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Reprinted from "International Dictionary of Geophysics"
PERGAMON PRESS

OXFORD · LONDON · EDINBURGH · NEW YORK
TORONTO · SYDNEY · PARIS · BRAUNSCHWEIG

ENERGETICS OF ATMOSPHERIC CIRCULATION.

Introduction. The Earth's atmosphere affords an ideal example of a fluid which is in continual circulation, and which therefore possesses kinetic energy. This energy is being continually dissipated by friction, and so must be continually supplied by other processes if circulation is to be maintained.

The ultimate source of nearly all atmospheric energy is the Sun. The kinetic energy present in the atmosphere is equal to the solar energy reaching the Earth in about one hour, and also to the kinetic energy dissipated by friction, and generated by other processes, in about one week. Hence the atmosphere may be viewed as an enormous engine, driven by solar energy, and producing kinetic energy with an efficiency of less than one per cent.

A complete theory of atmospheric energetics would have to contain the answers to two general questions:

1. Through precisely what chain of events is some solar energy ultimately converted into the kinetic energy of the observed atmospheric motions?

2. Why does the atmosphere choose this particular chain of events, rather than some other conceivable chain, to accomplish this energy conversion?

The first question is essentially the one posed prior to World War II by Brunt (1939). At that time no suitable answer could be given. By now the question has been reasonably well answered, thanks to many careful observational studies. The second question is a special problem in the theory of fluid motion, and the answers so far given are incomplete and somewhat speculative.

About one third of the solar radiation intercepted by the Earth and its atmosphere is reflected back to space, largely by clouds and by the Earth's surface, and plays no further part in the energetics of the atmosphere. Of the remaining two thirds, a large fraction passes through the atmosphere and heats the underlying water or land surface, which in turn may heat the atmosphere. A general study of atmospheric energetics must therefore include the energetics of the atmosphere-ocean-Earth system.

The hot interior of the Earth also represents a tremendous reservoir of energy, but the heat escaping from it is a negligible fraction of the heat received from the Sun, and appreciably affects only small local atmospheric circulations, in regions of volcanic activity. For the purposes of this exposition, the interior of the Earth need not be included in the atmosphere-ocean-Earth system.

In a complex process where energy appears in many forms, it is not always possible to say which form of energy is being directly converted into which other form. If, however, the process can be resolved into a combination of simpler processes, with each simpler process affecting only two forms of energy, the rate of conversion of each form into each other form may be specified. The greater the distinction made between different forms of energy, the greater must be the distinction between processes, in order that each process may affect only two forms of energy. Occasionally these distinctions may seem somewhat artificial.

The ensuing discussion is divided into three sections. The first section deals with the relevant physical forms of

energy—kinetic energy (KE), (gravitational) potential energy (PE), (thermal) internal energy (IE), and the latent energy (LE) of water vapour—and the processes which affect these forms. The concept of available potential energy (APE) is introduced, and the basic energy cycle of the atmosphere—the generation of APE by heating and cooling, the conversion of APE into KE by reversible processes, and the dissipation of KE by friction—is described. The next section deals with the extensive work performed since World War II, in which further details of the energy cycle are obtained by expressing the fields of motion and temperature as sums of simpler fields, and treating the KE and APE associated with the simpler fields as separate forms of energy. The final section presents the mathematical development needed for a quantitative study. References to specific equations in this section follow some of the qualitative statements in the earlier sections.

Basic energy conversions. Among the physical processes affecting the atmosphere-ocean-Earth system, only radiation, which adds or removes IE, appreciably alters its total energy. The remaining processes are internal. Work done by or against the pressure forces converts IE into KE, or KE into IE. Likewise work done by or against the force of gravity converts PE into KE, or KE into PE. There is no direct conversion of IE into PE, or PE into IE, but either form may be indirectly converted into the other by first being converted into KE. Friction converts KE into IE.

Although LE is not converted directly into KE, nor KE into LE, indirect conversions occur, since condensation converts LE into IE, while evaporation converts IE into LE. Only insignificant amounts of other physical forms of energy are converted into KE, directly or indirectly. Lightning discharges, for example, convert electrical energy into IE, and may play a role in the dynamics of thunderstorms.

