

1953

THE VERTICAL EXTENT OF JUPITER'S ATMOSPHERE

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It has long been recognized that Jupiter's visible surface is not a solid surface like that of the moon. Rather it appears to be something drifting in the atmosphere, like a layer of clouds. Perhaps the surest evidence for this conclusion is provided by its period of rotation, which varies with both latitude and time. An outstanding feature is the so-called equatorial acceleration, a region extending several degrees north and south of the equator whose rotation period is some five minutes shorter than that of the remainder of the planet, and whose eastward speed is, therefore, some 100 m sec^{-1} with respect to its immediate surroundings.

Astrophysical evidence suggests that the atmosphere may extend to great depths below the visible surface. There is some evidence, based upon Jupiter's observed low density, that the planet is composed largely of hydrogen. The pressures at which hydrogen would no longer be gaseous could exist only at very great depths. In this paper it will be shown that a simple consideration of Jupiter's variable rotation period also yields a rough approximation to the depth of the atmosphere below the visible surface.

For the purpose of computation, it will be assumed that between 12° N and 12° S the visible surface has the so-called System I rotation period of 9 hr, 50 min, 30 sec and the remaining portion of the visible surface has the so-called System II rotation period 9 hr, 55 min, 40 sec (see Shapiro (1951)). This assumption is, of course, a great simplification, but the variations of rotation period which have been neglected are of secondary importance compared to the variation which is taken into account. These periods apply, of course, to the atmosphere and not to the underlying solid surface. They may therefore be interpreted as indicating different wind speeds in different portions of Jupiter's atmosphere.

In the development which follows, these symbols appear:

- y horizontal distance, measured northward
- z vertical distance, measured upward
- p pressure
- T temperature
- u eastward component of wind velocity, with respect to rotating solid surface
- g apparent acceleration of gravity, $2.59 \times 10^8 \text{ cm sec}^{-2}$
- m molecular weight of atmosphere
- R universal gas constant
- ω angular velocity of rotation of solid surface
- a Jupiter's mean radius, 69,900 km
- ϕ latitude, equal to y/a .

* The research reported in this paper was sponsored by the Geophysics Research Directorate, AF Cambridge Research Center, under Contract No. AF 19(122)-162.

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It is natural to regard y and z as independent variables, and p , T and u as dependent variables; but it is sometimes more convenient to treat y and p as independent variables, and z , T and u as dependent variables. The subscripts y , z or p may be used outside partial derivatives to indicate which variable is being held constant.

It seems impossible that the average pressure, temperature and wind fields on Jupiter are not approximately in equilibrium with each other. The hydrostatic equation, expressing equilibrium between the vertical forces in the atmosphere, and relating pressure and temperature, may be written

$$\frac{1}{p} \left(\frac{\partial p}{\partial z} \right)_y = \frac{-gm}{RT}, \quad (1)$$

provided that the atmosphere behaves as an ideal gas. The geostrophic wind equation, expressing equilibrium between the horizontal forces, and essentially relating pressure and wind, may be written

$$u = - \frac{g}{2\omega \sin \phi} \left(\frac{\partial z}{\partial y} \right)_p. \quad (2)$$

The thermal wind equation, relating temperature and wind, and obtained by combining Eqs. (1) and (2) may be written

$$\left(\frac{\partial u}{\partial z} \right)_y = - \frac{g}{2\omega \sin \phi} \frac{1}{T} \left(\frac{\partial T}{\partial y} \right)_p. \quad (3)$$

