Annu. Rev. Fluid Mech. 1991. 23: 179–96

THE THEORY OF HURRICANES

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KEY WORDS: tropical cyclones, convection, moist convection, finite-amplitude instability

INTRODUCTION

The hurricane remains one of the outstanding enigmas of fluid dynamics. This is so, in part, because the phenomenon is comparatively difficult to observe and because no laboratory analogue has been discovered. To this it must be added that hurricanes have received surprisingly little attention from the theoretically inclined fluid dynamicist, perhaps owing to an understandable tendency to avoid problems that involve complex thermodynamics and lack laboratory analogues. Yet hurricanes involve a rich spectrum of fluid-dynamical processes, including rotating, stratified flow dynamics, boundary layers, convection, and air-sea interaction; as such, they provide a wealth of interesting and consequential research problems. This article reviews recent developments in the theory of hurricanes and delineates the important remaining scientific challenges.

THE MATURE HURRICANE: A NATURAL CARNOT ENGINE

About 80 rotating circulations known generically as tropical cyclones form over the tropical oceans each year. Of these, roughly 60% reach an intensity (maximum winds in excess of 32 m s^{-1}) that qualifies them as hurricanes, a term applied only in the Atlantic and eastern Pacific. (Similar storms in other parts of the world go by different names.) An excellent review of the climatology and observed characteristics of these storms is provided by Anthes (1982). Here we use the term *hurricane* in place of the generic term *tropical cyclone*.

0066–4189/91/0115–0179$02.00

The mature hurricane may be idealized as an axisymmetric vortex in hydrostatic and rotational balance. The cyclonic azimuthal flow reaches its maximum intensity near the surface and decreases slowly upward, becoming anticyclonic near the top of the storm, roughly 15 km above the surface. This flow configuration corresponds to a warm core structure with maximum temperature perturbations on isobaric surfaces well in excess of 10°C, highly concentrated at high levels near the center of the vortex. The radius at which the azimuthal winds peak ranges from 10 to 100 km near the surface and generally increases with height. Inside the radius of maximum winds the core is nearly in solid-body rotation, while outside the core the winds fall off gradually with radius, obeying approximately an $r^{-1/2}$ law. No low-level circulation can be detected outside a finite radius ranging from 100 to 1000 km. While the geometric size of hurricanes ranges over an order of magnitude, their intensity, as measured by maximum wind speeds or central pressure deficit, bears no perceptible relation to their size (Merrill 1984). While axisymmetry is a good approximation for the cyclonic flow, the upper anticyclone is usually highly asymmetric, with the bulk of the flow confined to one or two anticyclonically curving jets.

The transverse circulation of a mature hurricane is thermally direct (except in the eye) and consists of radial inflow within a frictional boundary layer roughly 1–2 km deep, ascent mostly within a narrow, outward-sloping eyewall 5 to 100 km from the center, and radial outflow in a thin layer at the storm's top. The eyewall looks like a coliseum of convective clouds surrounding an eye that is often nearly free of clouds. The transverse circulation in the eye is mechanically maintained and thermally indirect, with warm air slowly subsiding near the center. Clouds and precipitation outside the eyewall are usually organized in one or more cyclonically curved spiral bands of order 10 km in width, extending to a height of from 3 to 15 km. The axisymmetric structure of the mature hurricane is summarized in Figure 1.

Kleinschmidt (1951) first recognized that the energy source of hurricanes resides in the thermodynamic disequilibrium between the tropical atmosphere and oceans. This is reflected not in an actual temperature difference between air and sea, which in the tropics is usually less than 1°C, but rather in the undersaturation of near-surface air. The evaporation of water transfers heat from the ocean, whose effective heat capacity is enormous in comparison with the overlying atmosphere. To bring the troposphere into thermodynamic equilibrium with the ocean would require the transfer of roughly 10^8 J m^{-2} of energy from the ocean.

The rate of transfer of heat from the ocean to the atmosphere is a function of the surface wind speed. If the ocean were a flat surface, the

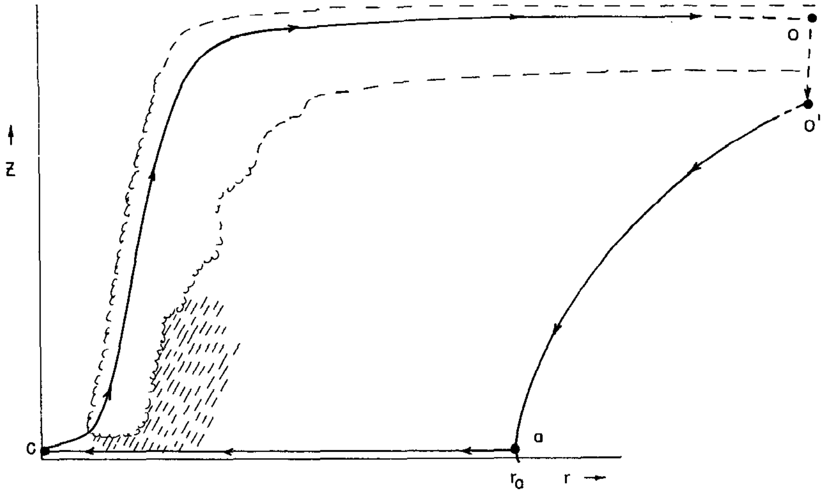

Figure 1 The hurricane Carnot cycle. Air begins spiraling in toward the storm center at point *a*, acquiring entropy from the ocean surface at fixed temperature T_s. It then ascends adiabatically from point *c*, flowing out near the storm top to some large radius, denoted symbolically by point *o*. The excess entropy is lost by export or by electromagnetic radiation to space between *o* and *o′* at a much lower temperature T_0. The cycle is closed by integrating along an absolute vortex line between *o′* and *a*. The curves *c–o* and *o′–a* also represent surfaces of constant absolute angular momentum about the storm's axis.