Some writers (e.g., Brunt 1939) prefer to regard only the KE of the larger-scale organized circulations as KE. The KE of the superposed smaller-scale motions is regarded as a separate form of energy—turbulent energy. Organized KE is then converted into turbulent energy by turbulent friction, while molecular friction, which has a minor effect upon organized KE, converts turbulent energy into IE. Viewed closely, turbulent friction consists of a combination of imperfectly understood processes which bring about the ultimate decay of KE into turbulent energy. The scale of motion separating organized motion from turbulence is somewhat arbitrary. This exposition deals almost entirely with the energetics of large-scale motions, which are assumed to include cyclones and anticyclones, but not individual thunderstorms and smaller systems.

When one distinguishes the energy of the atmosphere from that of the underlying ocean or land, the physical process of conduction enters the picture, converting atmospheric IE into the IE of the underlying surface, or vice versa. Evaporation from the ocean converts oceanic IE into LE, while condensation generally converts LE into atmospheric IE, so that evaporation and condensation together form another effective means of transferring IE

from the ocean to the atmosphere. Friction converts some atmospheric KE into the KE of ocean currents and ocean waves, rather than into IE.

Often one distinguishes the kinetic energy of the vertical component of motion (vertical KE) from that of the horizontal components (horizontal KE), whereupon work done by gravity converts PE into vertical KE only. Work done by the vertical component of the pressure forces, converting IE into vertical KE, is then distinguished from work done by the horizontal components, converting IE to horizontal KE. Direct conversions from horizontal KE to vertical KE are minor.

An ever-present feature of large-scale atmospheric systems is hydrostatic equilibrium—the approximate balance between gravity and the vertical pressure forces. Evidence that the amounts of work done by these forces nearly cancel is the lack of accumulation of vertical KE; nearly all the KE of the atmosphere is horizontal KE.

Under hydrostatic equilibrium, changes in PE and IE are proportional, in the approximate ratio 5:2 (equations 8 and 9). Thus, whenever heating adds one unit of IE, or when one unit of horizontal KE is converted into IE, the resulting vertical motions are such that the vertical pressure forces convert $\frac{2}{5}$ of a unit of IE into vertical KE, while simultaneously gravity converts a like amount of vertical KE into PE.

It is therefore convenient to drop the distinction between IE and PE, and, following Margules (1903), to regard the two as a single physical form of energy, called total potential energy (TPE). At the same time vertical KE is disregarded altogether. Horizontal pressure forces then convert TPE into KE, or KE into TPE. If the definition of TPE is extended to include the IE of the underlying surface, TPE affords the only major direct source or sink for

Friction is an irreversible process; it always converts KE into TPE. Work done by the horizontal pressure forces is reversible, but it must on the average offset the depletion of KE by friction. Since a force must have a positive component in the direction of motion of a system, in order to do work upon the system, the atmosphere must, in some over-all sense, be moving horizontally toward lower pressure (equation 16). Equivalently, in some over-all sense, there must be horizontal divergence of velocity in regions of higher pressure, and horizontal convergence in regions of lower pressure (equation 17). Starr (1951) has noted the significance of divergence in the subtropical high-pressure systems, and convergence in the subpolar lows, as a factor in maintaining the general atmospheric circulation.

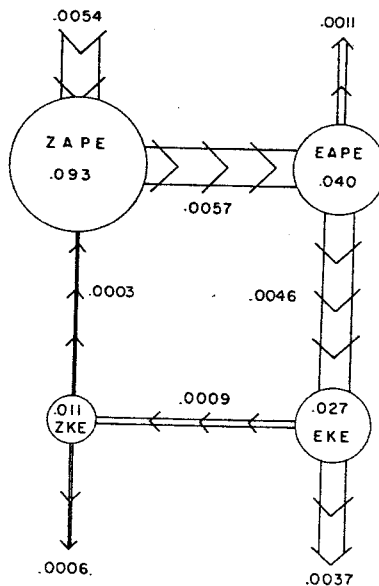
Likewise, in some over-all sense, individual pressure rises must be occurring in the denser air, and individual pressure falls in the lighter air (equation 13). Since, within most of the atmosphere, large pressure changes signify large changes of elevation, it is often stated loosely that, in some over-all sense, cold air must be sinking and warm air must be rising. This statement appears reasonable, but it cannot be rigorously defended.

There is no net conversion of TPE into LE, since TPE affords the only direct source or sink for LE. Hence, be-

yond those processes already cited, the one remaining process affecting TPE, namely radiation, must in the long run neither add nor remove TPE from the atmosphere-ocean-Earth system.

