

On Estimating Hurricane Return Periods

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ABSTRACT

Interest in hurricane risk usually focuses on landfalling events of the highest intensity, which cause a disproportionate amount of hurricane-related damage. Yet assessing the long-term risk of the most intense landfalling events is problematic because there are comparatively few of them in the historical record. For this reason, return periods of the most intense storms are usually estimated by first fitting standard probability distribution functions to records of lower-intensity events and then using such fits to estimate the high-intensity tails of the distributions. Here the authors attempt a modest improvement over this technique by making use of the much larger set of open-ocean hurricane records and postulating that hurricanes make landfall during a random stage of their open-ocean lifetime. After testing the validity of this assumption, an expression is derived for the probability density of maximum winds. The probability functions so derived are then used to estimate hurricane return periods for several highly populated regions, and these are compared with return periods calculated both from historical data and from a set of synthetic storms generated using a recently published downscaling technique. The resulting return-period distributions compare well to those estimated from extreme-value theory with parameter fitting using a peaks-over-threshold model, but they are valid over the whole range of hurricane wind speeds.

1. Introduction

Estimation of the return periods of hurricane winds is an important component of hurricane risk assessment. For many applications, it is important to quantify the return periods of the most extreme winds, which return periods may be comparable to or longer than the historical record at the site in question, making direct inference from historical storms problematic. Early risk assessments (e.g., Georgiou et al. 1983; Neumann 1987; Jagger et al. 2001) fitted standard distribution functions, such as lognormal or Weibull distributions, to the distribution of maximum intensities of all historical storms coming within a specified radius of the point of interest, and then, drawing randomly from such distributions, used standard models of the radial structure of storms, together with translation speed and landfall information, to estimate the

maximum wind achieved at the point of interest. A clear drawback of this approach is that estimates of the frequency of high-intensity events are sensitive to the shape of the tail of the assumed distribution, for which there are very little supporting data. This limitation was, to some extent, circumvented in the work of Darling (1991) and Chu and Wang (1998), who used empirical global distributions of relative intensity (the ratio of actual to potential intensity) together with climatological values of potential intensity to infer local intensity distributions. A similar approach was taken by Murnane et al. (2000), who used global estimates of hurricane actual (rather than relative) wind intensity cumulative probability distributions.

Another approach to the difficulty of defining the tails of the intensity distributions is to fit the shorter, historically inferred return periods to a limited set of functions that conform to expectations based on generalized extreme-value (GEV) theory (Jagger and Elsner 2006). This fit provides an extrapolation to very long return periods. A third approach is to generate large sets of synthetic events and use those to infer return periods

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(Vickery et al. 2000; Emanuel et al. 2006). Although this second approach can be used directly to infer arbitrarily long return periods, the computational requirement increases with the desired return period. Here, too, one could fit GEV functions to the calculated return periods and use those functions to extrapolate to higher return periods. One limitation of this approach is that the GEV functions will not be generally valid over the whole range of return periods, since they are designed to describe the extreme tails of distributions, and so it is not always clear whether the data being fitted lie far enough toward the distribution tail to be in a region for which the GEVs are valid. For this reason, it is desirable to derive functional forms of return periods that are valid over the whole range of hurricane wind speeds. An effort to do so is described here.

2. Empirical probability density functions for normalized hurricane wind speeds

Using best-track data, Emanuel (2000) calculated cumulative distributions of storm lifetime maximum wind speed, normalized by the potential intensity at the place and time each storm achieved its peak intensity. This was done for storms whose peak intensity was not limited by landfall or passage over cold water. These distributions were shown to be bilinear, with one slope for tropical cyclones of subhurricane intensity and another slope for hurricanes. These correspond to constant probability densities, one for hurricanes and a second, higher density for subhurricane-strength tropical cyclones. It was also shown that the probability that the (normalized) wind speed is any given fraction of the storm lifetime maximum (normalized) wind speed is a constant. These two empirical findings, taken together, were shown to imply that the probability density f of encountering a normalized hurricane wind speed x at any given point in a storm's lifetime is given by