transfer would increase linearly with wind speed, but the increasing roughness of the sea surface leads to a somewhat greater dependence on wind. The actual rate of heat transfer is a subject of much controversy and research. The dependence of the transfer rate on wind is the principal feedback mechanism that allows hurricanes to develop. In its essence, the hurricane may be thought of as a wind-induced surface heat exchange instability, in which increasing surface winds lead to increased heat transfer from the sea, which leads to intensification of the storm winds, and so on.

The energy cycle of the mature hurricane has been idealized by the author (Emanuel 1986) as a Carnot engine that converts heat energy extracted from the ocean to mechanical energy. In the steady state, this mechanical-energy generation balances frictional dissipation, most of which occurs at the air-sea interface. The idealized Carnot cycle is illustrated in Figure 1. Carnot's theorem may be easily derived from Bernoulli's equation and the first law of thermodynamics. The former states that along streamlines or absolute vortex lines in a steady system,

$$d\left(\frac{1}{2}|\mathbf{V}|^2\right)+d(gz)+\alpha dp+\mathbf{F}\cdot d\mathbf{l}=0, \tag{1}$$

where **V** is the vector velocity, g the acceleration of gravity, z the height above the surface, α the specific volume, p the pressure, **F** the frictional force per unit mass, and $d\mathbf{l}$ an incremental distance along a streamline or absolute vortex line. An energy equation may be derived by substituting from the first law of thermodynamics, which in a moist system may be written

$$T\,ds = C_p\,dT + d(L_v q) - \alpha\,dp, \tag{2}$$

where s is the specific total entropy content of air (including the water vapor), C_p is the heat capacity of air at constant pressure, L_v is the latent heat of vaporization, and q is the mass of water vapor per unit mass of air. The above neglects the heat capacity of water substance and the effect of water on the density of an air-water vapor mixture. [An exact treatment is provided in Emanuel (1988).]

Eliminating $\alpha\,dp$ between (1) and (2) gives

$$d\left(\frac{1}{2}|\mathbf{V}|^2 + gz + C_p T + L_v q\right) - T\,ds + \mathbf{F}\cdot d\mathbf{l} = 0. \tag{3}$$

This can be integrated around a closed circuit (Figure 1), the first three branches of which are streamlines. The fourth branch is an absolute vortex line. Then

$$\oint T\,ds = \oint \mathbf{F}\cdot d\mathbf{l}, \tag{4}$$

which illustrates that in the steady state, heating balances friction.

Most of the heat input to a hurricane is from the sea surface. As air flows radially inward along the surface, its temperature is observed to be held nearly constant by a combination of turbulent fluxes and radiative transfer from the ocean. Thus, in the first branch, one has

$$\int_a^c T\,ds = T_s \Delta s, \tag{5}$$

where $\Delta s (\equiv s_c - s_a)$ is the difference between the entropies of air near the storm center and in the ambient environment. The first law of thermodynamics can be used to derive the entropy s (neglecting the heat capacity of water substance and other small terms):

$$s = C_p \ln T - R \ln p + \frac{L_v q}{T}, \tag{6}$$

where R is the gas constant for air. Then

$$T_s \Delta s = RT_s \ln \frac{p_a}{p_c} + L_v(q_c - q_a), \tag{7}$$

where the subscripts c and a refer to quantities evaluated along the surface at the storm center and at the starting point, respectively. The saturation of air near the storm center limits the entropy increase to

$$T_s \Delta s_{\max} = RT_s \ln \frac{p_a}{p_c} + \frac{L_v}{T_s}(q_c^* - q_a), \tag{8}$$

where q_c^* is the saturation mixing ratio at the storm center and is a function of p_c and T_s. From an approximate integration of the Clausius-Clapeyron equation, we have, to an excellent approximation,

$$q_c^* = \frac{3.802\,\text{mbar}}{p_c} \exp\left[\frac{17.67 T_s}{243.5 + T_s}\right], \tag{9}$$

in which T_s is expressed in degrees Celsius.