In summary, the work done by horizontal pressure forces is positive. If, as in the usual statement of the first law of thermodynamics, "heat" means everything affecting IE except work, the net heating of the atmosphere-ocean-earth system is positive. However, heat so defined includes heating by friction; the net heating by processes other than friction is zero.

Brunt (1926) has estimated the frictional dissipation of KE to be two per cent of the rate at which solar energy reaches the Earth. Lettau (1954) has reduced the figure to



The energy cycle of the atmosphere. Arrows show normal directions of generation, conversion, and dissipation processes, according to consensus of investigators. Areas of circles are proportional to amounts of indicated forms of energy, and widths of arrows are proportional to rates of conversion processes, as estimated by Oort (1964a). Numerical values are estimates for entire atmosphere. Unit for energy is 1.5×10^{29} ergs, equal to solar energy reaching Earth in one day. Unit for energy generation, conversion, and dissipation is 1.5×10^{29} ergs per day. Algebraic sum of two generations, conversions from APE to KE, or dissipations, 0.0043, is estimated efficiency of atmospheric engine.

six tenths of one per cent. Because the laws relating turbulent friction to large-scale motions are not well known, and the small scale motions themselves are not

well observed, estimates of frictional dissipation remain rather crude.

In theory an independent estimate should be obtainable by computing the rate at which reversible processes convert TPE into KE. Unfortunately such computations involve the nearly unmeasurable vertical motion, horizontal divergence, or horizontal cross-isobar flow. Recent indirect estimates of vertical motion have led to estimates of the conversion of TPE into KE which are consistent with estimated frictional dissipation (see figure). Fair estimates of typical cross-isobar flow within the lowest kilometre of the atmosphere (the "friction layer") are available; indeed, Brunt's estimate of dissipation seems to be really an estimate of conversion of TPE into KE within the friction layer.

The third method—measurement of the total heating of the atmosphere—is now seen to be useless. Total heating includes heating by friction, and any estimate of total heating would not be independent of the frictional dissipation of KE, but would in fact be identical with it.

This difficulty may be overcome through the further resolution of TPE into available potential energy (APE) and unavailable potential energy (UPE). Among those hypothetical states of the atmosphere obtainable from the existing state by rearranging the mass, without altering the statistical distribution of potential temperature, there is a unique reference state of minimum TPE. The fraction of the atmosphere's mass whose potential temperature exceeds a given value is the same in the existing state and the reference state, but in the reference state the surfaces of constant potential temperature are horizontal, and the potential temperature never decreases with elevation.

Following Lorenz (1955, 1960), UPE is defined as the TPE of the reference state, while APE is the excess of TPE over UPE. Since reversible processes do not alter individual potential temperatures, they do not alter UPE, and can convert only APE into KE.

There is no requirement in the definition of APE that the envisioned rearrangement of mass be dynamically possible. Hence APE is an upper bound to the amount of TPE available for conversion into KE.

An approximation obtained by Lorenz (1955) expresses APE as a weighted average of the variance of temperature within quasi-horizontal (isobaric) surfaces. At a given elevation, the weighting function is inversely proportional to the horizontally averaged static stability.

This approximation is consistent with the approximate rule that kinetic energy is produced when cold air sinks and warm air rises. If the stratification is stable, the temperature at a fixed elevation will fall in the rising air and rise in the sinking air, so that the horizontal temperature contrast will be reduced, and will ultimately disappear. Moreover, the less stable the stratification, the more the cold air must sink and the warm air must rise before the temperature contrast is destroyed. Thus there is more APE when the horizontal temperature contrast is greater, and when the stratification is less stable.

The detailed distribution of heating by friction is not well known; however, in converting KE into TPE, friction raises the potential temperature of some portions of the

atmosphere, and lowers the potential temperature of none. It thus raises the TPE of the reference state, i.e. the UPE, so that the APE produced, if any, must be less than the TPE produced, and is presumably much less.

Herein lies the principal importance of APE. Since work done by the pressure forces removes as much APE as TPE, there must be a net production of APE by heating processes other than friction. A consideration of these processes leads to an independent estimate of the rate at which the energy cycle proceeds.

There are two general methods by which heating can produce APE; by heating warmer regions and cooling cooler regions at the same level, thus increasing the temperature contrast, and by heating the lower levels and cooling the upper levels, thus decreasing the stability. In the winter in particular, the heating at low latitudes together with the cooling at high latitudes effectively maintains the supply of APE. Estimates of the generation of APE by heating and cooling (see Fig. 1) are consistent with the estimates of frictional dissipation, within the limits of data accuracy.