$$f(x) = -c \ln(x), \quad (1)$$

where $x \equiv v/v_p$, with v being the maximum circular component of the wind and v_p being the potential maximum wind speed. This was shown to be valid over $x_t \leq x \leq 1$, where x_t is a threshold loosely corresponding to the (dimensional) hurricane threshold wind speed of 64 kt (1 kt $\approx 0.5 \text{ m s}^{-1}$). The quantity c is defined as

$$c \equiv \frac{1}{1 - x_t + x_t \ln x_t}. \quad (2)$$

This ensures that the integral of the probability density over x in the interval $x_t \leq x \leq 1$ is unity.

The complementary cumulative probability distribution derived from (1), for normalized circular winds X in excess of normalized wind speed x , is just the integral of (1) over x :

$$P(X > x) = \int_x^1 f(x) dx = c[1 - x + x \ln(x)], \quad (3)$$

where it is understood that $x_t \leq x \leq 1$. This should be interpreted as the probability, given that an event exists, that its normalized wind speed X exceeds x .

Now suppose that individual events are independent of each other, and occur at an annual rate r . Then the annual rate of encountering normalized circular winds in excess of X is $rP(X > x)$ and the annual probability of encountering normalized circular winds in excess of X is

$$\begin{aligned} P_{\text{year}}(X > x) &= 1 - \exp[-rP(X > x)] \\ &= 1 - \exp\{-rc[1 - x + x \ln(x)]\}. \end{aligned} \quad (4)$$

The return period is, by definition, just the inverse of the probability given by (4).

To test the new assumption we are making here, that landfall occurs during a random phase of a storm's life cycle, we compare the predictions of (4) with deductions from best-track data and from large sets of hurricane events synthesized using the method of Emanuel et al. (2006). This method begins by drawing randomly from a space-time probability distribution of genesis events deduced from best-track data (Jarvinen et al. 1984) and then carries the track forward using either a Markov chain based on the best-track data or on a beta-and-advection model (Marks 1992) driven by reanalysis winds (we use the second approach here). Storm intensity is calculated using a deterministic coupled ocean-atmosphere model (Emanuel 1995).

In normalizing the maximum circular wind component of a landfalling storm, it is not always clear what value of potential intensity to use, since it varies along the storm track. This is particularly problematic in the case of storms passing over cold water before making landfall, as in the case of New England hurricanes; here the potential intensity can be small or even zero in the hours before landfall. Thus, here we treat the normalizing potential intensity as an empirical parameter for each location at which hurricane risk is assessed. First we estimate the probability density $f(v)$ of the dimensional maximum circular wind component of historical or synthetic storms. From (1) and the definition of x , we have

$$f(v) = -c \ln(v) + c \ln(v_p). \quad (5)$$

Linear regression on observed distributions of f can then be used to determine the optimal value of v_p as well as