In the second leg of the Carnot cycle, air ascends within deep convective clouds in the eyewall of the storm and then flows out to large radius. It is important to note that when water vapor is properly included in the description of the thermodynamic state of the system, this leg is very nearly reversible and adiabatic, so that $ds = 0$. Some researchers define a dry entropy [without the last term in (6)] and are forced to deal with very large sources of dry entropy in the ascent region, where there is large conversion of latent to sensible heat. The problem with such an approach is that the diabatic source is purely a function of the flow itself and cannot be properly regarded as external. Attempts to regard the condensation heat source as external lead to the oft-repeated statement that hurricanes are driven by condensation of water vapor, a view rather analogous to that of an engineer who proclaims that elevators are driven upward by the downward acceleration of counterweights. Such a view, though energetically correct, is conceptually awkward; it is far more natural to consider the elevator and its counterweight as a single system driven by a motor. Here we adopt a similar strategy by dealing with the most conserved thermodynamic variable available, the total specific entropy.

The altitude that the outflow asymptotically approaches is determined by the requirement that it be neutrally buoyant with respect to the environment. That is, the temperature in the outflow blends smoothly into the ambient temperature profile without shocks. The ability of the ambient atmosphere to control the interior structure of the vortex through this requirement is consistent with the fact that hurricanes are *subcritical* vortices (i.e. internal gravity-inertia waves may propagate inward against the outflow).

In the third leg of the Carnot cycle, air descends slowly in the lower stratosphere, retaining a nearly constant temperature T_o while losing heat by electromagnetic radiation to space. In this leg, then,

$$\int_{o}^{o'} T\,ds = -T_o \Delta s, \tag{10}$$

with Δs given by (7).

Real hurricanes are open systems that continually exchange mass with their environments. Nonetheless, the Carnot cycle can be closed by integrating the Bernoulli equation along a fourth branch that is an absolute vortex line of the system, which is also a surface of constant absolute angular momentum about the storm center. Since we are no longer following air parcels, the relationship implied by the first law of thermodynamics is not strictly valid. It has been shown by Emanuel (1988), however, that there is little thermodynamic contribution from this last leg, owing to the convective neutrality of the ambient atmosphere. Thus adding (7) and (10) gives an expression for the frictional work from (4):

$$\varepsilon T_s \Delta s = \oint \mathbf{F} \cdot d\mathbf{l}, \tag{11}$$

where ε is the thermodynamic efficiency of the Carnot cycle, given by

$$\varepsilon \equiv \frac{T_s - T_o}{T_s}.$$

For typical atmospheric conditions in the tropics, $\varepsilon \sim 1/3$.

Most of the frictional energy loss in the cycle occurs in the surface boundary layer and at large radius in the outflow, where the air's original angular momentum must ultimately be restored. This latter loss is idealized as occurring at infinite radius. It may be estimated from conservation of absolute angular momentum (M) about the storm center:

$$M = rV + \frac{1}{2} f r^2, \tag{12}$$

where V is the azimuthal velocity, and f is twice the local vertical component of the Earth's angular velocity. From (12), it follows that

$$V^2 = \left(\frac{M}{r} - \frac{1}{2} f r\right)^2 = \frac{M^2}{r^2} - fM + \frac{1}{4} f^2 r^2. \tag{13}$$

The loss of kinetic energy in the third branch of the cycle is then

$$\int_{o}^{o'} \mathbf{F}\cdot d\mathbf{l} \simeq \lim_{r\to\infty}\frac{1}{2}\Delta V^2 = -\frac{f}{2}(M_c - M_a) = \frac{1}{4}f^2 r_a^2, \tag{14}$$

assuming that V is zero at the beginning of the cycle.

The frictional loss in the boundary layer may be related to the radial pressure drop by integrating the Bernoulli equation (1) inward along the first branch:

$$\int_{a}^{c} \mathbf{F}\cdot d\mathbf{l} = -\int_{a}^{c} \alpha\, dp = -\int_{a}^{c} RT\, d\ln p = RT_s \ln\frac{p_a}{p_c}, \tag{15}$$

where we have made use of the ideal gas law, $\alpha = RT/p$. Thus, the frictional loss is directly proportional to the logarithm of the pressure deficit.

Substituting (15) and (14) into (11) then gives

$$\varepsilon T_s \Delta s = RT_s \ln\frac{p_a}{p_c} + \frac{1}{4}f^2 r_a^2, \tag{16}$$

with an upper bound for Δs provided by (8). The last term in (16) reflects the energy put into the upper anticyclone; this always detracts from the intensity of the surface cyclone, as reflected in the surface pressure deficit. The stipulation that $\ln(p_a/p_c)$ must be positive leads to a restriction on the magnitude of r_a.

Given r_a, p_a, q_a, T_s, and T_o, a lower bound on p_c is obtained by using (8) [with (9)] in (16). Unless r_a is unusually large, it has little effect on this estimate. Figure 2 shows this lower bound (neglecting r_a) as a function of T_s and T_o, using a standard mean surface pressure p_a and assuming an ambient near-surface relative humidity of 75%. One curiosity of this calculation is that there is no solution for sufficiently large T_s or small T_o. In this regime (the *hypercane* regime) the Carnot cycle becomes unstable owing to a very large heat input from isothermal expansion: the more intense the storm, the lower the central pressure, giving greater isothermal heat input, which intensifies the storm, and so on.

An estimate of the minimum central pressure p_c from (16) using September climatological conditions is shown in Figure 3, together with locations and central pressures of the most intense hurricanes on record. Clearly, a few hurricanes reach the predicted upper bound on intensity, but the vast majority (not shown in Figure 3) do not.