The sum of APE and KE resembles negative entropy, in that internal processes either leave it unaltered or decrease it, while only external effects can increase it. It is a separate concept, however; entropy depends only upon the thermodynamic state, while the sum of APE and KE also involves the field of motion.

There remains the question as to why the atmosphere should be no more than one per cent efficient. Lorenz (1960) used a simple model to estimate the rate of generation of APE as a function of the temperature field, and found the maximum possible generation to be not far above the estimated actual rate. He then postulated that the atmosphere is somehow constrained to operate at nearly its maximum efficiency; specifically, if several modes of behaviour are dynamically possible, the less efficient modes may be unstable, and may give way to the more efficient modes. These ideas are rather speculative, and confirmation must await more refined treatments, incorporating the distribution of clouds and water vapour as well as temperature.

Modes of energy conversion. The foregoing description of the energy cycle does not relate the energy conversions to particular types of weather systems. For example, KE could conceivably be produced by a general over-all rising motion in warm latitudes and sinking in cold latitudes, or by rising motion in the warmer portions of cyclones and anticyclones and sinking in the colder portions. Further resolution of the forms of energy yields a more detailed description of the energy cycle.

The field of motion may be resolved into a zonal flow, symmetric with respect to the polar axis, and a field of superposed eddies. The KE may then be resolved into zonal kinetic energy (ZKE)—the KE which would exist if only the zonal flow were present—and eddy kinetic energy (EKE)—the excess of KE over ZKE. Because KE is quadratic in the velocity field, EKE also equals the KE which would exist if only the eddies were present (equations 27 and 28).

Likewise, the temperature field may be resolved into a

zonally symmetric field and a field of superposed disturbances, and the APE may be resolved into zonal available potential energy (ZAPE)—the APE which would exist if only the zonal temperature field were present—and eddy available potential energy (EAPE)—the excess of APE over ZAPE. Since APE is approximately quadratic in the temperature field, EAPE approximately equals the APE which would exist if only disturbances were present (equations 29 and 30).

The process which converts APE into KE, loosely described as a sinking of colder air and rising of warmer air, may now be resolved into two processes—an over-all sinking of colder zones (latitude circles) and rising of warmer zones of air, and a sinking of colder air and rising of warmer air within zones (equations 39 and 40), as suggested at the commencement of this section. The former process can convert ZAPE into ZKE, while the latter can convert EAPE into EKE. If sufficient care is taken in distinguishing between physical processes, no process will affect both ZAPE and EKE, nor both EAPE and ZKE.

If the entire atmospheric process is to be resolved as far as possible into simpler processes, each affecting only two forms of energy, there should be a process which can convert ZKE into EKE without altering the total KE. Such a process consists of a horizontal or vertical transport, by the eddy motions, of angular momentum toward zones of lower average angular velocity (equation 42; cf. Reynolds 1894). This process tends to equalize the angular velocities of the different zones, thereby reducing the over-all KE of the zonal flow, since among all velocity fields with the same total angular momentum, the field of constant angular velocity has the least KE. The only sink for this ZKE is the KE of the eddies themselves.

Likewise, a process which can convert ZAPE into EAPE, without altering the total APE, is a horizontal or vertical transport, by the eddies, of sensible heat toward zones where the temperature is low, relative to the horizontally averaged temperature (equation 41). This process tends to equalize the temperatures of the different zones at the same elevation, thereby reducing the ZAPE. The only sink for this ZAPE is EAPE.

Both ZKE and EKE are continually dissipated by friction. There must therefore be a net conversion of ZAPE into ZKE, or EAPE into EKE, but both conversions need not proceed in this direction, since one form of KE could serve as the source for the other form. Likewise there must be a net generation of ZAPE or EAPE by heating, but both generations need not take place, since one form of APE could be the source for the other form. In short, unlike the basic conversions deduced in the previous section, some of the modes of conversion can be ascertained only by recourse to observations.

