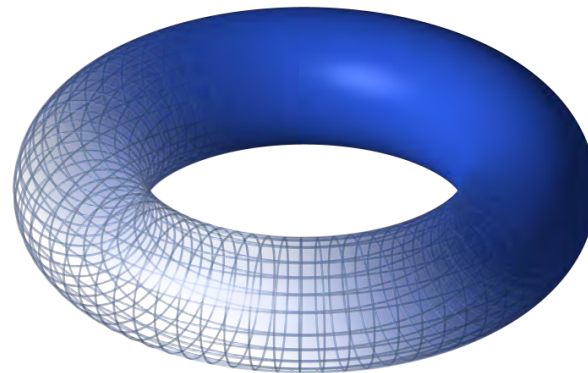
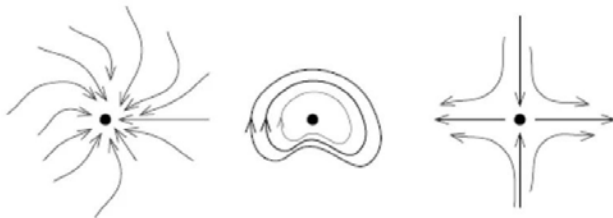
Fixed Point Attractors

Periodic Attractors

STABLE

MARGINALLY STABLE

UNSTABLE



■ Strange Attractors:

■ Fractal geometry

Lorenz Attractor:



Attractor Summary

Continuous motion	Poincaré section
Equilibrium	One point
Periodic orbit	Finite set of points
Quasi-periodic orbit	Closed curve
Chaotic orbit	Fractal set

Define a predictability time scale, τ , as the time scale beyond which two solutions that initially differ by a measure δ , diverge to be as different as any two arbitrary solutions.

For Lorenz (1963) and other discrete chaotic systems,

$$\lim_{\delta \rightarrow 0} (\tau) = \infty$$

This is a *regular limit*, in the sense that setting δ to zero gives the same answer.

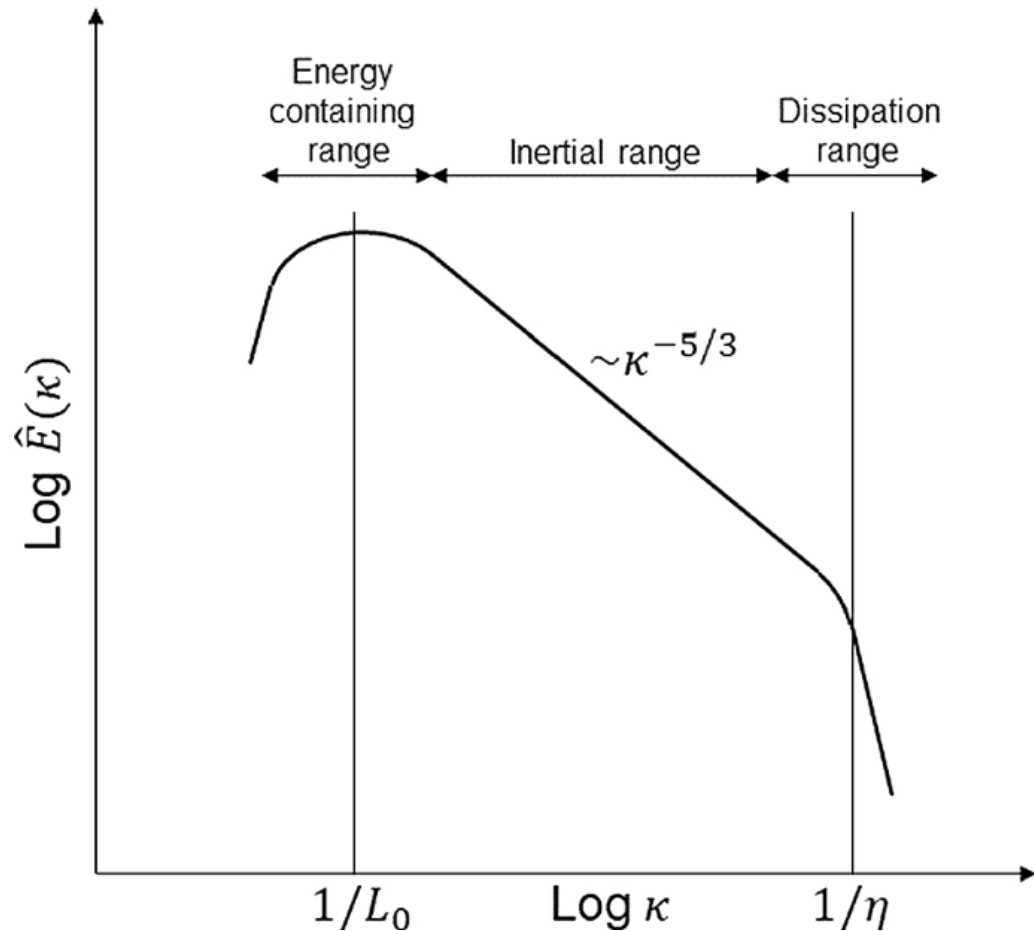
Lorenz, E. N. ,1969: The predictability of a flow which possesses many scales of motion. *Tellus*, 21(3), 289–307.

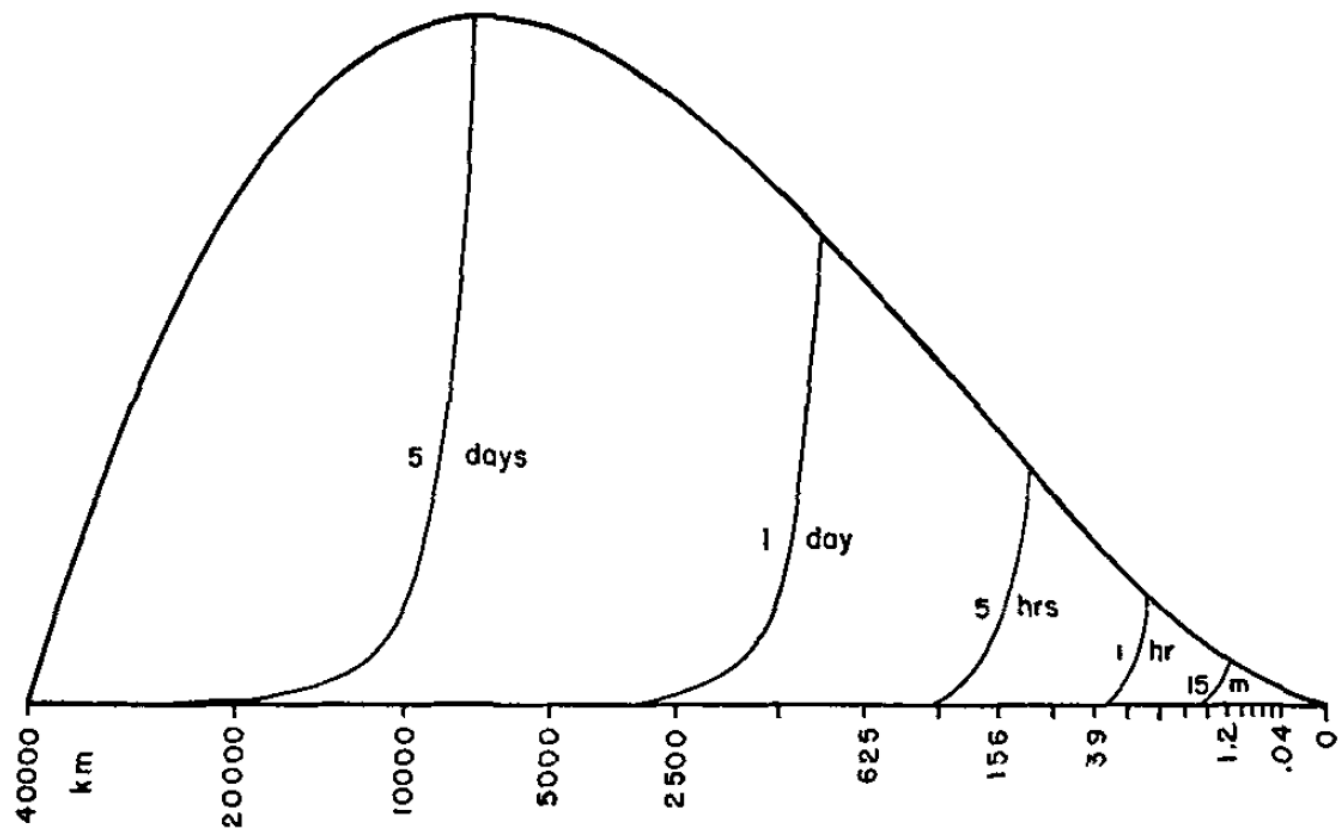
Continuum of scales in
turbulent flows

Simplest case:
Homogeneous, isotropic
turbulence

Kolmogorov scaling:
Energy per unit
wavenumber

$$E(\kappa) \sim \kappa^{-5/3}$$





Lorenz (1969) showed that if the exponent of the relationship between energy and wavenumber is less than -3,

$$\lim_{\delta \rightarrow 0} (\tau) = T,$$

where T is a finite number. This is a *singular limit*. **The predictability horizon of such systems cannot be lengthened by decreasing δ indefinitely.**

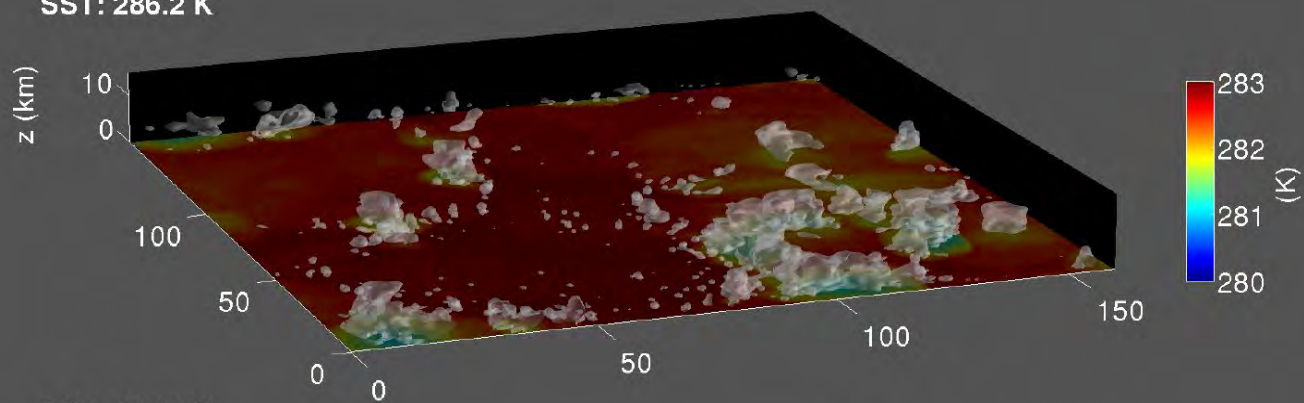
- T depends on scale

- For deep moist convection, $T \sim 1 \text{ hour}$

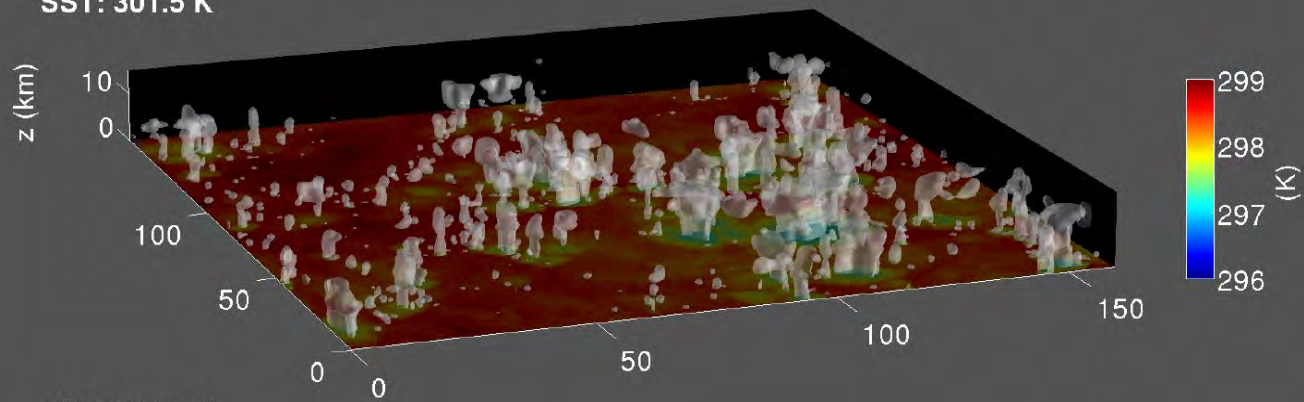
Simplest Paradigm: Moist Radiative-Convective Equilibrium