Two enigmas emerge from Figure 3: Given that the energy potential for hurricanes is large over much of the tropical oceans, why are hurricanes so rare? And even when a hurricane occurs, why do so few reach the theoretical upper bound on intensity?

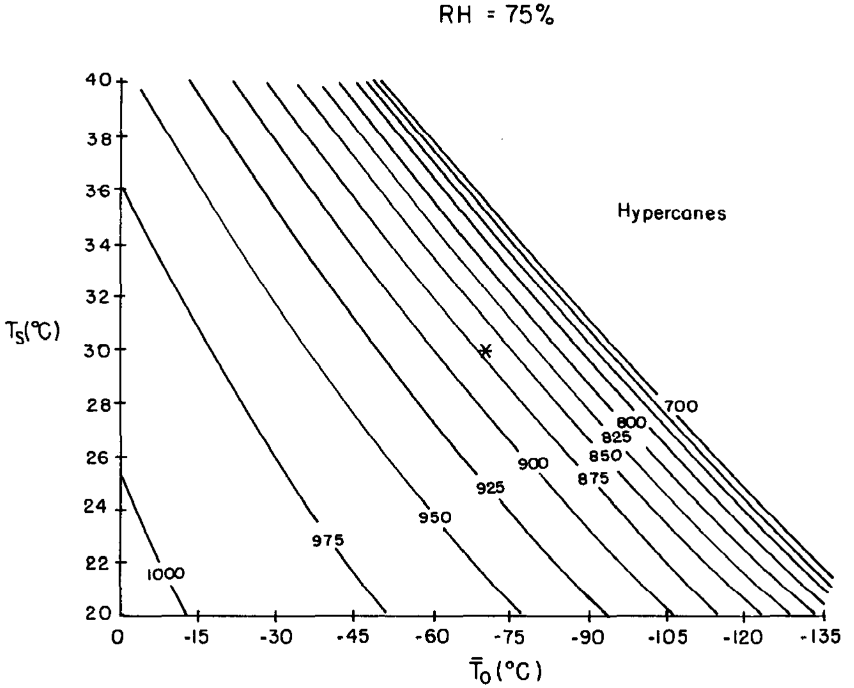

Figure 2 The minimum sustainable central pressure (in millibars) as a function of sea-surface temperature (T_s) and mean outflow temperature ($\bar{T}_o$), assuming an ambient surface pressure of 1015 mbar and an ambient near-surface relative humidity (RH) of 75%.

HURRICANE GENESIS AS AN EXAMPLE OF FINITE-AMPLITUDE INSTABILITY

Forecasters have known for some time that hurricanes never arise spontaneously, even if the environmental conditions are considered favorable. Rather, they always emerge from preexisting circulations of presumably independent dynamical origin. Yet some of the early theories attempted to treat tropical cyclogenesis using linear theory.

The Failure of Linear Stability Theory

Early investigations focused exclusively on the dynamics of moist convection (see the review by Yanai 1964). These studies suffered the result that disturbances of the smallest horizontal scale should develop most rapidly and so cannot explain the scale of hurricanes. Charney & Eliassen (1964) recognized this defect and proposed a theory called Conditional

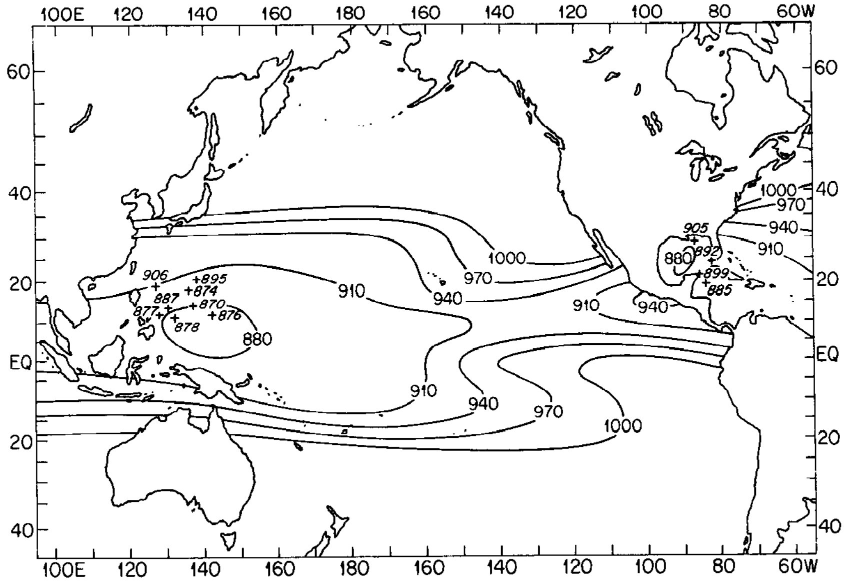

Figure 3 Minimum sustainable central pressure of tropical cyclones (in millibars) under September climatological conditions. The central pressures of some of the most intense tropical cyclones on record are shown by italicized numbers and crosses.