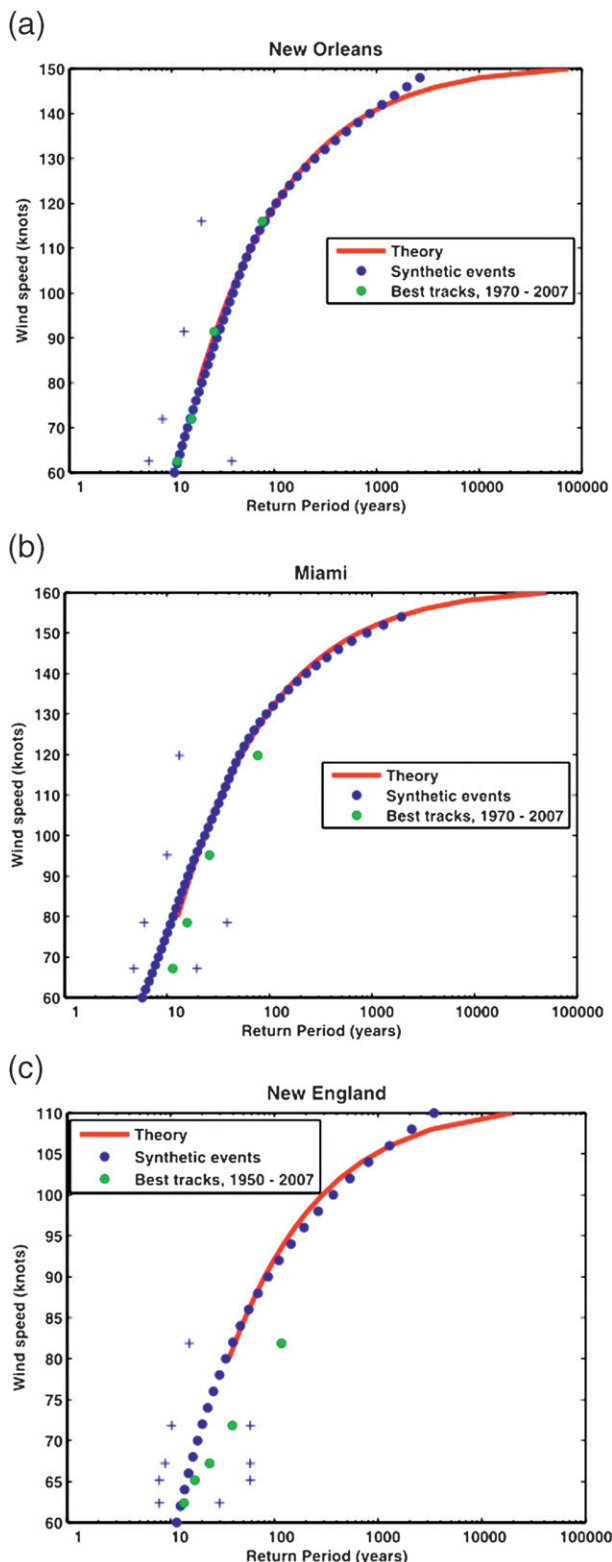


FIG. 1. Return periods of maximum circular wind component within 100 km of (a) New Orleans, (b) Miami, and (c) intercepting the New England coast. Blue dots show return periods deduced from 3000 synthetic events near New Orleans and Miami, and over

the constant c , which, through (2), also yields an optimal value of x_i . We conduct this regression over synthetic events whose circular wind speeds exceed 100 kt to maximize the fit near the important high-intensity end of the distribution, but we limit the upper end of the regression to circular wind speeds for which there are at least 10 events in the synthetic set. We note that fitting to the distribution of storms whose circular wind speeds exceed 100 kt also avoids the problem that the character of the distribution changes for wind speeds below nominal hurricane strength, as mentioned previously.

Figure 1 compares return periods of circular wind component calculated from the inverse of (4) (red curve) with those from sets of synthetic tracks passing within 100 km of New Orleans, Louisiana, and Miami, Florida, and also with tracks passing over the coast of New England, from New York City, New York, to Eastport, Maine (blue dots). The circular wind component is directly calculated in the synthetic track technique. In each case, the regression to find the optimal potential intensity yielded r^2 (correlation coefficient squared) values in excess of 0.9 and significant at the 1% level. Return periods from the synthetic tracks line up reasonably well with the formula given by the inverse of (4). We also show in Fig. 1 return periods estimated from historical hurricane data available from the National Oceanic and Atmospheric Administration National Hurricane Center (Jarvinen et al. 1984). For these historical tracks, we subtract 60% of the translation speed to estimate the circular wind component (Emanuel et al. 2006). To quantify the degree of agreement between return periods derived from the best-track and synthetic event sets, we first estimate the total number of events expected over the period of time that the best tracks are drawn from, assuming that the annual frequency indicated by the synthetic method is correct. We then assume that the actual number of events over that time interval is a random draw from a Poisson distribution with the aforementioned mean. We then derive from the Poisson distribution the integer number of events corresponding to the 5% and 95% quantiles of the distribution. Thus, if the best tracks are indeed drawn from the same distribution as the synthetic tracks, then 90%