- Infinite, homogeneous horizontal plane represented numerically by doubly periodic box
- Calculated or specified radiative cooling of the troposphere
- Specified or interactive sea surface temperature
- Run long enough to achieve statistical equilibrium

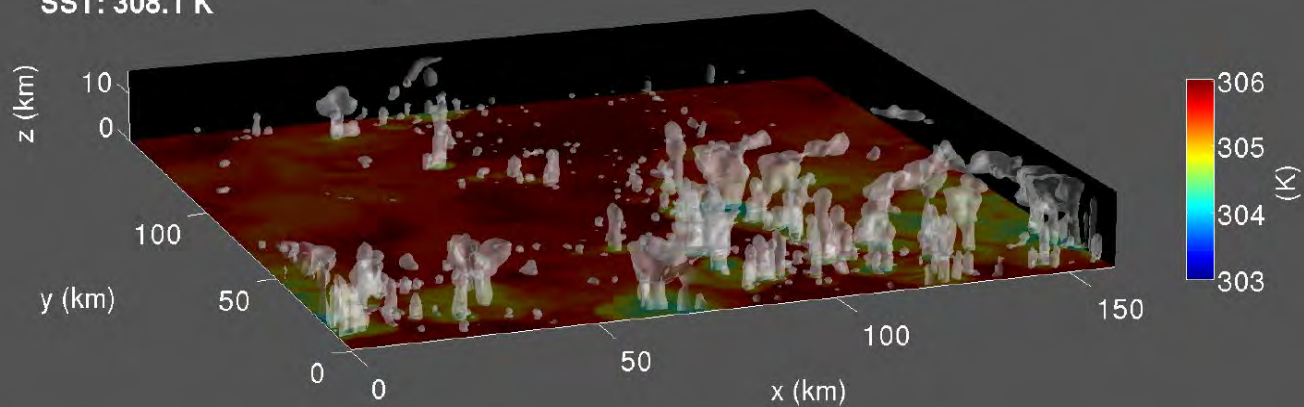
SST: 286.2 K



SST: 301.5 K

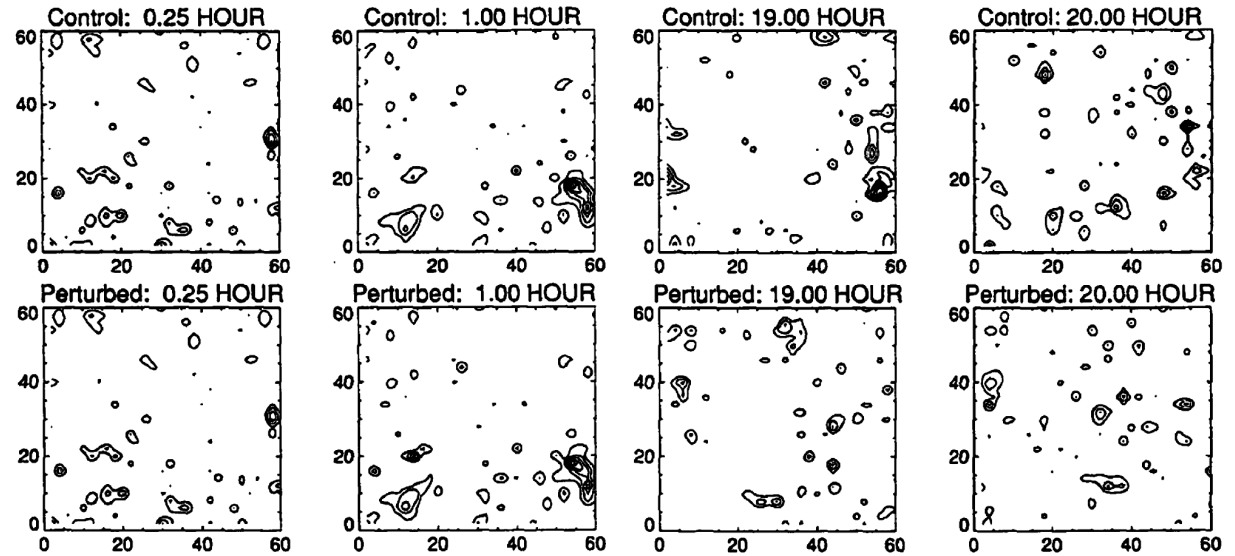


SST: 308.1 K

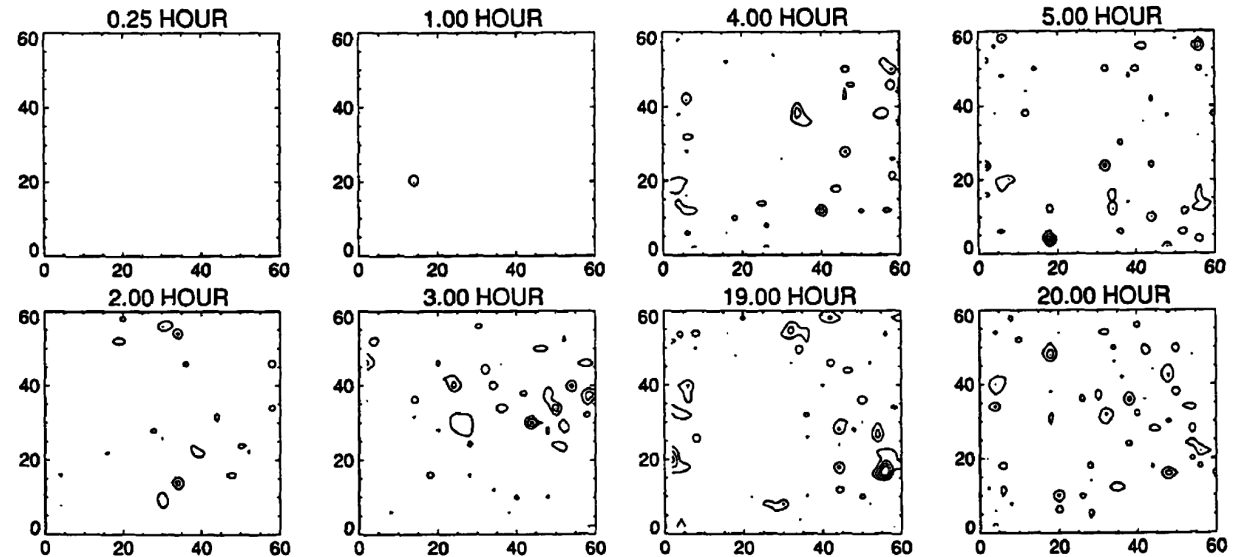


Surface Rainfall Intensity

Control experiment



Perturbed experiment



Difference

**Decorrelation time
scale is a few hours**

Islam, S., R. L. Bras, and K. Emanuel, 1993: Predictability of mesoscale rainfall in the tropics. *J. Appl. Meteor.*, **32**, 297–310.

Radiative-convective equilibrium states have most of the characteristics of self-organized critical states

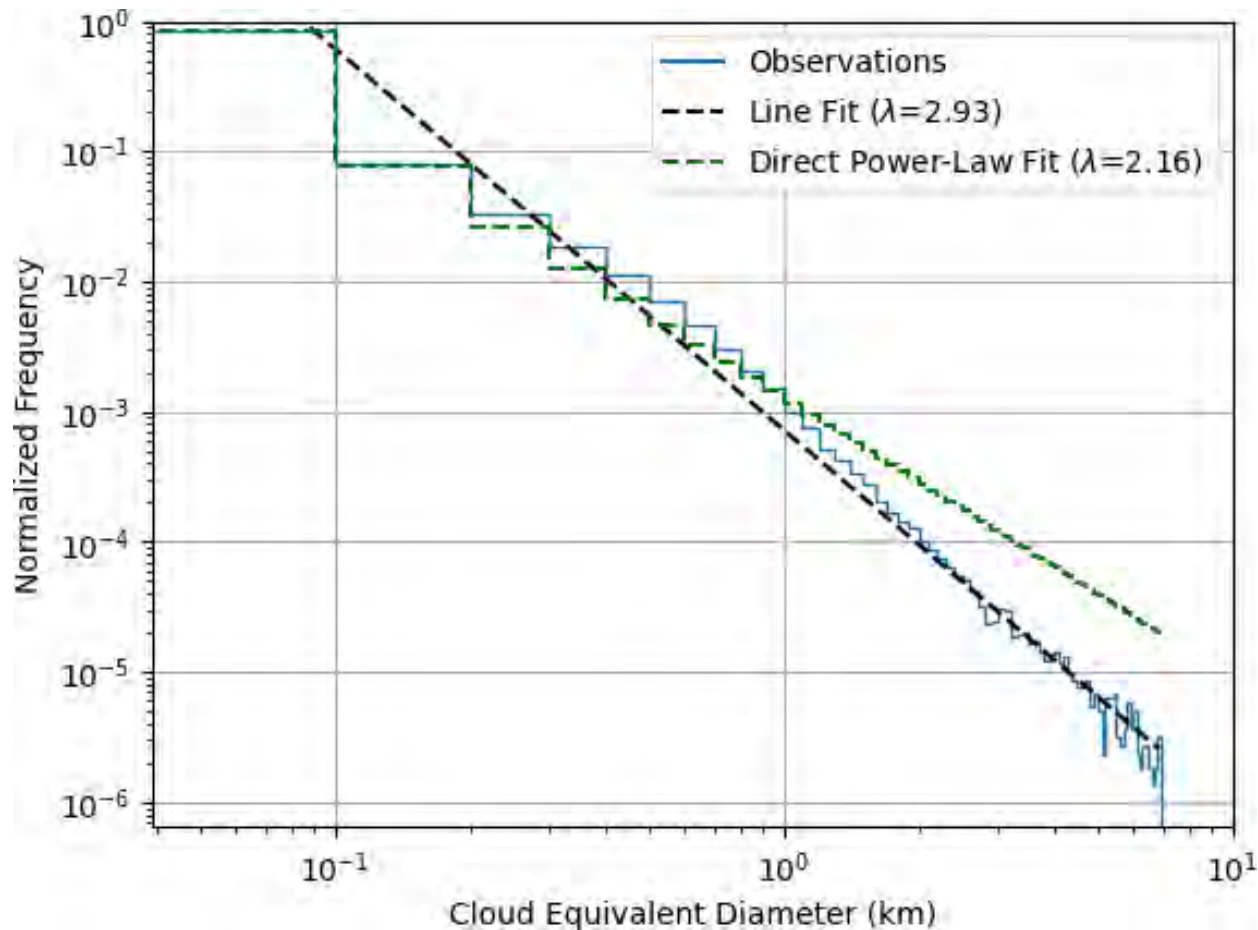
- A self-organized critical system is one that lies on the boundary between stability and instability. The “events” that result (convective clouds in this case) are “scale invariant”, so their sizes (as measured for example by cloud mass flux) follow power law distributions
- Examples of self-organized critical systems include earthquakes, forest fires, neural activity, and financial markets