Instability of the Second Kind (CISK) that sought to explain the scale of hurricanes by requiring the convection to occur in proportion to the upward motion induced by the frictional boundary layer of a vortex. Here, as in other similar works, dry thermodynamics was used and the heating was externalized as much as possible—in this case by making it proportional to the frictionally induced vertical velocity and by distributing it in the vertical by an arbitrary function. While not explicitly related to the concept of convective instability, CISK implicitly requires a reservoir of convective energy to operate. By choosing the right ad hoc parameters, a large-scale amplifying disturbance can be obtained.

While always controversial, CISK has remained popular among a subset of meteorologists, in part because of the mathematical simplicity of the representation of cumulus convection suggested by Charney & Eliassen (1964), but perhaps also because of the enormous appeal of the notion of the latter authors that "cumulus clouds and the large scale circulation cooperate, rather than compete." CISK entirely disregards the importance of enhanced heat fluxes from the sea, emphasized by earlier researchers such as Kleinschmidt (1951) and Riehl (1954), and can be shown to rely energetically on stored potential energy in the tropical atmosphere. This stored energy would be reflected in a thermodynamic stratification of the atmosphere that allows boundary-layer air to become positively buoyant when displaced upward a sufficient distance; this is called a state of conditional instability. Recent work by Betts (1982) and Xu & Emanuel (1989), however, demonstrates that mean thermodynamic profiles over tropical oceans are almost precisely constant, as is true of most other forms of high-Rayleigh-number convection, so that there is very little variation of free atmospheric temperature in the absence of boundary-layer entropy variations. Stored energy occurs occasionally over midlatitude continents as a result of particular arrangements of topographical features. Thus CISK predicts, in direct contradiction to observation, that nascent hurricanes should be common over midlatitude continents during the warmer months but absent over the oceans.

The weight of evidence suggests that CISK be rejected as a useful hypothesis and that attempts be undertaken to find a finite-amplitude instability based on the thermodynamic interaction of the atmosphere and ocean.

Results of Fully Nonlinear Integrations

There have been a number of successful numerical simulations of hurricanes, dating back to the work of Ooyama (1969). Such modeling has advanced to the point where cumulus clouds themselves can be explicitly (albeit crudely) resolved, at least in axisymmetric models. Virtually all numerical simulations contain representations of the sea-air heat flux necessary to maintain hurricanes, but nearly all begin with decidedly unstable thermal stratification. This creates an ambiguity in interpreting the initial spin-up of the cyclone, since unstable convection artificially constrained to two dimensions will lead to upscale energy transfer, as shown originally by Fjørtoft (1953). This no doubt occurs in the axisymmetric numerical simulation by Yamasaki (1977), whose integration begins with highly unstable stratification but whose model allows no surface heat fluxes. The result is a thunderstorm-scale cyclone that reaches maturity in 40 hr, considerably less time than is generally observed in nature. At the other extreme, Rotunno & Emanuel (1987) integrate an axisymmetric, nonhydrostatic model that resolves cumulus convection, starting from an initial state that is neutral to cumulus convection. The evolution of their simulated cyclone is portrayed in Figure 4, which shows three experiments that differ only in the amplitude and geometry of the initial vortex. It is evident that amplification occurs only when the initial vortex is sufficiently intense and concentrated. In this simulation, the amplification may be considered to result from a finite-amplitude instability, in accord with nature; the resting atmosphere is metastable. The mature model cyclone

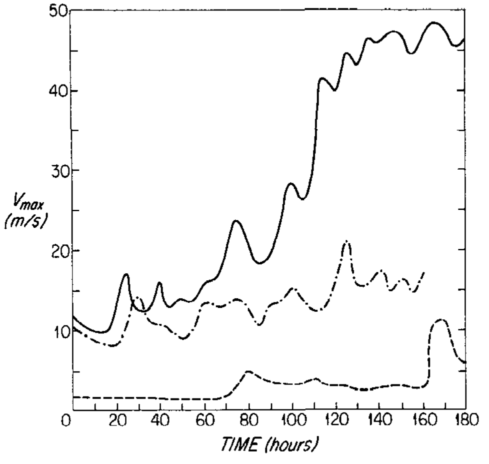

Figure 4 Maximum azimuthal velocity as a function of time (in hours) for three numerical simulations of hurricanes from Rotunno & Emanuel (1987). The *solid curve* denotes the control simulation. The *dashed curve* shows the results of an experiment identical to the control but starting with an amplitude of 2 m s^{-1}. The *dash-dot curve* denotes a third experiment in which the initial vortex has twice the radial dimensions of the control.

has a structure and amplitude that are quite compatible with observed hurricanes, and the amplitude conforms nearly exactly with the theoretical prediction based on the Carnot cycle, as discussed previously. Assuming that the physical reasons for the finite-amplitude nature of the instability in the numerical model and in nature are similar, we may inquire about the latter by using the fields produced by the model. As is often the case, however, the output of complex numerical models is as inscrutable as nature herself.

A Minimal Nonlinear Model

One approach to understanding complex phenomena is to reduce them to their barest essence in idealized mathematical models. In doing so, one hopes to retain the fundamental physics while discarding embellishments, taking care not to throw away the baby with the bath water. (A worse fate yet is to end up with the wrong baby.)