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17 000 New England events. The green dots show return periods based on best-track data, and the blue crosses to the right and left denote return periods within which 90% of the best-track return periods are expected to fall if the mean rate given by the synthetic tracks is correct. (The absence of crosses to the right where they exist on the left denotes an infinite return period, or zero count.) The red curve shows return periods based on (4) with a constant, empirically determined potential intensity v_p , the values of which are 149 kt for New Orleans, 161 kt for Miami, and 111 kt for New England.

of them would fall between these limits, which are indicated by crosses in Fig. 1.

Return periods from the synthetic tracks do line up reasonably well with the formula given by the inverse of (4) using the empirically chosen constant potential intensity, and the historical events are mostly contained within the 90% confidence interval. At the high-intensity, long-return period end of the distribution, however, there are so few historical events that they often lie at or near the limits of the 90% confidence range.

The good agreement between the shape given by (4) and the shape of the synthetic-track return periods suggests that, at least for these events, using a constant empirical potential intensity for each landfall location and assuming that landfall occurs during a random phase of the storm's lifetime works well, even for the New England storms. Although the resulting probability distributions are to some extent empirical, they retain the important property that the maximum circular wind component is bounded by a peak potential intensity.

To predict the true maximum wind speed experienced at a point of interest, it is necessary to account for the contribution of the translation velocity to the total storm wind. Here, following Emanuel et al. (2006), we add 60% of the translation velocity to the circular wind component. The probability of experiencing a net wind v_n will be the convolution of the probability of experiencing a circular wind component v with the probability that 60% of the storm's translation velocity is u :

$$P(v_n) = P(v + u) = \int_0^\infty P_v(v_n - u)P_u(u) du, \quad (6)$$

where P_v is the probability density for circular wind component v and P_u is the probability density for 60% of the translation speed, and we have assumed that the translation velocity is independent of the circular wind speed v .¹ The probability density function $P_u(u)$ for 60% of the translation speed is calculated directly from the synthetic tracks, discretized to intervals of 2 kt; following the preceding development, the probability density $P_v(x_e)$ of the modified normalized circular wind component is given by

$$P_v(x_e) = -c \ln(x_e), \quad (7)$$

where $x_e \equiv v/v_p$, with v_p being the empirically determined constant potential intensity and c being given

by (2). As before, the cumulative probability is given by integrating (6) over all values of v_n exceeding a threshold. Figure 2 compares these return periods (the inverse of the cumulative probabilities) with those calculated directly from the synthetic event sets and from historical data, with the confidence interval calculated as with the circular wind component. The agreement between the return periods estimated from the synthetic event set and those calculated from the cumulative distribution based on (7) is very good, and thus the quasi-empirically derived probability functions serve as reasonable continuations of the event data to very long return periods. Agreement with return periods based on historical best-track data is also reasonably good.

3. Comparison with GEV approximations

Generalized extreme-value theory builds on the central limit theorem to derive functional forms for probability distributions near the extreme tails of the distributions. A comprehensive review of the application of GEV theory to the estimation of hurricane return periods using historical hurricane data is provided by Jagger and Elsner (2006), who show that reasonable fits to historical data are provided by the GEV probability distribution. This distribution, given by (8), is the probability of having at least one event in any given year whose wind speed W_n exceeds v_n . In other words, the probability that the maximum wind speed during any given year exceeds v_n is given by

$$P(W_n > v_n) = 1 - \exp \left\{ -r \left[1 + \xi \left(\frac{v_n - \mu}{\sigma_\mu} \right) \right]^{-1/\xi} \right\}, \quad (8)$$

where r is the annual rate of all events, μ is a threshold wind speed, and σ_μ and ξ are the scale and shape parameters. (A separate functional form is obtained for the case $\xi = 0$.) Jagger and Elsner (2006) show that the threshold velocity μ may be estimated from the data using mean residual life plots. We fit the rate, scale, and shape parameters using a peaks-over-threshold model. In this model formulation, we estimate the parameters using the subset of data whose winds exceed μ . Figure 3 shows the results of such an exercise, in which the parameters in (8) have been chosen to optimize agreement with the synthetic track data; the best-fit values of these parameters are listed in Table 1.² In all cases, the fit is