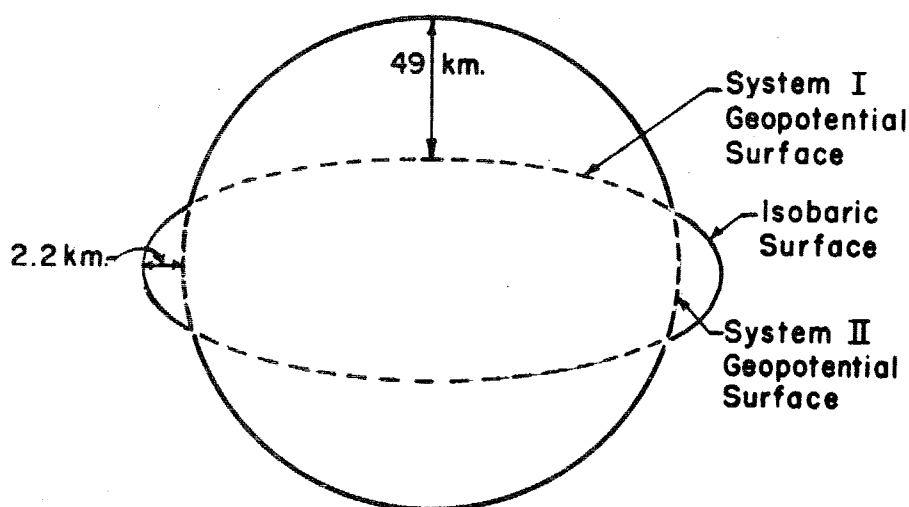
These equations, or their equivalent forms, may be found in standard textbooks on meteorology, (see Haurwitz (1941)).

It is evident that the definitions of ω and u depend on the period of rotation of the solid core. Since this period is not known, actual values of ω and u cannot be determined. They may be determined, however, and Eqs. (1), (2) and (3) will still be equally valid, if the quantities are defined with respect to some standard period of rotation, which need not be that of the core.

It is perhaps less obvious, but equally true, that the definitions of y and z depend on the period of the core. On the earth, horizontal and vertical directions are defined as directions parallel and perpendicular to the geopotential surfaces. The earth's geopotential actually consists not only of a gravitational potential, but also of a potential associated with the centrifugal force due to the rotation of the solid earth (not the rotation of the atmosphere). Thus the geopotential surfaces are ellipsoidal. To determine Jupiter's geopotential similarly, the rotation period of the solid surface must be known. For example, if the core rotated with the System II period, the geopotential surfaces would be ellipsoids of a certain eccentricity, and if it rotated with the System I period, they would be ellipsoids of slightly greater eccentricity. A pair of these hypothetical geopotential surfaces is illustrated in Fig. 22.

On the other hand, the definition of p is independent of the period of the core, and the form of the isobaric surfaces can be precisely determined. From Eq. (2) it follows that if $u = 0$, the isobaric and the geopotential surfaces coincide, a result which is almost self-evident. Therefore, within the region of the equatorial acceleration, the isobaric surfaces coincide with the System I geopotential surfaces, since the wind there would vanish if the core rotated with the System I period. Similarly, over the remainder of the planet, the isobaric surfaces coincide with the System II geopotentials. A typical isobaric surface near the visible surface, therefore, consists of portions of two ellipsoids, intersecting at 12° N and 12° S, as shown in Fig. 22. From the geostrophic point of view, the kinks in the isobaric surfaces at latitude 12° are identical with the abrupt changes of wind speed there.

Fig. 22. An isobaric surface near Jupiter's visible surface, composed of portions of two hypothetical ellipsoidal geopotential surfaces.



The geostrophic wind equation (2) also shows that the ellipsoids in Fig. 22, which intersect at 12° N and 12° S, are separated by 2.2 km at the equator, and by 49 km at the poles. The height, 2.2 km, of the equatorial bulge of the isobaric surfaces will prove to bear an important relation to the depth of the solid core below the visible surface.

In trying to determine the depth of Jupiter's atmosphere, one must bear in mind the possibility of a very shallow atmosphere, in which the winds just above a shallow friction layer at the core are equal to those at the visible surface. The winds at the top of the friction layer exert torques upon the core. The law of conservation of angular momentum shows that the total angular momentum of the core plus the atmosphere must be constant, but it is equally true that over periods of several years, the angular momenta of the core and the atmosphere must each remain nearly constant, since no progressive changes in the rotation periods have been observed. The total torque exerted on the core by the atmosphere must therefore be zero. Under the assumption of a very shallow atmosphere, this can occur only if the core rotates with a period intermediate to Systems I and II, so that the equatorial winds blow from the west relative to the core, and try to drag the core ahead, while the remaining winds blow from the east, and try to hold the core back.