A minimal hurricane model has been constructed by the author (Emanuel 1989). (The model may be run on personal computers and is well documented. It is available on request.) The flow in the model is taken to be axisymmetric and in hydrostatic and gradient wind balance, except in a frictional boundary layer. The equations of the model are transformed into a coordinate system in which one of the coordinates, R, is proportional to the absolute angular momentum per unit mass about the storm center:

$$\frac{1}{2}fR^2 \equiv rV + \frac{1}{2}fr^2, \tag{17}$$

where f is considered constant. Since R is conserved in the absence of friction, this formulation eliminates radial advections, except in the frictional boundary layer. Moreover, the theory of slantwise convection (e.g. Emanuel 1983) shows that the relevant convectively neutral state is one in which air parcels are neutrally buoyant along angular-momentum surfaces, rather than along vertical surfaces.

The structure of the minimal model is summarized in Figure 5. The dependent variables are the physical radius r, predicted at the top of the model and at the top of the boundary layer; the mass streamfunction ψ, predicted at the middle level and diagnosed at the top of the boundary layer; temperature, represented by a saturation entropy s^*, predicted at the middle level and in the lower layer; and actual entropy s, predicted in the boundary layer and in the lower layer. (The saturation entropy is the entropy the air would have if it were saturated at the same temperature and pressure. It is a state variable.)

The essence of the finite-amplitude nature of the hurricane instability appears to rely on the existence of a spectrum of different types of wet convection, represented in the model by only two categories: shallow convection, which penetrates only to the lower layer; and deep convection, which extends through the depth of the model atmosphere. These two forms differ in that the former does not precipitate while the latter does. Within deep, precipitating clouds, the latent heat acquired from the sea is mostly converted into sensible heat, while shallow clouds produce no net

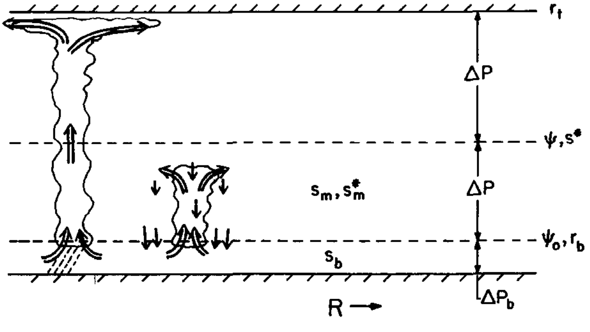

Figure 5 Vertical structure of the simple balanced model. The dependent variables are physical radii r_t and r_b, saturation entropies s^* and s_m^*, and actual entropies s_b and s_m. Deep clouds occur whenever $s_b > s^*$, while shallow clouds are present when $s_b > s_m^*$. The shallow clouds do not precipitate and thus produce no net heating.

heating, since all the condensed water is ultimately reevaporated. But shallow clouds do stabilize the atmosphere by exchanging the high-entropy, boundary-layer air with low-entropy air from the lower troposphere. The entropy minimum in the lower troposphere above the boundary layer, which results from the large subsaturation there, is a normal feature of the tropical atmosphere and is crucial for understanding the finite-amplitude nature of tropical cyclogenesis. In the minimal model, shallow clouds occur whenever there is local convective instability in the lower atmosphere (i.e. when $s > s_m^*$, where s_m^* is the saturation entropy of the lower layer). Deep clouds occur when $s > s^*$.

The fundamental dimensional parameters governing the behavior of the model are the Coriolis parameter f, the depth H of the troposphere, the dimensionless exchange coefficient C_D that appears in the bulk aerodynamic formulae by which heat and momentum fluxes from the ocean are calculated, and a measure of the sea-air thermodynamic disequilibrium:

$$\chi_s \equiv (T_s - T_t)(s_0^* - s_a), \tag{18}$$

where T_s is the ocean temperature, T_t the ambient temperature of the tropopause, s_0^* the saturation entropy of the ocean surface, and s_a the entropy of the normal tropical atmosphere near sea level. The quantity $s_0^* - s_a$ represents the thermodynamic disequilibrium of the atmosphere-ocean system. When the governing equations are suitably normalized, very few nondimensional parameters are needed to describe the system. The most important of these are those describing the initial vortex and the magnitude of the entropy minimum in the lower troposphere. The scaling factors for time, length, and velocity are listed in Table 1. Note that all of the scales depend on χ_s, and that these scales are quite different from those of baroclinic cyclones.

Table 1 Hurricane scales

Quantity	Scale[a]	Typical value
Length	$\chi_s^{1/2} f^{-1}$	1000 km
Time	$C_D^{-1} H \chi_s^{-1/2}$	16 hr
Azimuthal velocity	$\chi_s^{1/2}$	60 m s^{-1}
Radial velocity	$\frac{1}{2} C_D \chi_s f^{-1} H^{-1}$	10 m s^{-1}
Vertical velocity	$C_D \chi_s^{1/2}$	6 cm s^{-1}

[a] Scale parameters: $H \equiv$ depth of convecting layer, $f \equiv$ twice the local vertical component of Earth's angular velocity, $C_D \equiv$ dimensionless exchange coefficient for surface fluxes, $\chi_s \equiv$ thermodynamic disequilibrium parameter, defined by Equation (18).