The numerous observational investigations which have now been made are by and large in qualitative agreement. First, the heating in the warm latitudes and the cooling in the cooler latitudes account for a net generation of ZAPE, while apparently EAPE is removed rather than added by heating. Next, the transport of heat by eddies is predominantly toward zones of lower temperature, so that

ZAPE is converted into EAPE by eddy motions. On the other hand, the transport of angular momentum by eddies is predominantly toward zones of *higher* angular velocity, so that EKE is converted into ZKE by the eddies. The last result, first substantiated by Kuo (1951), implies that in a sense the large-scale eddying process is an unmixing process. Any attempt to deduce the general atmospheric circulation by regarding the effect of eddies as a sort of large-scale turbulent friction would fail, unless it assumed a negative coefficient of friction (cf. Starr 1953). This phenomenon is now fairly well known among meteorologists, and is often cited by them as exemplifying the hazards involved in drawing hasty conclusions in a subject with which one is not thoroughly familiar.

From these results it may be deduced that EAPE is converted into EKE, there being no other source for EKE. Within zones, then, the colder air tends to sink while the warmer air tends to rise. No corresponding sinking of colder zones of air and rising of warmer zones is observed; apparently there is sinking in the cold polar regions but also in the rather warm subtropical regions, while there is rising in the warm equatorial regions but also in the rather cold subpolar regions.

The energy cycle is shown schematically in the figure. Oort (1964a) arrived at his numerical estimates after assessing all available quantitative results of other investigators.

Further details of the energy cycle have been deduced from further resolutions of KE and APE. Of special interest are the harmonic analyses of the eddy-fields of motion and temperature along individual latitude circles, performed by a few groups of investigators (Saltzman and Fleisher 1960, 1961; Wiin-Nielsen 1959; Wiin-Nielsen and Brown 1960; Wiin-Nielsen *et al.* 1963, 1964; Murakami and Tomatsu 1964).

It appears that all wave numbers contribute positively to the conversion of ZAPE into EAPE, and to the destruction (negative generation) of EAPE by heating, the greatest contribution in each case coming from the long waves (numbers 1 to 4). All waves act to convert EAPE into EKE, with a peak contribution from the longer cyclone waves (numbers 5 to 9), and a secondary peak from the long waves. The long waves act to convert EKE into ZKE, while it is not certain whether the longer cyclone waves contribute positively or negatively. Among the wave numbers, both the long waves and the shorter waves (numbers 10 and above) appear to gain KE at the expense of the longer cyclone waves, although here there is not complete agreement. In general the resolutions have not been carried beyond wave number 15.

The study of the energy cycle of a particular region of the atmosphere, or a particular weather system, is complicated by the possibility of exchanges of mass as well as energy across the boundary of the region. The energetics of the stratosphere has caused much speculation, following the discovery by White (1954) of a flow of heat from colder to warmer regions in the lower stratosphere. White and Nolan (1960) and Jensen (1961) found evidence of a sinking of warmer air and rising of colder air in the stratosphere. Oort (1964b) has studied the complete energy cycle

of the lower stratosphere (100–30 millibars), and has found that the processes within the stratosphere act to convert EKE into EAPE, and EAPE into ZAPE, while radiation from the stratosphere destroys ZAPE. The stratosphere therefore does not seem to be thermally driven, whence it must be mechanically driven, presumably by coupling with the troposphere. The stratospheric energy cycle thus proceeds in the opposite direction from that of the atmosphere as a whole, and the stratosphere acts as an engine in reverse, or a thermodynamic refrigerator.

Although no simple theory for the modes of energy conversion is known, there is reason to believe that a theoretical explanation exists; the manner of conversion does not seem to be accidental. In seeking a theory, meteorologists have been influenced and guided by the results of two special categories of studies. First there are the laboratory experiments (Fultz *et al.* 1959; Hide 1958) in which a rotating cylindrical vessel containing water or some other fluid is heated near its circumference and cooled near its centre, in a manner simulating the external heating of a hemisphere of the atmosphere. The resulting thermally forced circulation possesses its own energy cycle.

Low rates of rotation, and also higher rates with very weak or very strong heating and cooling, lead to symmetric circulations where no EAPE nor EKE are present. Higher rotation rates with moderate heating and cooling lead instead to the formation of eddies resembling those found in the atmosphere. Measurements have indicated conversions of ZAPE into EAPE and of EKE into ZKE in these cases. There are indications that even when eddies are observed, symmetric flow represents a mathematically stable but unstable state.

In other studies are the extended numerical solutions or somewhat simplified systems of equations resembling those governing the atmosphere. The prototype is the study of Phillips (1956), who obtained energy generations, conversions, and dissipations in qualitative agreement with the observational studies. Similar agreement occurs in the more detailed study by Smagorinsky (1963).