Since the region of the equatorial acceleration covers only one fifth of the area of the planet, the winds within this region must be stronger than the remaining winds under the proposed balance of torques. The balance requires a period of about 9 hours, 54 minutes for the core. Under such conditions, the average wind speed at the top of the friction layer would be about 35 m sec^{-1} . It is this proposed average wind speed which will be examined critically.

The wind speed just above the friction layer provides a rough measure of the rate at which kinetic energy is dissipated by friction. In view of the well-known formula $\tau = \kappa \rho c^2$ relating the frictional stress τ at the ground to the density ρ and the wind speed c at some level within the friction layer (Taylor (1916)), it is possible that the dissipation of kinetic energy varies as the cube of the wind speed. The ultimate source of at least a large portion of Jupiter's energy is the sun. The ratio of the energy dissipated by friction to the energy received from the sun is therefore a quantity of some interest. Perhaps it may be thought of as the efficiency of Jupiter's atmosphere, regarded as a thermodynamic engine doing work against the solid core.

It is easier to compare Jupiter to the earth than to compute this ratio directly. Jupiter receives about $1/27$ as much solar energy per unit area as the earth, and, having a somewhat higher albedo, absorbs an

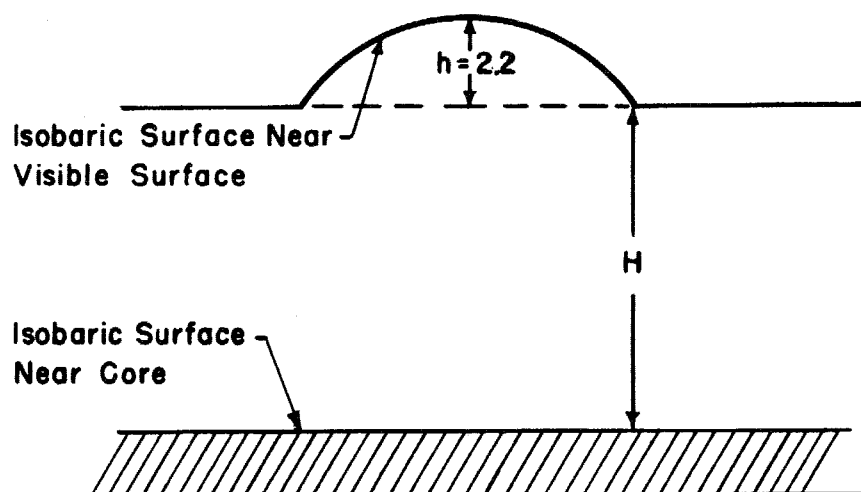


Fig. 23. Idealized isobaric surfaces near Jupiter's visible surface and near Jupiter's core in the vicinity of the equator.

even smaller fraction. If Jupiter and the earth have the same "efficiency," Jupiter should have a much smaller dissipation of energy per unit area, and hence have definitely weaker winds just above the friction layer, although the exact ratio depends upon the unknown density of Jupiter's atmosphere. The average wind speed just above the earth's friction layer may be 15 or even 20 m sec⁻¹, but can hardly approach 35 m sec⁻¹. The hypothesis that Jupiter has an average wind speed of 35 m sec⁻¹ near the core, therefore, seems to be untenable.

The only remaining possibility is that the winds just above the friction layer are considerably weaker than those at the visible surface, and may, as a first approximation, be considered negligible. One may then compute from the thermal wind equation (3) the vertical distance necessary for the winds to decrease from their speeds at the visible surface to zero, in terms of an assumed horizontal temperature gradient. Alternatively, one may compute the vertical distance necessary for the equatorial bulge in the isobaric surfaces to disappear. Such a computation will be carried out diagrammatically with the aid of Fig. 23.