Rice accumulating on a circular plate



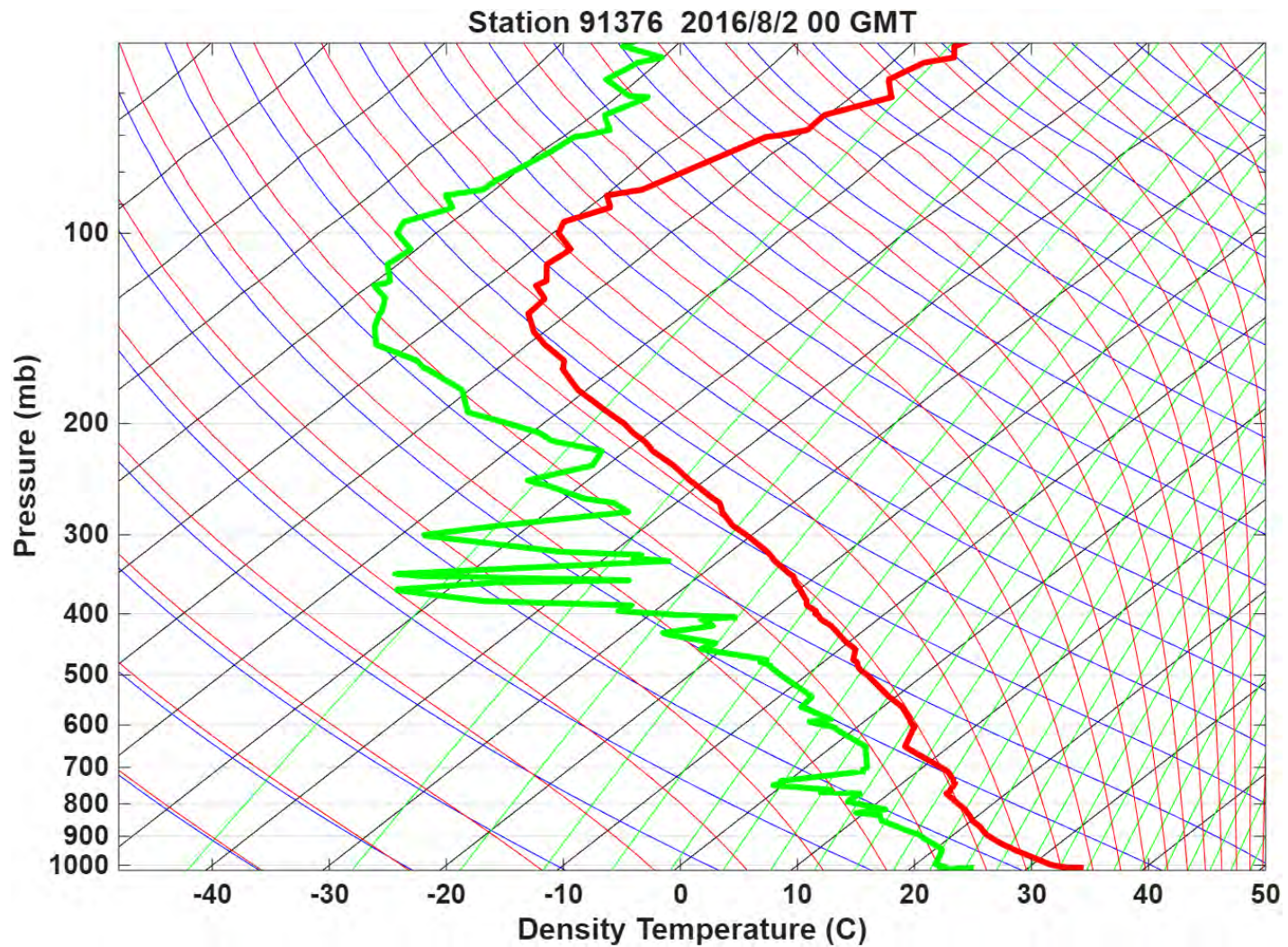


Observations of clouds over the North Pacific

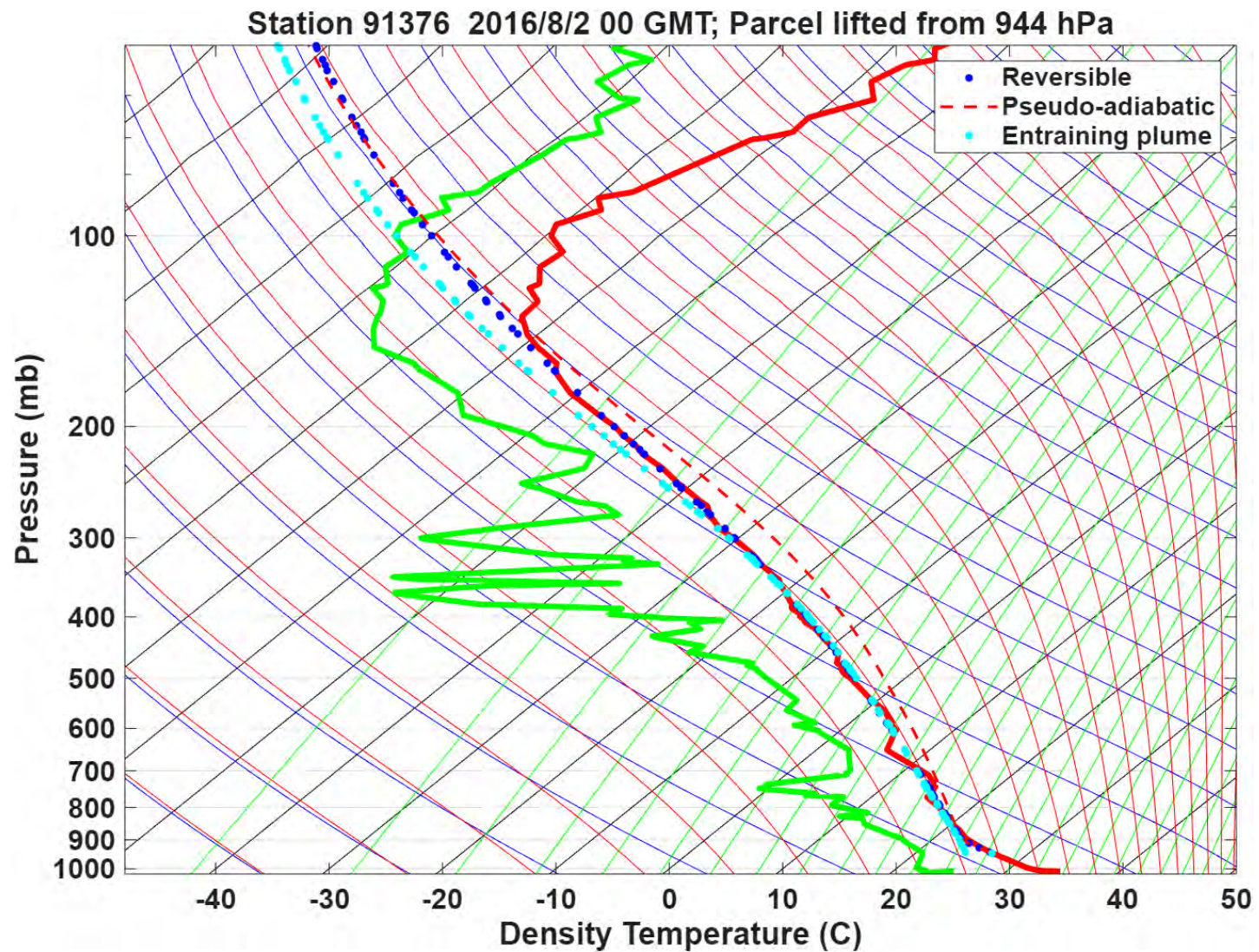


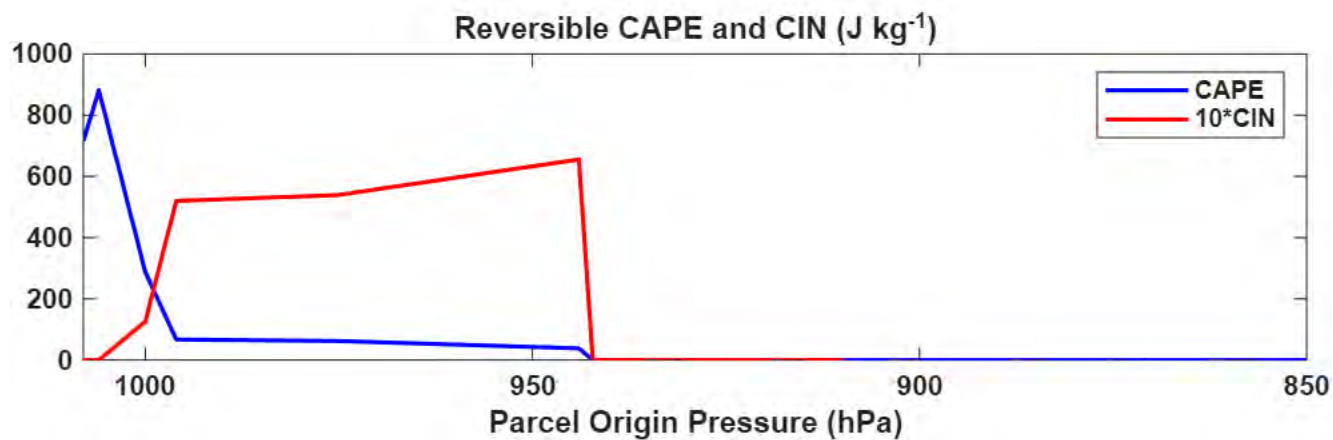
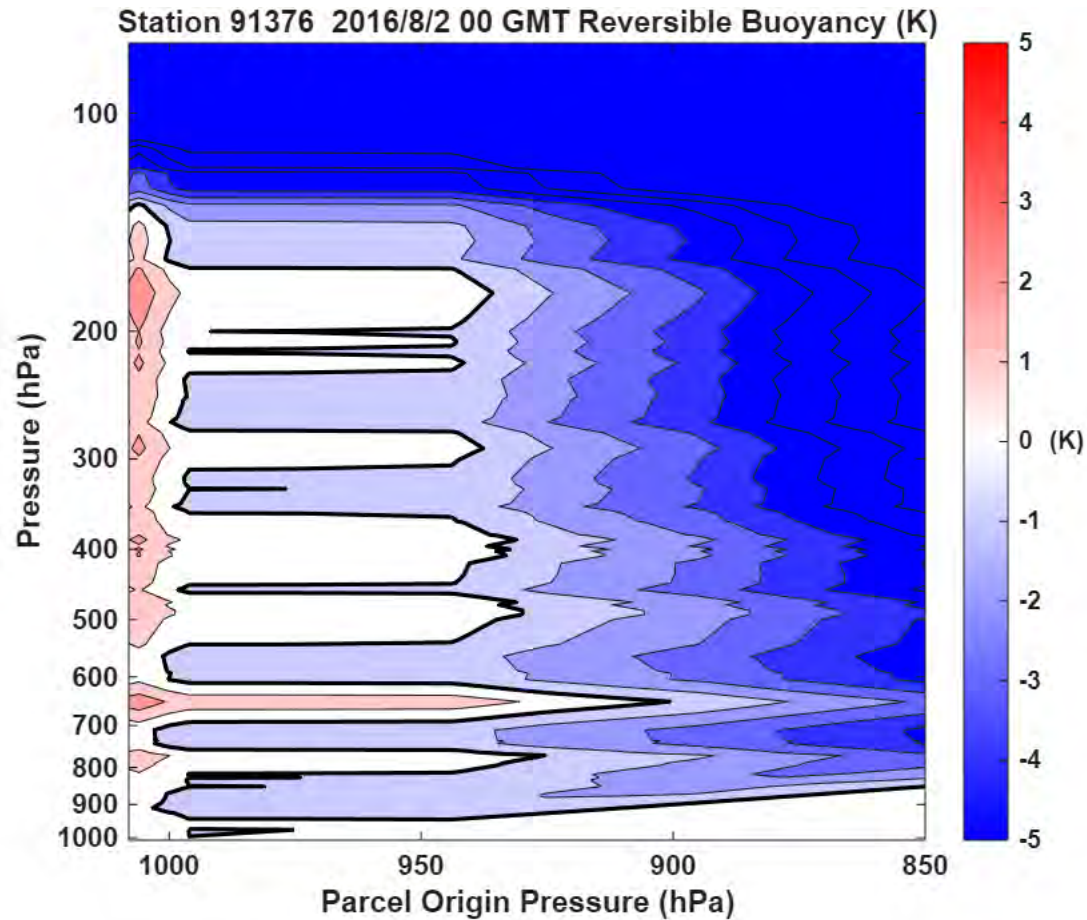
De Vera, M. V., L. Di Girolamo, G. Zhao, R. M. Rauber, S. W. Nesbitt, and G. M. McFarquhar, 2024: Observations of the macrophysical properties of cumulus cloud fields over the tropical western Pacific and their connection to meteorological variables. *Atmospheric Chemistry and Physics*, **24**, 5603–5623, <https://doi.org/10.5194/acp-24-5603-2024>.