These studies imply that the explanation for the observed modes of energy conversion may be found within the framework of the equations ordinarily assumed to govern the atmosphere. They further suggest that even a qualitative explanation requires quantitative considerations; the atmosphere might behave quite differently if the Earth were rotating at a different rate, or if the solar heating were more intense.

There is a strong suggestion that atmospheric eddies are an instability phenomenon, and that the existing scheme of energy conversion occurs because all other mathematically possible schemes of conversion are unstable. Nevertheless, calling a phenomenon "instability" falls short of explaining it; one is still entitled to ask why certain schemes of energy conversion are stable and others are unstable.

Quantitative considerations. This section contains the mathematical development needed for a quantitative treatment of atmospheric energetics. The following symbols are used:

a	Earth's radius
g	acceleration due to gravity
Ω	angular velocity of Earth's rotation
\mathbf{k}	vertical unit vector
c_v	specific heat of air at constant volume
c_p	specific heat of air at constant pressure
R	gas constant for air, $c_p - c_v$
κ	ratio R/c_p , approximately $2/7$
Γ_d	dry-adiabatic lapse rate, g/c_p
p_{00}	standard pressure, 1000 millibars
t	time
λ	longitude
φ	latitude
z	elevation
p	pressure
p_0	pressure at Earth's surface
α	specific volume
T	temperature
\mathbf{v}	horizontal wind velocity
u	eastward component of \mathbf{v}
v	northward component of \mathbf{v}
ω	individual pressure change, dp/dt
θ	potential temperature
Γ	lapse rate of temperature, $-\partial T/\partial z$
Q	heating per unit mass
\mathbf{b}	horizontal pressure gradient force, $-\alpha \nabla_z p = -g \nabla_p z$
\mathbf{F}	friction force per unit mass
dS	element of horizontal area
dV	element of volume, $dz dS$
dM	element of mass, $\alpha^{-1} dV$
∇_z	horizontal differential operator
∇_p	differential operator along isobaric surface

A location in the atmosphere may be identified by the values of λ , φ , z , or by λ , φ , p . Both coordinate systems are standard in meteorology. The latter is often more convenient when systems in approximate hydrostatic equilibrium are being treated, and is consistent with the current practice of preparing weather maps for surfaces of constant pressure.

Following the notation of Starr and White (1951), square brackets ($[]$) will denote an average value around a latitude circle, while an asterisk (*) will denote a departure from such an average. A wavy line (\sim) will denote an average over an isobaric surface, and a prime (') will denote a departure from this sort of average.

With sufficient accuracy, the physical laws governing the atmosphere may be expressed as the equation of state, the equation of continuity, the thermodynamic equation (first law), the equations of horizontal motion, and the hydrostatic equation. In a system with t , λ , φ , and p as independent variables, these equations may be written

$$p\alpha = RT, \quad (1)$$

$$\nabla_p \cdot \mathbf{v} + \partial \omega / \partial p = 0, \quad (2)$$

$$c_p dT/dt - \alpha \omega = Q, \quad (3)$$

$$d\mathbf{v}/dt = -2\Omega \sin \varphi \mathbf{k} \times \mathbf{v} + \mathbf{b} + \mathbf{F}, \quad (4)$$

$$\partial z / \partial p = -\alpha / g. \quad (5)$$

Equations 1-5 form a closed system, provided that Q and F are regarded as known functions of the dependent variables T, α, z, v and ω .

The presence of surface topography is neglected in the ensuing formulae.

The KE, PE, and IE of the whole atmosphere are given by the formulae

$$K = \int_M \frac{1}{2} v \cdot v dM, \quad (6)$$

$$P = \int_M gz dM, \quad (7)$$

$$I = \int_M c_p T dM. \quad (8)$$

With the aid of equation 1 and 5 it follows also that

$$P = \int_M RT dM, \quad (9)$$

so that

$$P + I = \int_M c_p T dM. \quad (10)$$

Hence the TPE is determined by the temperature field (as a function of λ, φ, p).