In Fig. 23, $h = 2.2$ km is the height of the equatorial bulge, and H is the distance between the visible surface and the core, at the edge of the equatorial bulge. If T_1 and T_2 are the mean temperatures throughout the depth of atmosphere under consideration, at the equator and 12° of latitude, respectively, it is evident from the hydrostatic equation (1) that

$$\frac{T_1}{T_2} = \frac{H + h}{H},$$

whence

$$H = h \left(\frac{T_1}{T_1 - T_2} \right).$$

Thus the depth of the atmosphere below the visible surface is equal to the height of the equatorial bulge, multiplied by the ratio of the absolute temperature to the temperature contrast associated with the bulge. Computation directly from the thermal wind equation (3) yields the same result.

The magnitude of the temperature contrast is a matter of speculation. The absolute temperature varies by roughly 10 percent across the earth's jet stream, near latitude 30°. In view of the nearness of Jupiter's zone of temperature contrast to the equator, a smaller value, perhaps 2 percent, might be expected. If the figure 2 percent is a correct guess, the depth of Jupiter's atmosphere below the visible surface is 110 km.

This result, 110 km, is probably of the correct order of magnitude. Beyond this it must be regarded as only an estimate, for it is obvious that some of the assumptions involved are not proven facts. First, an assumed temperature contrast greater or less than 2 percent would result in a computed depth smaller or larger than 110 km. There may be some justification for assuming a smaller temperature contrast, since the temperature may be nearly uniform at great depths, if little solar radiation reaches there directly. Next it was tacitly assumed in Fig. 23, that the core rotates with the period of System II. A more rapid rotation, making the lower curve in Fig. 23 concave downward, would result in a smaller computed depth, but would also require increasing temperatures poleward from 12° , to remove the polar bulge. The more reasonable assumption that the temperature decreases poleward from 12° would lead to a greater computed depth. Finally, if the winds near the core are merely somewhat lighter than those higher up, the computed depth should be decreased. If, however, as on the earth, there are equatorial easterlies near the core, the computed depth should be increased. Together, these considerations suggest that the true depth may be considerably greater than 110 km.

Even the rejection of the hypothesis that Jupiter has strong winds near the core may not be beyond question. Perhaps Jupiter's atmosphere is a more "efficient" engine than the earth's, or perhaps heat from Jupiter's interior plays an important part. Nevertheless, the result that Jupiter's atmosphere is at least 100 km deep seems as reasonable as any other conclusion which could be drawn from the same observations.

A complete discussion of the vertical extent of Jupiter's atmosphere should say something about the extent above the visible surface. No attempt will be made to define the top of the atmosphere; instead, the atmosphere will be compared to the earth's. For the earth, if the temperature is 273°K , the hydrostatic equation becomes

$$\frac{1}{p} \frac{\partial p}{\partial z} = \frac{1}{8\text{km}}.$$

For Jupiter, if the temperature is assumed to be 150°K , and if the molecular weight has its smallest possible value, that of hydrogen, the hydrostatic equation becomes

$$\frac{1}{p} \frac{\partial p}{\partial z} = \frac{1}{24\text{km}}.$$

If the molecular weight is greater, the pressure falls off somewhat more rapidly with elevation, but it would require the improbably high molecular weight $m = 6$ for the pressure to fall off as rapidly as on the earth. It seems safe to conclude, therefore, that Jupiter's atmosphere extends farther above the visible surface than does the earth's atmosphere above sea level, regardless of what definition one takes for the top of the atmosphere.

REFERENCES

- Haurwitz, B. (1941), *Dynamic Meteorology*, McGraw-Hill, New York; 365 pp.
- Shapiro, R. (1951), "A Quantitative Study of Bright and Dark Spots on Jupiter's Surface," Rept. No. 7, Contract No. AF 19(122)-162, AF Cambridge Research Center, Cambridge, Mass.
- Taylor, G. I. (1916), "Skin Friction of the Wind on the Earth's Surface," *Proc. Roy. Soc. Lon.* 92, 196-199.