¹ This may not always be justified; for example, high translation speeds may be correlated with strong wind shear, which would in turn usually be associated with smaller circular wind speeds.

² Bear in mind that the frequency given for New England represents the entire coastline, whereas the frequencies for Miami and Boston represent storms passing within 100 km of those points.

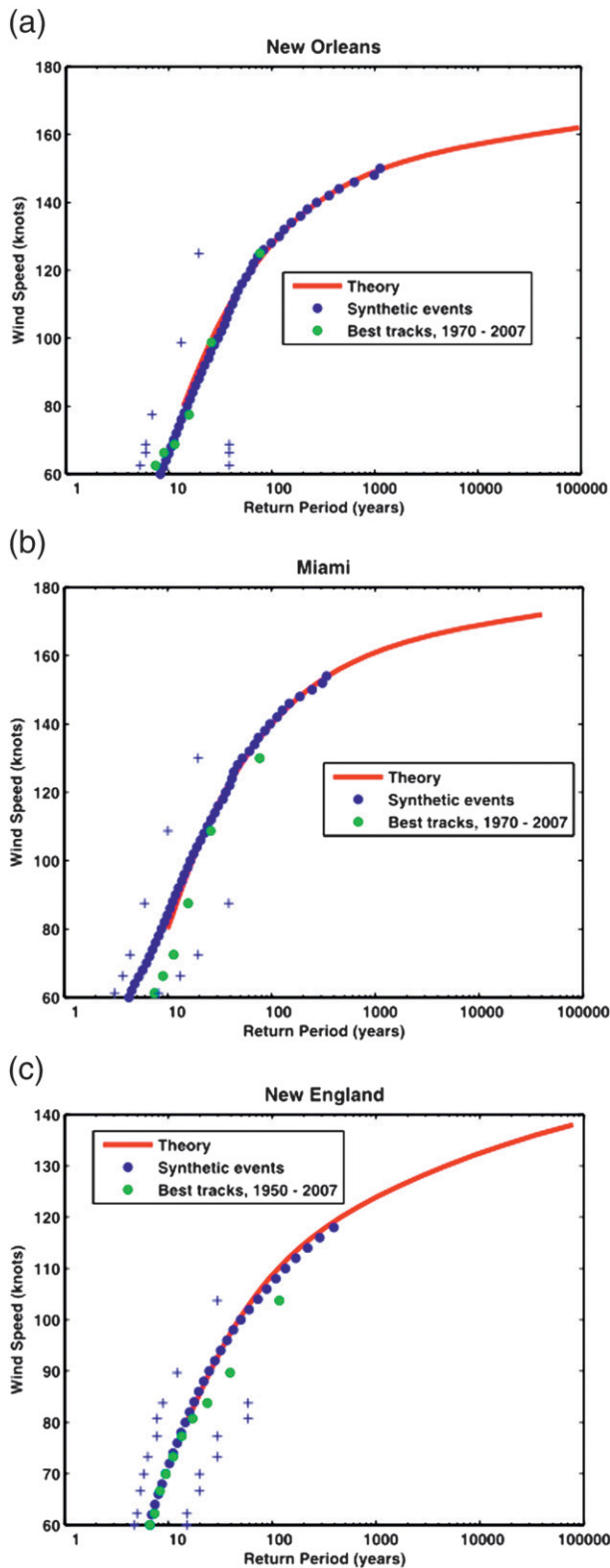


FIG. 2. As in Fig. 1, but for maximum total wind component and the red curve now shows return periods based on (4) and (6).