Sounding at Majuro, August 2 2016 0 GMT



Sounding at Majuro, August 2 2016 0 GMT





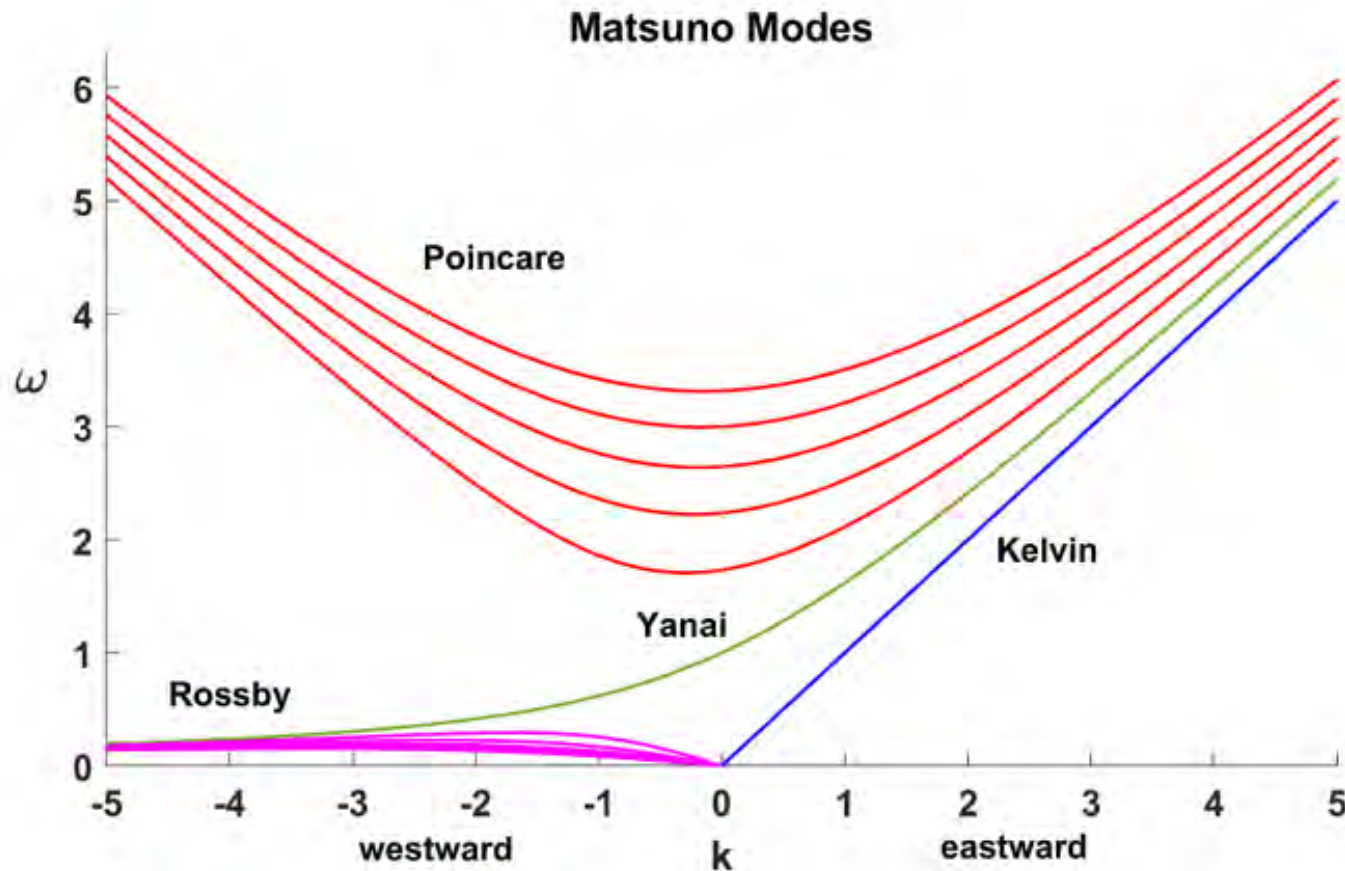
Moist RCE should be thought of as the natural base state for tropical dynamics. (Much of the older literature takes a dry atmosphere as the starting point.) For circulations larger than inter-cloud spacing:

- Circulations are hydrostatic and always preserve a moist adiabatic profile
- These two assumptions filter out all but a barotropic mode and a first baroclinic mode which obeys shallow water equations
- First baroclinic mode “feels” an effective stratification that is much smaller than the dry stratification but is still positive:

$$N_{eff}^2 = N^2 (1 - \epsilon_p)$$

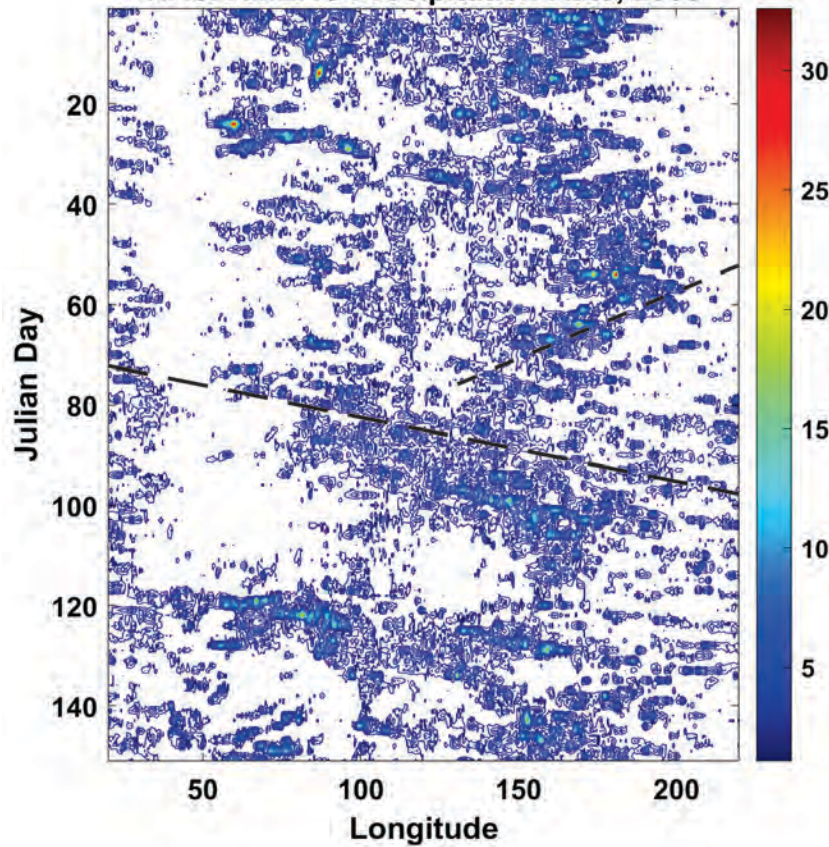

Convective cloud precipitation efficiency

Classical equatorially trapped waves on an equatorial beta plane, shallow water equations:

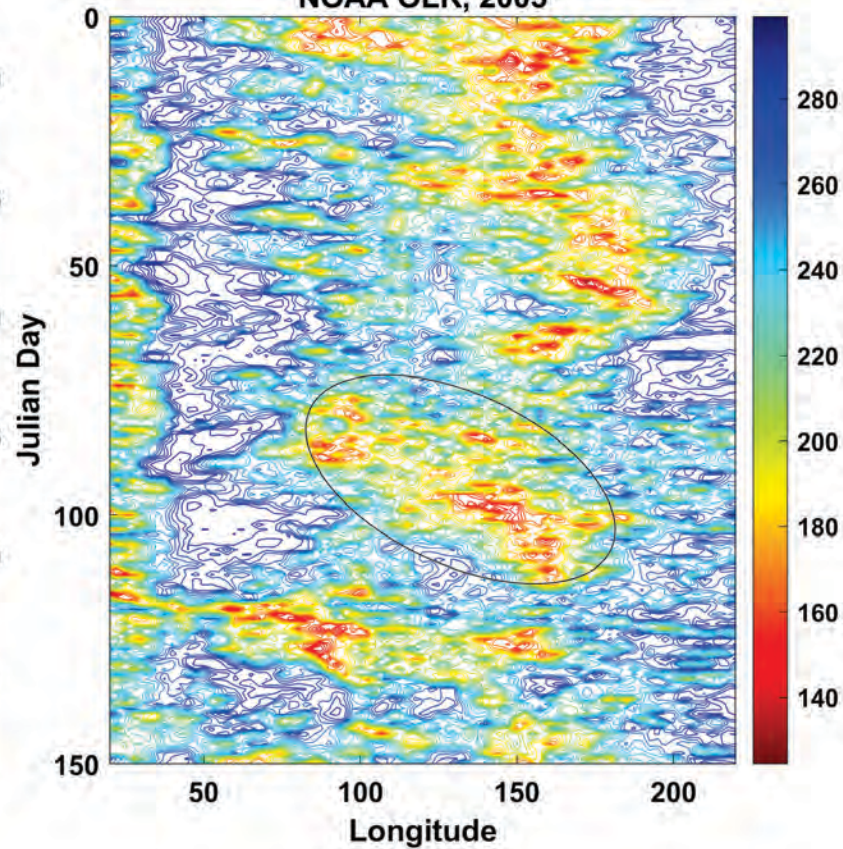


Matsuno, T., 1966: Quasi-geostrophic motions in the equatorial area. *J. Meteor. Soc. Japan*, **44**, 25–42.