Figure 6 shows the evolution of the maximum (dimensionless) wind with (dimensionless) time for four different experiments. The control run begins with a warm-core vortex of amplitude 0.22, which decays with time initially but ultimately amplifies to a quasi-steady intense cyclone. The second experiment is identical to the first except that the initial amplitude is 0.05. This vortex never amplifies, which demonstrates the finite-amplitude nature of instability in the simple model. The third and fourth experiments are identical to the first two except that shallow clouds are omitted. These simulations exhibit linear instability in the sense that small perturbations can amplify. Clearly, the finite-amplitude nature of cyclogenesis in the simple model depends on the existence of weakly precipitating convection. Why?

The Nature of Moist Convective Adjustment

Consider two extremely different convective processes. In the first, we allow only nonprecipitating convection. Suppose we cool the middle troposphere by, say, adiabatic lifting of the (dry) middle-troposphere air. This destabilizes the column to convection, but since the convection does not precipitate, its ensemble-average effects do not include heating and the adiabatic cooling is thus unopposed. But the convection *does* drive the column back toward neutrality by importing low-entropy air into the boundary layer from the middle troposphere (whose entropy is usually substantially less than that of the boundary layer). Hence, *nonprecipitating convection forces the boundary-layer entropy to follow changes in free-atmosphere temperature.* In the second process, we suppose that *all* of the water condensed in clouds falls out as rain. Now, when the middle troposphere is cooled, at least some of the cooling is opposed by the net heat released in precipitating clouds. *Precipitating convection drives the atmosphere toward neutrality, in part, by forcing the free-atmosphere*

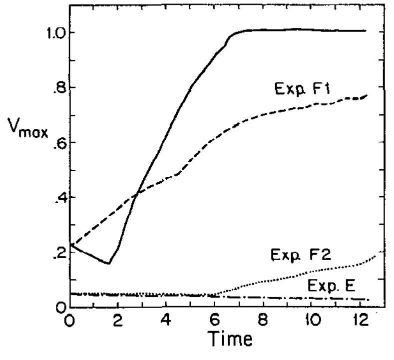

Figure 6 Evolution of maximum dimensionless velocity with dimensionless time for four experiments with the simplified model. The control run is shown by the *solid line*. Experiment E is identical to the control except that it begins with a smaller amplitude. Experiments F_1 and F_2 are identical to the first two experiments except that shallow clouds are omitted.

temperature to follow changes in the boundary-layer entropy. Generalizing from these two examples, we assert that only a fraction of forced cooling of the middle troposphere will be opposed by convective heating in a conditionally neutral mean atmosphere. This fraction is approximately equal to the precipitation efficiency, which is the fraction of condensed water that falls out of the system.

From this argument we may understand the essence of the finite-amplitude nature of tropical cyclogenesis. When a weak vortex is placed in contact with the sea surface, frictional inflow forces upward motion in the free atmosphere near the vortex core. (Other processes may force upward motion in real cyclones.) Since the initial convection will no doubt have a precipitation efficiency less than unity, not all of the adiabatic cooling associated with the ascent of dry (low-entropy), middle-tropospheric air will be opposed by convective heating, and the central core will cool. Moreover, the boundary-layer entropy near the core will decrease by the action of downdrafts so as to keep the column neutral to moist convection. Thus, we have no more than the classical spin-down of a balanced, stratified vortex on a rigid surface, but with the static stability N^2 replaced by an effective stability approximately equal to $N^2(1-\varepsilon_p)$, where ε_p is the precipitation efficiency.

Opposing this tendency is the anomalous flux of (mostly latent) heat from the ocean into the boundary layer, associated with the anomalous surface winds. Initially, this flux can only partially compensate for the reduction in boundary-layer entropy due to the import of low-entropy air from the middle troposphere by the convective downdrafts. The surface fluxes vary with approximately the first power of the surface wind speed, while the drag (and thus the frictionally induced vertical motion) increases more nearly as the square of the wind speed, so that, if anything, the net cooling increases with initial vortex amplitude. But after some time has elapsed, the entropy of the middle troposphere increases owing to the upward flux of high, boundary-layer entropy by low-precipitation-efficiency convection. *When the entropy of the middle troposphere becomes large enough, the import of low-entropy air into the boundary layer by downdrafts can no longer compensate for enhanced surface heat fluxes, and the boundary-layer entropy increases together with the free-atmospheric temperature.* The vortex amplifies. This is clearly a nonlinear effect. Examination of the thermodynamic fields of both the simple model and the complete model supports this heuristic view. The obvious observational test of this idea is to find out whether the saturation of a deep column of tropospheric air is a necessary and sufficient condition for the development of tropical cyclones.