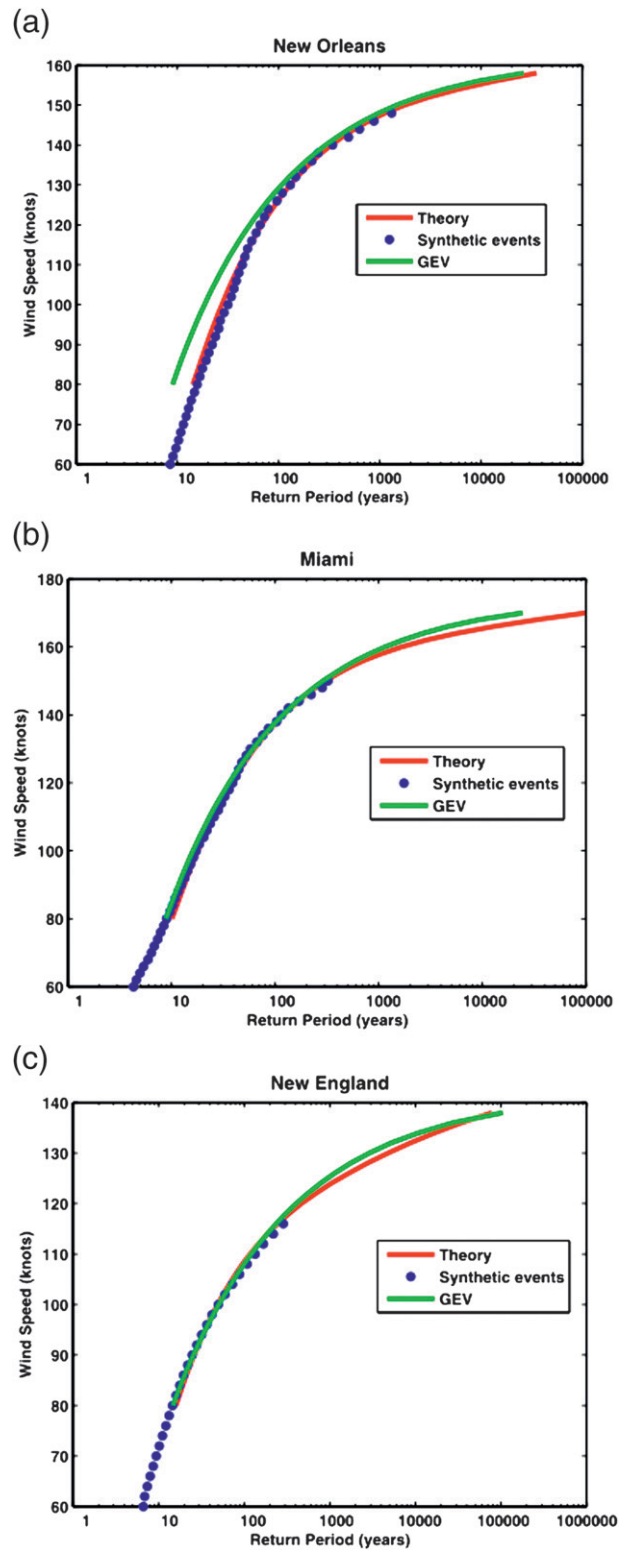


FIG. 3. As in Fig. 2, but omitting the best-track data and showing instead, in green, the best fit to the synthetic storm return periods of the extreme value function (8), with parameter values given in Table 1.

TABLE 1. Parameter values used in (8).

	Miami	New Orleans	New England
μ (kt)	83	64	64
r (events per year)	0.105	0.101	0.105
σ_μ (kt)	35	36	24
ξ	-0.38	-0.37	-0.31

very good and also very close to the probability functions derived here. The rationale for this exercise is to show that the synthetic track data generate realistic data from a statistician's perspective. In our case, we showed that for winds exceeding μ the maximum yearly wind speed distribution matches what one would expect from an actual physical process where the maximum yearly wind speed generally follows a GEV distribution.