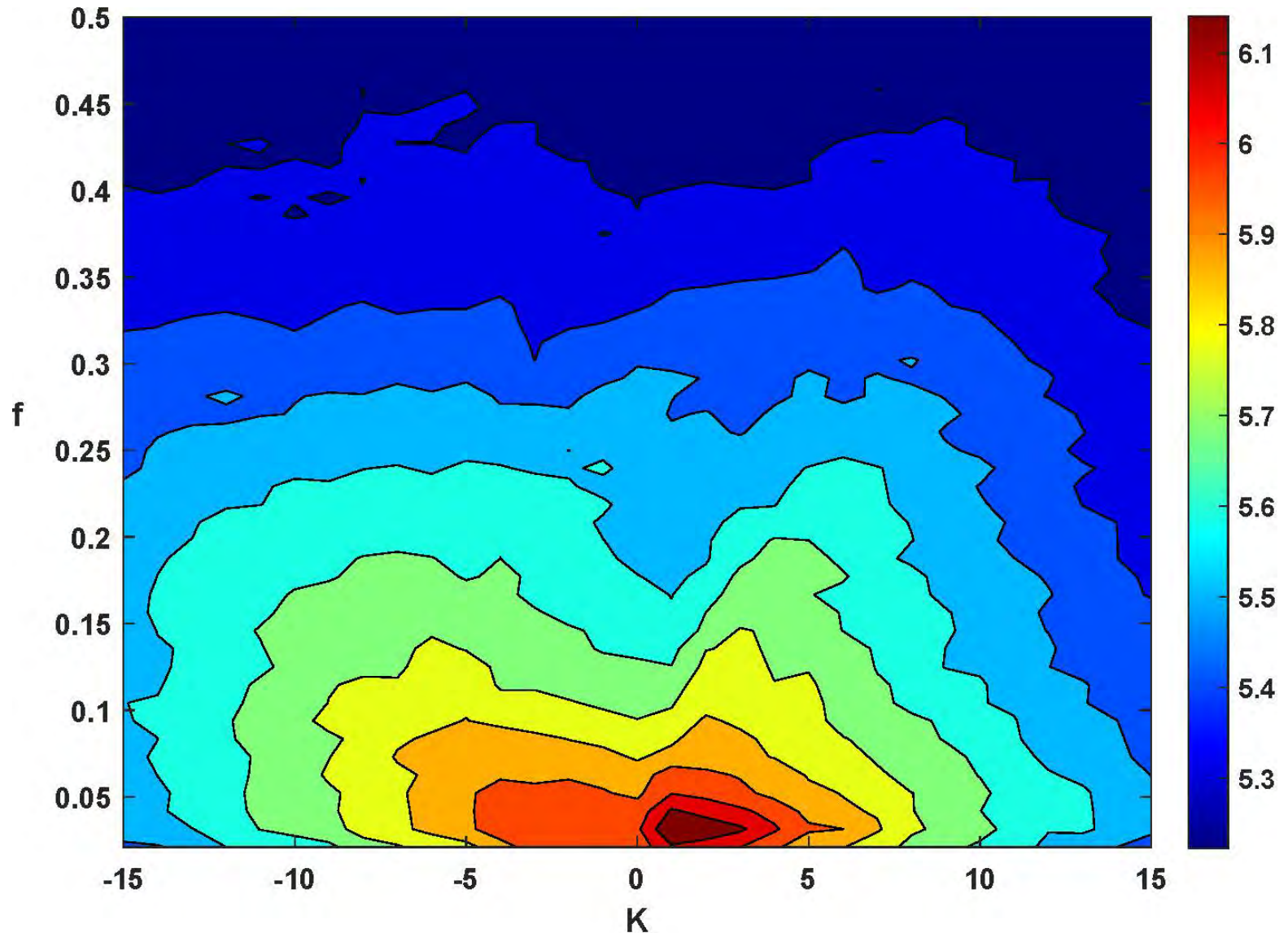
NASA IMERG Precipitation Rate, 2005



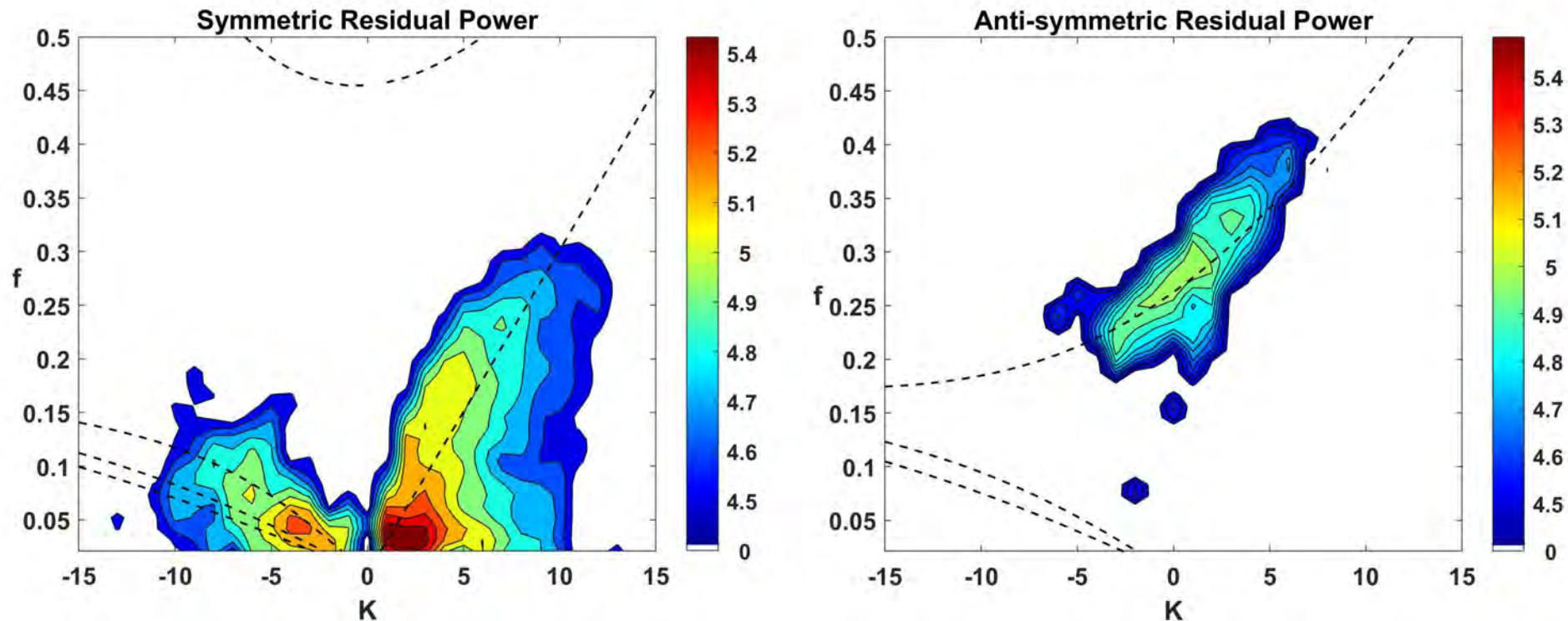
NOAA OLR, 2005



Wavenumber-frequency power spectrum of CLAUS brightness temperatures:

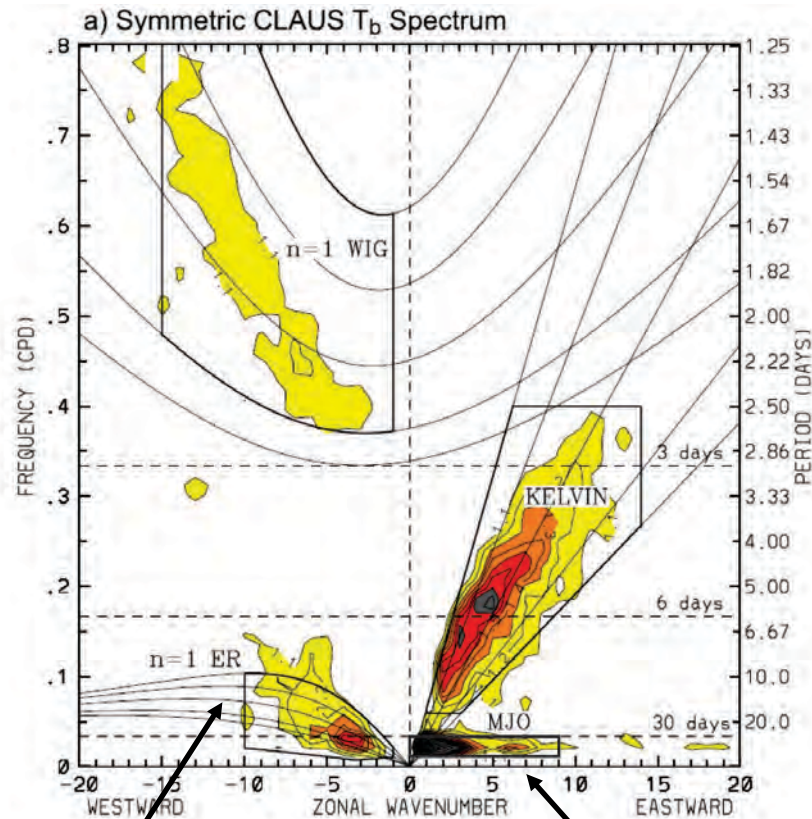


Break into equatorially symmetric and antisymmetric parts
and subtract smoothed red-noise spectrum from total:



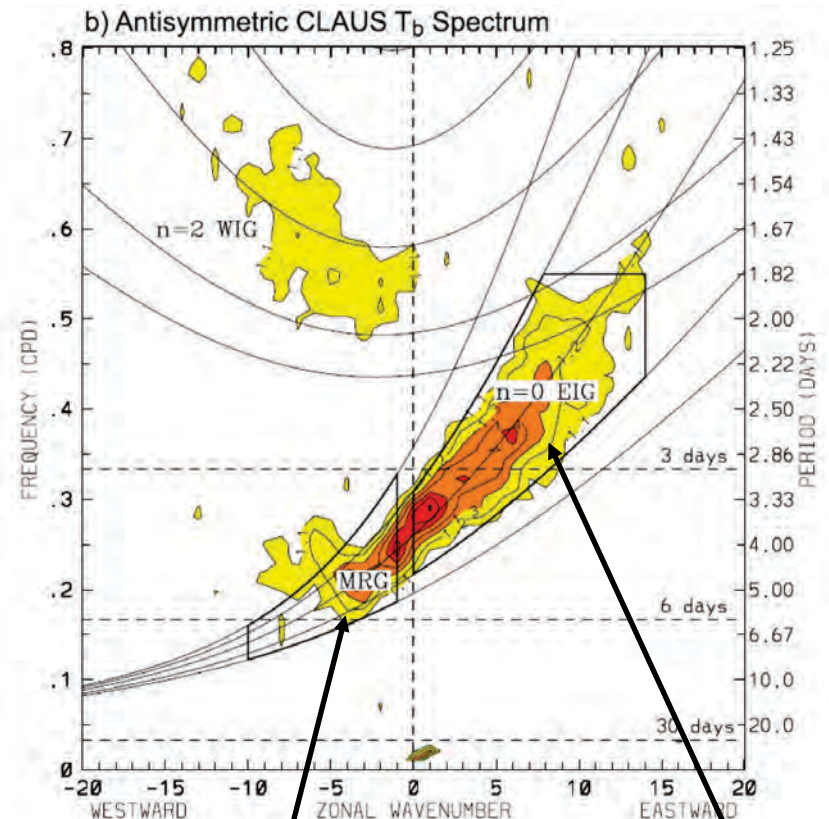
After Wheeler, M., and G. N. Kiladis, 1999: Convectively coupled equatorial waves: Analysis of clouds and temperature in the wavenumber-frequency domain. *J. Atmos. Sci.*, **56**, 374–399.