From equation 3 it follows that

$$d(P + I)/dt = H - C \quad (11)$$

where

$$H = \int_M Q dM \quad (12)$$

is the total heating of all sorts, and

$$C = - \int_M \alpha \omega dM. \quad (13)$$

Likewise, from equation 4 it follows that

$$dK/dt = C - D, \quad (14)$$

where

$$D = - \int_M v \cdot F dM, \quad (15)$$

is the total dissipation, and

$$C = \int_M v \cdot b dM. \quad (16)$$

That expressions (13) and (16) for C are equivalent, and represent the conversion from TPE to KE by reversible processes, follows from equations 3 and 5, with some integration by parts.

It is a positive average value of $-\alpha\omega$ in equation 13 which is often described as a sinking of denser air and rising of lighter air. Such a description is strictly valid only if "sinking" and "rising" are interpreted to mean moving downward and moving upward with respect to surfaces of constant pressure, which may be moving downward or upward themselves. It is not possible to

transform C into another expression involving only the sinking of colder air and rising of warmer air, in the usual sense of "sinking" and "rising".

Expression (16) may also be written

$$C = \int_V p \nabla_z \cdot v dV \quad (17)$$

after integration by parts, indicating that a positive correlation between pressure and horizontal divergence is needed for the production of KE.

It is sometimes convenient to use the potential temperature in place of p as a vertical coordinate. In this case

$$P + I = (1 + \kappa)^{-1} c_p g^{-1} p_{00}^{-\kappa} \int_S \int_0^\infty p^{1+\kappa} d\theta dS. \quad (18)$$

The difficulties that might be expected because the lowest value of θ in a vertical column is not the value 0 appearing as a limit of integration in equation 18, and because the same value of θ may appear at several elevations in a column, may be circumvented by treating p in equation 18 as the weight per unit area of that portion of a column whose potential temperature exceeds θ . Thus p assumes its usual meaning within a column where θ always increases with elevation, and reduces to p_0 for values of θ lower than those occurring within a column. If

$$P(\theta) = \frac{1}{S} \int_S p(\lambda, \varphi, \theta) dS \quad (19)$$

denotes the average value of p over an isentropic surface, P becomes the weight per unit area of that portion of the atmosphere whose potential temperature exceeds θ . In essence P is a probability measure for θ .

The UPE is now obtained by replacing p by P in equation 18 thus replacing the existing state of the atmosphere by a reference state where $p = P$. It then follows that the APE is given by

$$A = (1 + \kappa)^{-1} c_p g^{-1} p_{00}^{-\kappa} \int_S \int_0^\infty (p^{1+\kappa} - P^{1+\kappa}) d\theta dS. \quad (20)$$

Equation 20 is the so-called exact expression for the APE, although actually the presence of topography renders it somewhat inexact.

With θ as the vertical coordinate, the equation of continuity becomes

$$\frac{\partial}{\partial t} \frac{\partial p}{\partial t} + \nabla_\theta \cdot \left(\frac{\partial p}{\partial \theta} v \right) + \frac{\partial}{\partial \theta} \left(c_p^{-1} p_{00}^\kappa p^{-\kappa} \frac{\partial p}{\partial \theta} Q \right) = 0. \quad (21)$$

It follows, after some manipulation of formulae, that

$$\frac{dA}{dt} = G - C, \quad (22)$$

where C retains its former meaning, and

$$G = \int_M (1 - P^\kappa p^{-\kappa}) Q dM. \quad (23)$$

Unlike the generation of TPE, which depends only upon the total heating H , the generation of APE is favoured by heating where p exceeds P and cooling where P exceeds p ; i.e. by heating in warm regions, and heating near the Earth's surface.

Since it is sometimes inconvenient to work in a $(\lambda, \varphi, \theta)$ coordinate system, various approximations to equation 20 have been generally used in computations and theoretical studies. Equivalent approximations given by Lorenz (1955) are

$$A = \frac{1}{2} \kappa c_p p_{00}^{-\kappa} \int_M p^{1+\kappa} \left(\frac{\partial \theta}{\partial p} \right)^{-1} \tilde{\theta}^2 dM, \quad (24)$$

and

$$A = \frac{1}{2} \int_M c_p \Gamma_d (\Gamma_d - \tilde{\Gamma})^{-1} \tilde{T}^{-1} \tilde{T}^2 dM. \quad (25)$$

From equation 25 comes the approximation

$$G = \int_M \Gamma_d (\Gamma_d - \tilde{\Gamma})^{-1} \tilde{T}^{-1} \tilde{T}^2 dM. \quad (26)$$

Only the influence of heating upon the factor \tilde{T}^2 has been included. A closer approximation would include additional terms involving the effect of heating on \tilde{T} and $\tilde{\Gamma}$, but even this approximation would not agree with the "exact" expression (23).