Although agreement between the empirical probability functions derived here and GEV distributions is excellent, there may nevertheless be some advantages to the former. Given enough events to make reasonable estimates of $P(u)$ in (8), the quasi-empirical approach used here requires only one free parameter, the potential intensity v_p , which can be estimated from observed distributions of the circular wind component of historical storms. Extreme-value approaches are more sensitive to the maximum observed value in the data to which they are fit than is the approach presented here.

4. Application

To apply the technique described herein to a given tropical-cyclone dataset, one would take the following steps to estimate return periods of maximum storm winds:

- 1) From a dataset of maximum tropical-cyclone wind speeds in storms affecting a particular region, first estimate the circular component of the wind speeds by subtracting 60% of the translation speed from the maximum ground-relative wind speed.
- 2) Retain from above only those events whose maximum circular wind speeds exceed about 65 kt.
- 3) Estimate the probability density $f(v)$ of circular winds from step 2.
- 4) Solve the regression equation $\log(v) = a + bf(v)$ for the coefficients a and b . The optimum potential intensity is then just $\exp(a)$.
- 5) Use (7) and the optimum potential intensity to estimate the normalized probability of maximum circular storm winds.
- 6) Estimate the probability density of 60% of the storm translation speeds.

- 7) Convolve the probability density from step 5 with that from step 6 using (6).
- 8) Find the cumulative probability by summing the result of step 6.
- 9) Find the annual probability P_{year} by multiplying the cumulative probability $P(X > x)$ from step 8 by the observed overall annual rate r from the event set and then find the annual probability using

$$P_{\text{year}}(X > x) = 1 - \exp[-rP(X > x)].$$

- 10) If desired, the return period in years is then the inverse of the result of step 9.
- 11) Pointwise confidence limits can be found by first finding the confidence intervals on the annual rate r assuming that the rate is drawn from, say, a Poisson distribution, and then repeating steps 9 and 10 using the limiting rates. Calculate the difference between these confidence limits and the best estimate found in step 9. Then, in solving the regression equation in step 4, calculate confidence limits of the potential intensity and repeat steps 5–10 using the limiting values of potential intensity; again calculate the difference between these limits and the best estimate found originally in step 9. Estimate the net confidence limits as the square root of the sum of the squares of the differences found above and use step 10 to find the associated return periods.

Figure 4 shows the results of applying this procedure to the historical data only for New Orleans, Miami, and the coast of New England. These results can be compared with those based on the much larger numbers of synthetic tracks shown in Fig. 2. In general, the estimates based strictly on historical data line up well with those based on the larger set of synthetic tracks, except in the case of New England storms, for which deductions based only on historical data yield longer return periods of high-intensity events. Note also that the most intense event in the New England case lies outside the 95% confidence bounds. This was a storm that intercepted a coastal gate that extends well east of Cape Cod, and represents Hurricane Gerda of 1969, which passed well to the southeast of New England but qualifies as a New England coastal event under the selection criteria we used.

5. Summary

Open-ocean hurricane statistics suggest that the circular component v of storm winds obeys a probability distribution proportional to $\ln(v_p/v)$, where v_p is an upper bound, which may be determined empirically from