Wheeler, M., and G. N. Kiladis, 1999: Convectively coupled equatorial waves: Analysis of clouds and temperature in the wavenumber-frequency domain. *J. Atmos. Sci.*, **56**, 374–399.



Equatorial Rossby Waves (ER)

Madden-Julian Oscillation (MJO)



Mixed Rossby Gravity Waves (MRG)

Equatorial Inertia-Gravity modes (EIG)

What Drives These Disturbances?

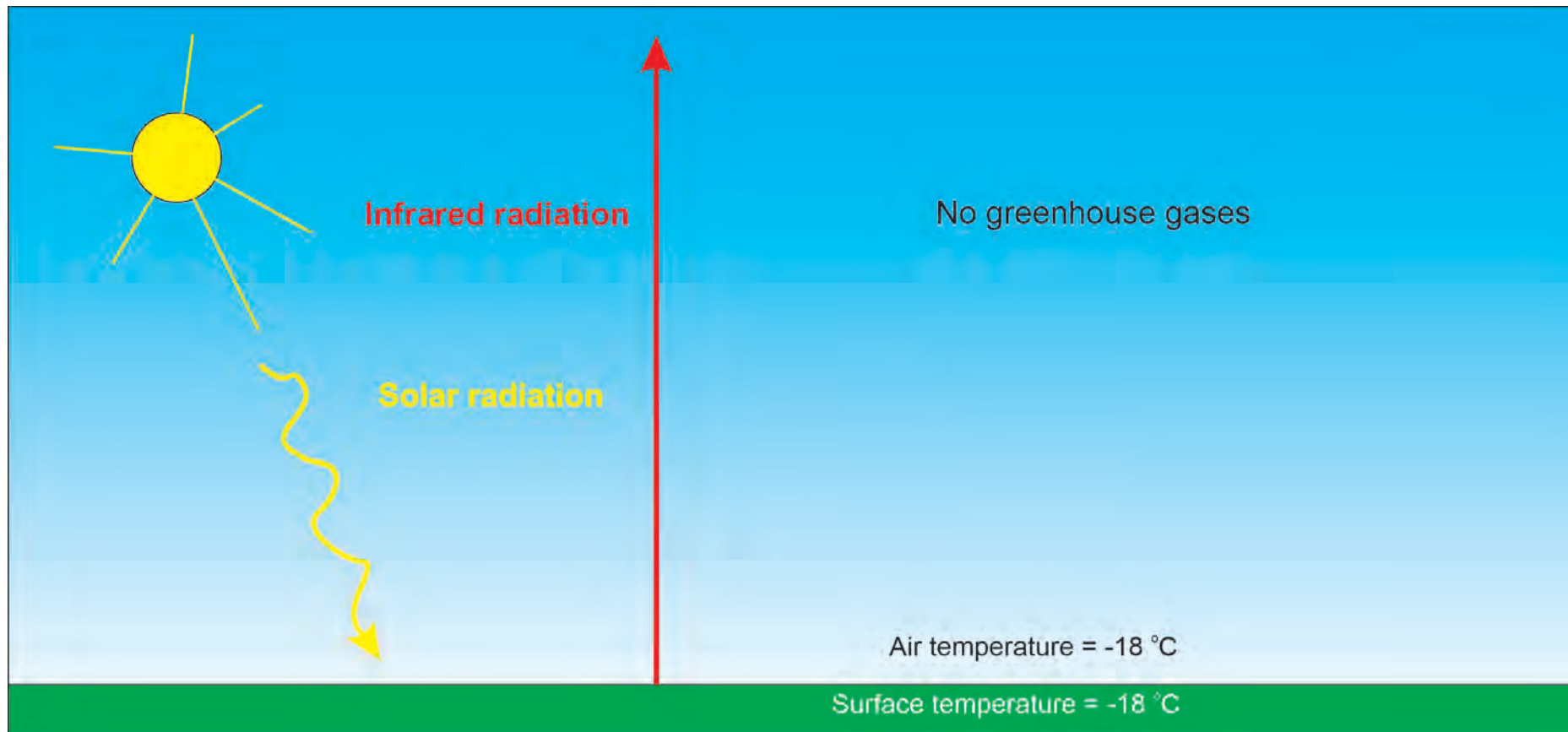
- Candidate Mechanisms:
 - Cloud-radiation interaction
 - Wind-Induced surface heat exchange (WISHE)
 - Response to local convective noise and/or leakage of extratropical disturbance energy into the tropics
- Many theories; no generally accepted explanation

The Importance of Thermodynamic Disequilibrium

- Ilya Prigogine (1917-2003)
 - Showed that organized structures arise from thermodynamic disequilibrium and that structures only arise if enough disequilibrium is present
- Radiative equilibrium involving discontinuous opacity, such as at a liquid-gas interface, entails thermodynamic disequilibrium

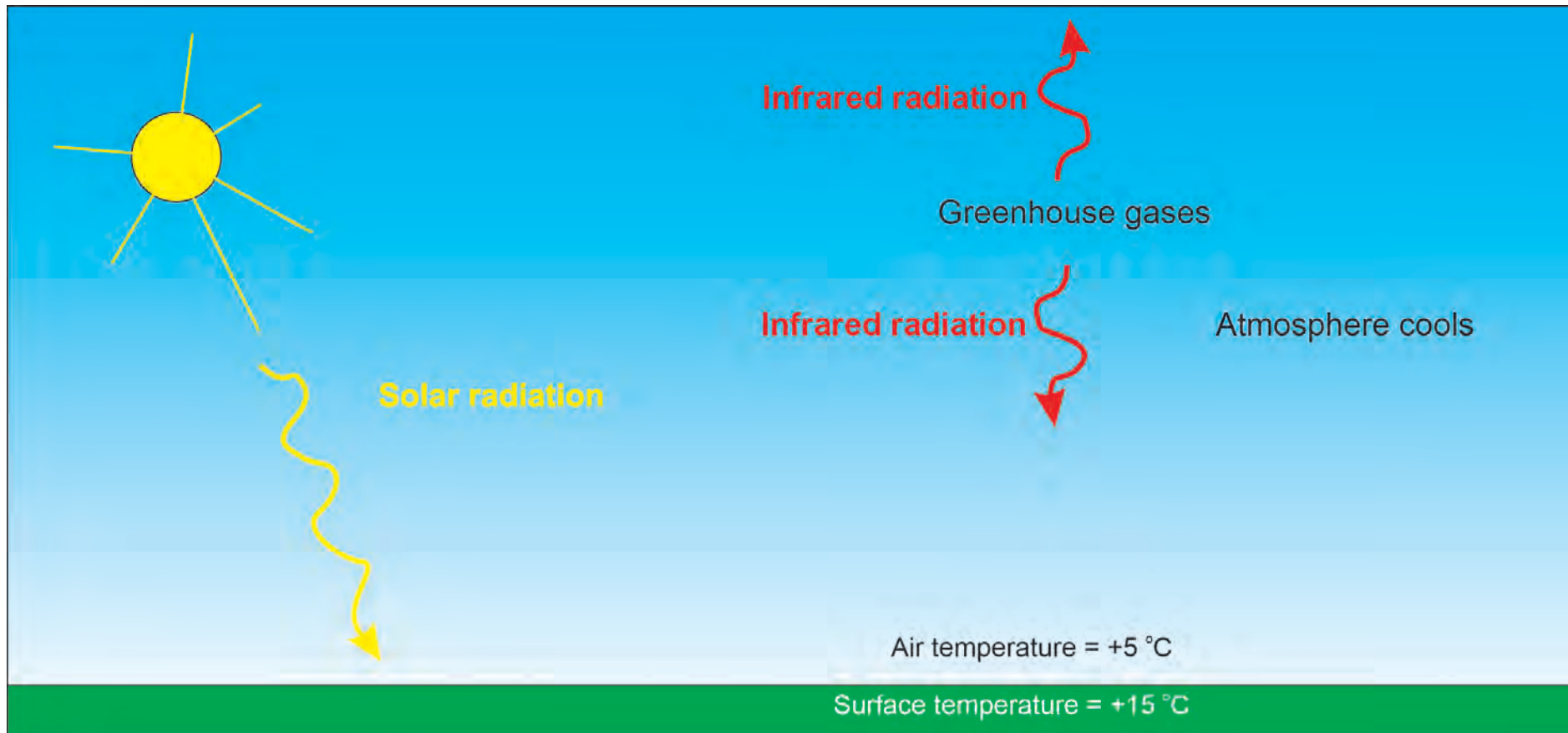
What causes disequilibrium in earth's climate?