SUMMARY AND REMAINING PROBLEMS

The mature hurricane is a Carnot engine driven by the thermodynamic disequilibrium between the tropical oceans and atmosphere. Air spiraling radially inward in the boundary layer is brought closer to thermodynamic equilibrium with the ocean by large wind-induced heat fluxes; it then rises nearly (moist) adiabatically to great altitudes, where the excess heat is exported or lost by electromagnetic radiation to space. The mechanical energy available from this cycle is a thermodynamic efficiency ε, multiplied by the surface temperature and by the difference between the saturation entropy of the ocean surface (at the central pressure of the hurricane) and the undisturbed boundary-layer entropy. The efficiency is

$$\varepsilon \equiv \frac{T_s - T_o}{T_s},$$

where T_s is the sea-surface temperature, and T_o is the mean temperature at which heat is exported by or lost from the storm's high-level outflow. Its typical magnitude is 1/3.

While the energy source for mature hurricanes has been recognized since at least the time of Kleinschmidt (1951), controversy remains about the energetics and dynamics of hurricane genesis. Observational evidence and forecasting experience favor the idea that hurricanes result from a finite-amplitude instability: Weak disturbances are often observed to decay even under favorable environmental conditions, and hurricanes are, after all, rare despite the nearly ubiquitous presence of an energy reservoir (see Figure 3).

In the early 1960s the theory known as Conditional Instability of the Second Kind (CISK) was proposed to explain hurricane genesis. In this theory, the ocean surface serves as a sink for momentum but not as a source of heat; the spin-up relies on convective energy stored in the atmosphere. This theory is suspect for a variety of reasons. First, careful analyses of the maritime tropical atmosphere show little stored convective energy, and the few locations (such as central North America in spring) that exhibit large amounts of stored convective energy are not known to produce incipient cyclones. Also, CISK is a fundamentally linear instability; taken at face value, it predicts that incipient cyclones should be ubiquitous features of all convectively unstable atmospheres.

The author (Emanuel 1986) has proposed an alternative theory that regards tropical cyclogenesis as resulting from finite-amplitude Wind-Induced Surface Heat Exchange (WISHE) instability of the tropical atmosphere, relying on a positive feedback between surface heat fluxes and surface wind. WISHE modes are regarded as occurring in ambient atmospheres that are neutral to adiabatic displacements of boundary-layer air and hence have no stored convective energy. The reevaporation of condensed water in the low-entropy air of the middle troposphere appears to be the reason for the finite-amplitude nature of the instability: The resulting downdrafts import low-entropy air into the boundary layer at a rate that exceeds the enthalpy flux from the ocean surface. Intensification occurs when the entropy of the middle-tropospheric air has been raised enough to substantially weaken the low-entropy flux into the boundary layer by downdrafts. While these ideas are consistent with complex numerical simulations, they have yet to be systematically tested in real tropical cyclones.

Even if the reasons for the finite-amplitude nature of tropical cyclogenesis are correctly identified, the problem of genesis would remain as one of explaining the initiating disturbance. Observations indicate that a variety of circulations of dynamically independent origin may initiate tropical cyclones. These include easterly waves, which are wavy disturbances in the east-to-west flow of air in the tropics, especially over and west of sub-Saharan Africa and in the central Pacific; baroclinic developments in the subtropics; and continental mesoscale thunderstorm complexes that occasionally drift out over open water. A complete finite-amplitude theory of hurricanes could presumably specify the required characteristics of the initiating disturbances.

In addition to the problem of genesis, several aspects of hurricane behavior remain poorly understood. Given that "ignition" has occurred and a hurricane develops, very few reach the upper bounds on intensity implied by Figure 3. On the other hand, many numerical simulations (e.g. those of Rotunno & Emanuel 1987) consistently intensify storms right to the upper bound. Why is nature different? One possibility is that most hurricanes are limited by the cold water that they invariably mix upward from beneath the oceanic seasonal thermocline. Observed sea-surface temperature changes are as large as 5°C; only about 2.5°C of cooling is needed to reverse the air-sea thermodynamic disequilibrium. Preliminary studies show that the induced cooling may indeed limit the intensity of many hurricanes, though a comprehensive simulation with a coupled ocean-atmosphere model remains to be performed.

The spiral bands of convective clouds that give hurricanes their characteristic appearance in satellite photographs are not well understood. Extant theories include Ekman-layer instability and inertia-gravity waves propagating outward from the eyewall. These theories have not been rigorously tested against observations. Some strong hurricanes exhibit concentric eyewalls that undergo a characteristic evolution in which the eyewalls contract inward, the inner eyewall dissipates, and a new eyewall forms at a larger radius (Willoughby et al 1982). There is no well-accepted theory of this phenomenon.

Finally, the issue of hurricane steering remains the focus of lively research. Most theories pertain to the drift of barotropic vortices on β-planes (on which the vertical component of the Earth's rotation rate varies linearly with latitude). These theories predict that hurricanes should drift westward and poleward with respect to the mean wind. They do not account for the nonuniformity of the background potential-vorticity gradient in which hurricanes are embedded, nor do they recognize hurricanes as strongly baroclinic vortices with anticyclones in the upper troposphere. It seems likely that accounting for such effects will radically alter our understanding of hurricane motion.

Hurricanes present a large number of fascinating and unresolved problems that have received surprisingly little attention from theorists. As such, they remain a fertile and important subject of research in fluid dynamics.

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Annual Review of Fluid Mechanics
Volume 23, 1991

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