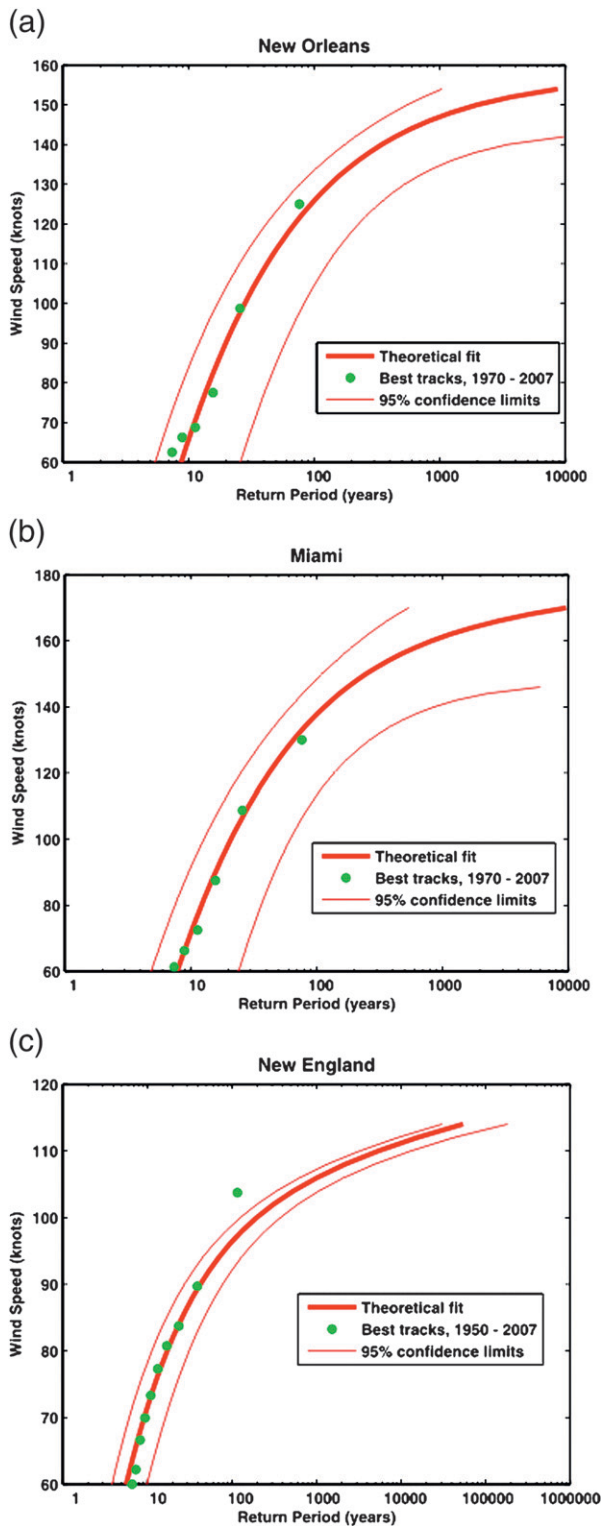


FIG. 4. Results of using the proposed procedure to estimate from limited historical data return periods of maximum total wind component within 100 km of (a) New Orleans, (b) Miami, and (c) intercepting the New England coast. The green dots represent direct estimates from historical data, as in Fig. 2. The thick red curve and associated 95% confidence limits (thin red curves) are calculated according to the procedure outlined in section 4.

historical hurricane data. This will also describe the probability density of winds at landfall, if it can be assumed that landfall occurs at a random phase of the storm's lifetime. Convolving this with an empirically determined probability that an empirical fraction of the storm's translation speed is u yields an expression for the probability density of maximum landfall wind speed, from which a return period for that wind speed can be derived. Such an expression should cover the entire range of hurricane wind speeds, and the results, presented in Figs. 2 and 4, suggest that the underlying assumptions are valid. This provides a means of extrapolating finite hurricane wind data to large return periods and, as shown in Fig. 3, is fully consistent with extreme-value theory, whose application to hurricane return periods has been described in detail by Jagger and Elsner (2006). The technique described here should, however, yield probability densities that are valid over the whole range of hurricane winds and requires only a single quasi-empirical parameter—the upper bound on the circular wind speed v_p . The procedure for estimating return periods using our technique is summarized in section 4.

The results presented here pertain to maximum winds within storms passing through certain defined regions; return periods for winds at a fixed point can be derived if the distribution of surface winds within a storm is known.

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