First case: Dry surface, no greenhouse gases



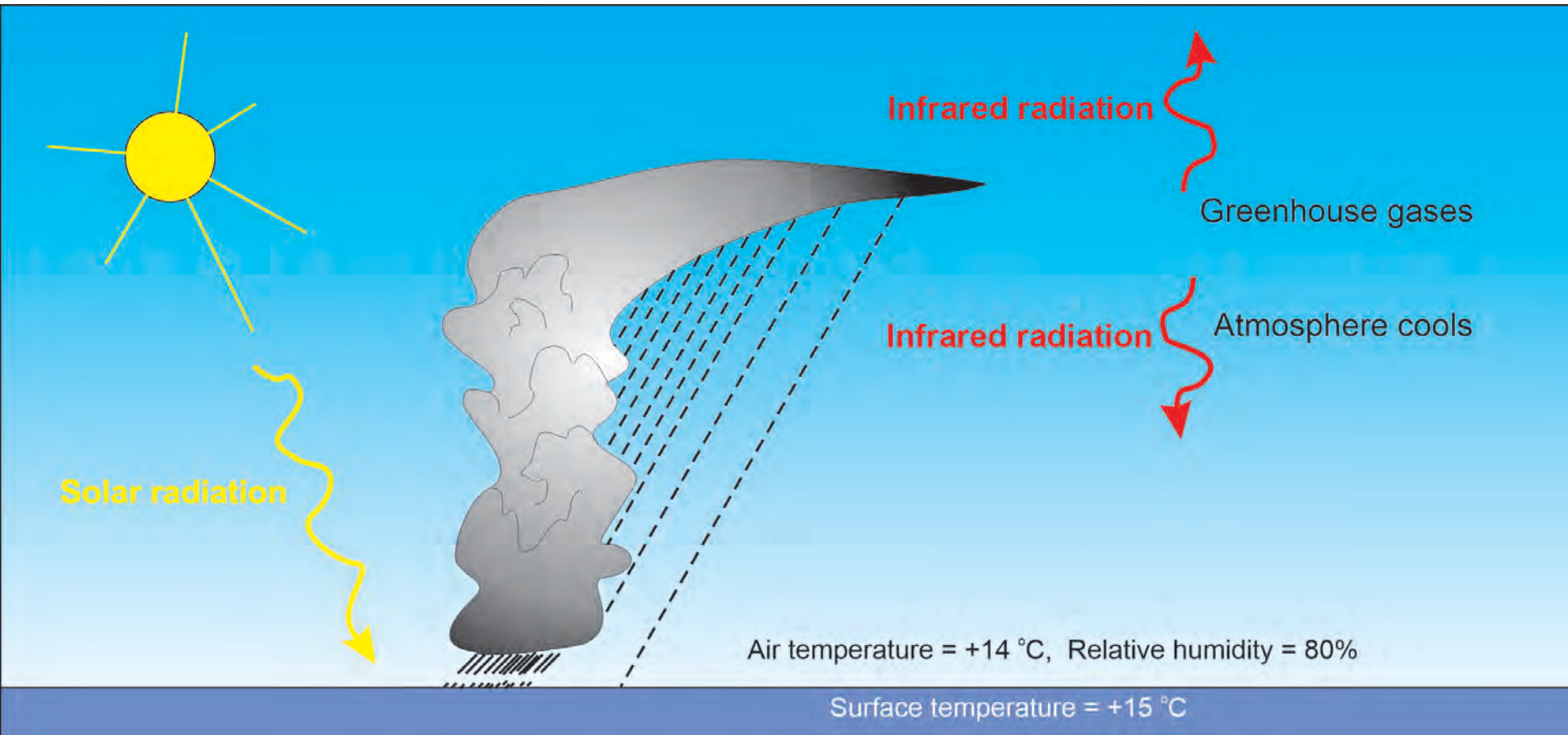
No Thermodynamic Disequilibrium

Second case: Dry surface, greenhouse gases



Thermodynamic Disequilibrium: Temperature jumps
between surface and atmosphere

Third case: Ocean surface, greenhouse gases



Thermodynamic Disequilibrium: Moisture jumps between surface and atmosphere. The more greenhouse gas, the greater the disequilibrium

"Warmest climes but nurse the cruelest fangs."

-- Herman Melville, *Moby Dick*

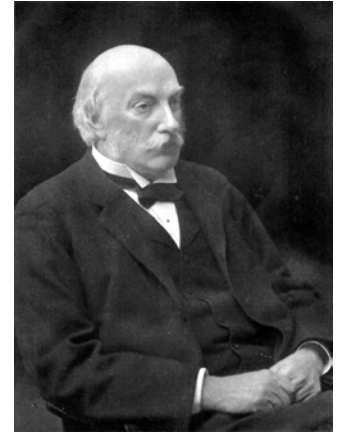
The Earth's most benign climate
breeds its most vicious storms

Start with a simple example: Convection in a box

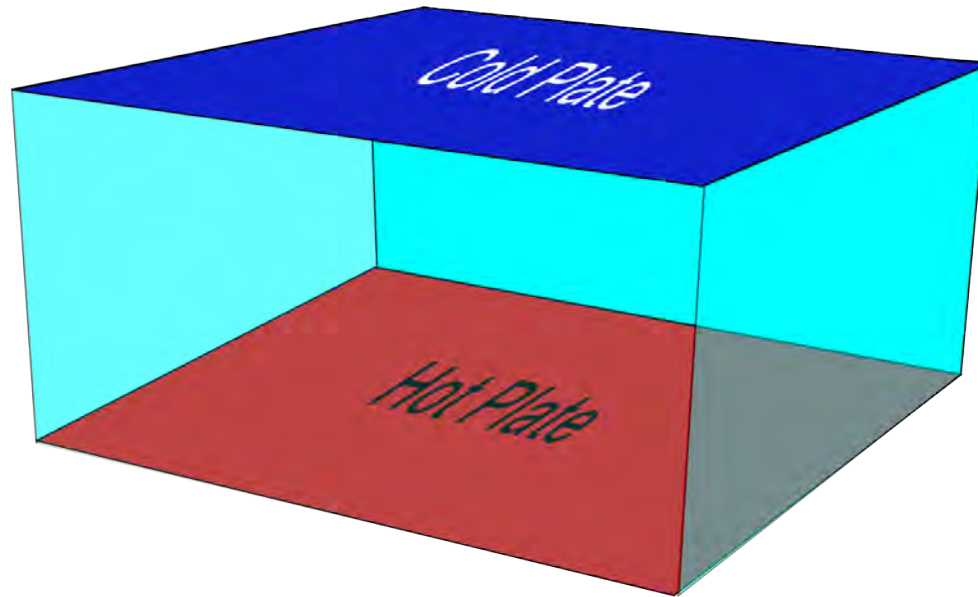
Rayleigh–Bénard convection



Henri Bénard
Experiments, 1900



John Strutt, Lord Rayleigh
Theory, 1916



Direct Numerical Simulation of Rayleigh–Bénard convection

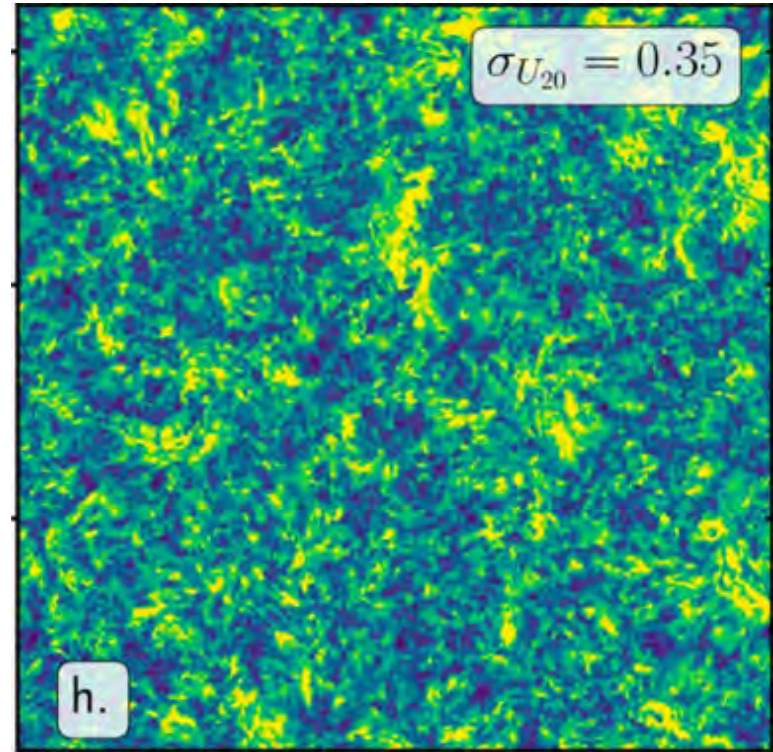
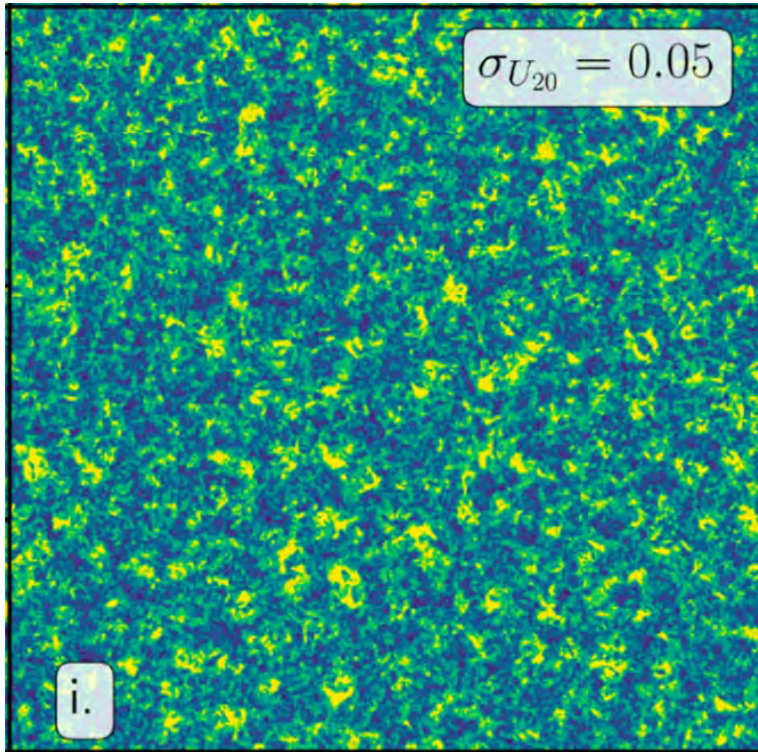


Computer-simulated convection in a rotating box

Fluid temperature must equal plate temperature at bottom

Fluid temperature must equal plate temperature also at top

Fluid cooled at constant rate everywhere



Horizontal wind speed near the bottom of the box

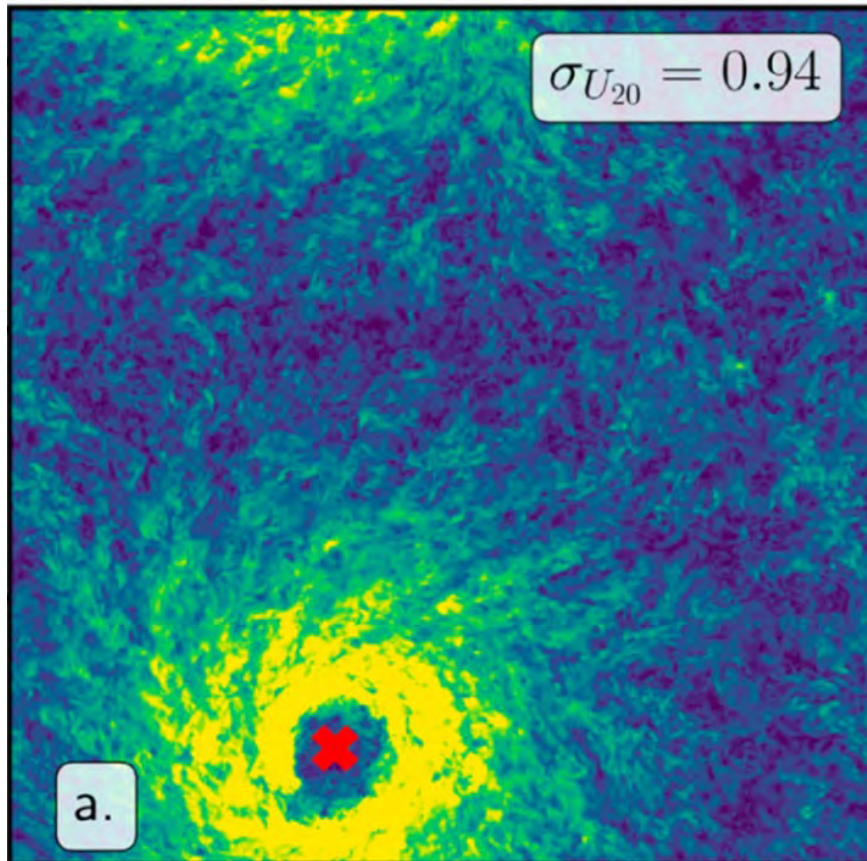
Mathematicians and laboratory experimentalists had assumed perfectly smooth boundaries. The rate of escape of heat from the plate to the fluid is always limited by molecular diffusion.

But geophysicists assumed that the earth's surface is rough. The rate of escape of heat from a rough surface is limited by turbulence, and molecular diffusion does not limit the flux of heat.

In this case, the flow of heat depends on the difference in temperature between the surface and the fluid, and the wind speed just above the surface.

So let's assume now that the surface is rough and replace the lower boundary condition by the geophysicist's formulation:

Fluid cooled at constant rate
everywhere



TCs

No latent heat!!

Conventional View of TC Energetics:

Me: *What drives tropical cyclones?*

Claude:

1. Warm, moist surface air rises rapidly, releasing latent heat as water vapor condenses
2. This latent heat warms the upper atmosphere, lowering surface pressure
3. Lower pressure draws in more warm, moist air from the surface
4. That air rises, releases more latent heat, further deepens the low — a positive feedback loop
5. The Coriolis effect organizes this inflow into a rotating spiral (cyclonic rotation)

Note: Claude goes on to cite sea-air heat transfer as necessary for further intensification

Typical undergraduate textbook:

~~*Hurricanes are driven by the latent heat released when water vapor condenses in clouds.*~~



TCs and some other tropical disturbances are driven by the wind-dependent flow of heat from the ocean to the atmosphere under conditions of strong thermodynamic disequilibrium.

There is evidence that many tropical disturbances, including the MJO and Matsuno-like modes, may arise from the radiatively driven disequilibrium between the tropical atmosphere and ocean, accessed through wind-dependent turbulent surface enthalpy fluxes

These disturbances result from instabilities of the RCE state that may serve to drive the ocean-atmosphere system in the tropics toward another self-organized critical state

Numerical simulation of Radiative-Convective Equilibrium

Note: Simulation jumps from day 16 to day 84

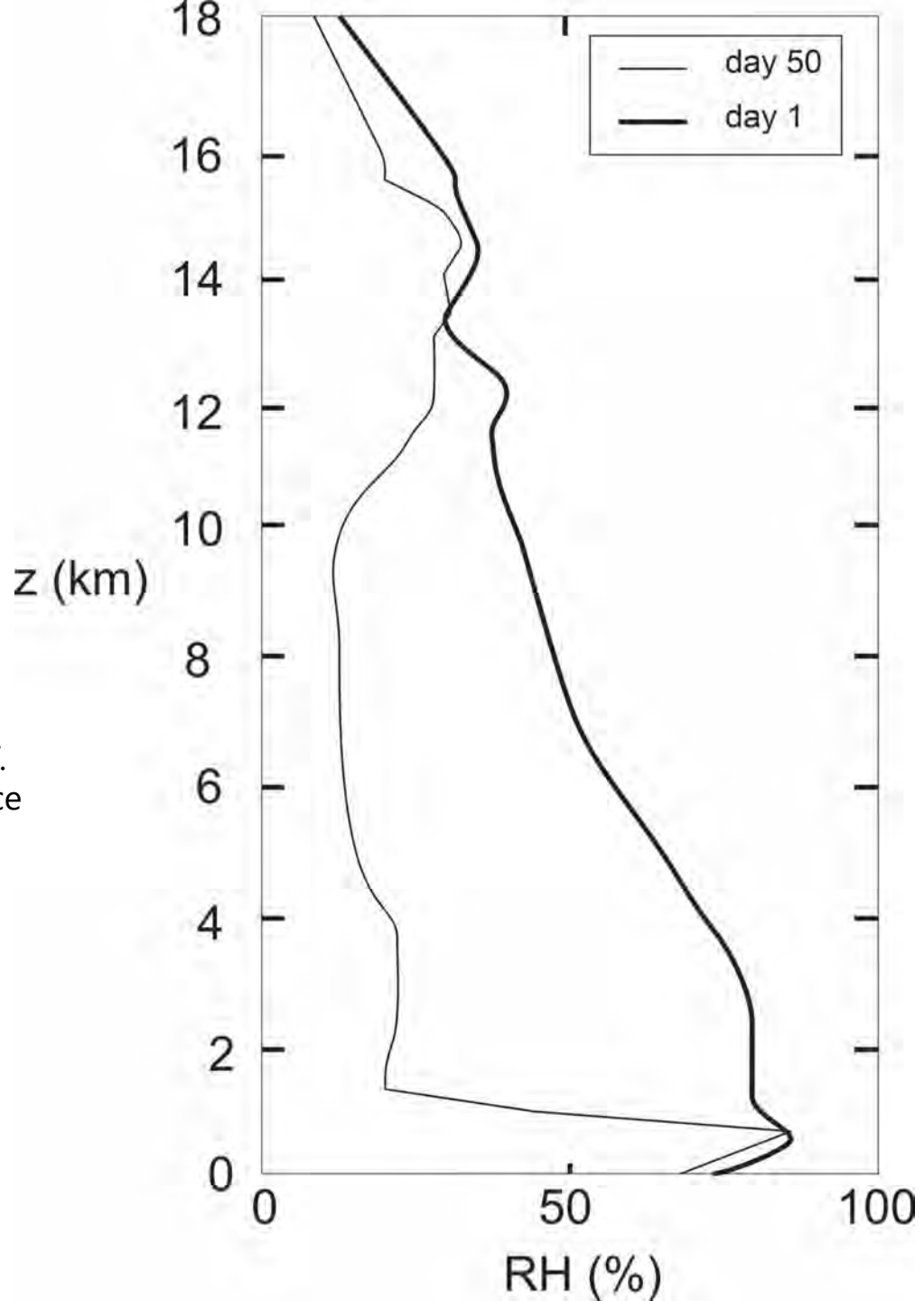


“Self-aggregation” of convection appears to be driven by cloud-water-vapor-radiation interactions and WISHE

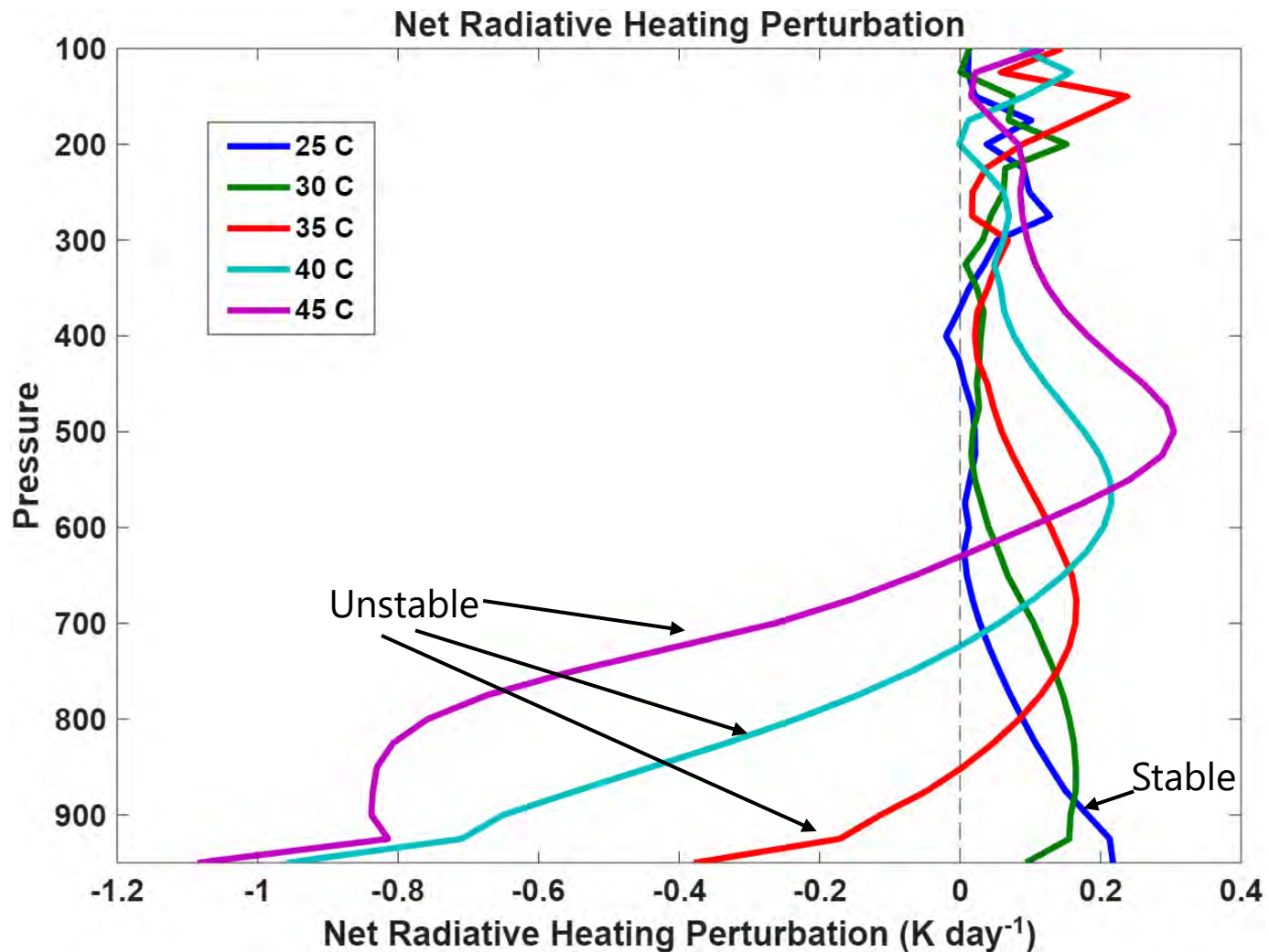
- On non-rotating plains, takes the form of cloud clusters
- On f -planes takes the form of tropical cyclones
- On equatorial beta-plane takes the form of the MJO and other Matsuno-like modes
- These modes are almost certainly much more predictable than moist convection

Self-aggregation profoundly dries the atmosphere, allowing more infrared emission to space. This has been observed from satellite. Such an increase in IR to space will cool the system.

Bretherton, C. S., P. N. Blossey, and M. F. Khairoutdinov, 2005: An energy-balance analysis of deep convective self-aggregation above uniform SST. *J. Atmos. Sci.*, **62**, 4273–4292



Instantaneous response of a radiative heating in a single column, initially in moist RCE, to a step function 20% decrease in specific humidity throughout the column



Higher system temperature leads to greater aggregation of moist convection, hence to a drier atmosphere and more infrared emission to space, cooling the system. Tropical climate may tend toward the critical temperature separating stability from instability.

Summary

- The predictability time scale of moist convection is a few hours
- It is a mistake to think of this convection as driving large-scale circulations (except squall lines)
- It is far more productive and natural to think of large-scale tropical circulations as perturbations (not necessarily linear) to a state of moist radiative-convective equilibrium

- The RCE state is held by the fast process of convection to be very nearly critical (moist adiabatic)
- Assuming criticality and hydrostatic balance reduces the tropical troposphere to a shallow water system
- The shallow water hypothesis is supported by wavenumber-frequency analysis of equatorial disturbances
- Predictability horizons of such disturbances are on the order of days to weeks, not hours

- Moist RCE states can be destabilized by interactions among clouds, water vapor, and radiation, and by the feedback between surface wind and surface enthalpy flux
- This instability may drive tropical climate toward an additional self-organized critical state that is nearly neutral to these instabilities, with implications for tropical climate