Van Mieghem (1956) has derived another expression for APE by assuming that the existing state is obtainable from the reference state by a dynamically possible process, and estimating the increase in KE during the process. His expression contains two terms, one of which is formally the same as equation 24, except that it contains the variance of θ within horizontal surfaces, which is generally smaller than the variance of θ within isobaric surfaces. The correction term, which is smaller in magnitude, involves the variance of pressure within horizontal surfaces.

The ZKE and EKE of the whole atmosphere are given by

$$K_z = \int_M \frac{1}{2} [\mathbf{v}] \cdot [\mathbf{v}] dM, \quad (27)$$

$$K_E = \int_M \frac{1}{2} \mathbf{v}^* \cdot \mathbf{v}^* dM. \quad (28)$$

Likewise the ZAPE and EAPE are given approximately by

$$A_z = \int_M \frac{1}{2} c_p \Gamma_d^{-1} (\tilde{\Gamma}_d - \tilde{\Gamma}) \tilde{T}^{-1} [\tilde{T}]^2 dM, \quad (29)$$

$$A_E = \int_M \frac{1}{2} c_p \Gamma_d^{-1} (\tilde{\Gamma}_d - \tilde{\Gamma}) \tilde{T}^{-1} [\tilde{T}^*]^2 dM. \quad (30)$$

It should be noted that T^* is not simply replaced by T to obtain (30) from (25); the temperature averaged around each latitude circle is subtracted out, but then the temperature averaged over each isobaric surface is added

back. More exact expressions for ZAPE and EAPE could be derived from equation 20.

From equations 27-30, a set of expressions suitable for approximate computations may be derived. These are

$$dA_z/dt = G_z - C_z - C_A, \quad (31)$$

$$dA_E/dt = G_E - C_E + C_A, \quad (32)$$

$$dK_z/dt = -D_z + C_z - C_K, \quad (33)$$

$$dK_E/dt = -D_E + C_E + C_K, \quad (34)$$

$$G_z = \int_M \Gamma_d (\Gamma_d - \tilde{\Gamma})^{-1} \tilde{T}^{-1} [\tilde{T}] [\tilde{Q}] dM, \quad (35)$$

$$G_E = - \int_M \Gamma_d (\Gamma_d - \tilde{\Gamma})^{-1} \tilde{T}^{-1} [\tilde{T}^*] [\tilde{Q}^*] dM, \quad (36)$$

$$D_z = - \int_M [\mathbf{v}] \cdot [\mathbf{F}] dM, \quad (37)$$

$$D_E = - \int_M \mathbf{v}^* \cdot \mathbf{F}^* dM, \quad (38)$$

$$C_z = - \int_M [\mathbf{x}] \cdot [\boldsymbol{\omega}] dM, \quad (39)$$

$$C_E = - \int_M \mathbf{a}^* \boldsymbol{\omega}^* dM, \quad (40)$$

$$C_A = - \int_M c_p \left(\frac{p}{p_{00}} \right)^{-\kappa} \times \\ \times \left(\frac{1}{a} [\mathbf{T}^* \mathbf{v}^*] \frac{\partial}{\partial \varphi} + [\mathbf{T}^* \boldsymbol{\omega}^*] \frac{\partial}{\partial p} \right) \times \\ \times \left(\frac{\Gamma_d}{\Gamma_d - \tilde{\Gamma}} \frac{[\tilde{T}]}{\tilde{\theta}} \right) dM, \quad (41)$$

$$C_K = - \int_M \cos \varphi \left(\frac{1}{a} [\mathbf{v}^* \mathbf{v}^*] \frac{\partial}{\partial \varphi} + [\mathbf{v}^* \boldsymbol{\omega}^*] \frac{\partial}{\partial p} \right) \times \\ \times \left(\frac{[\tilde{v}]}{\cos \varphi} \right) dM. \quad (42)$$

Since the zonally averaged northward component $[v]$ is ordinarily small compared to the zonally averaged eastward component $[u]$, the vector \mathbf{v} may for practical purposes be replaced by u in equation 42.

Equations suitable for treating the resolution of the eddy processes into wave numbers have been given by Saltzman (1957).

Most quantitative studies have been based upon equations 27-42. By the very nature of atmospheric observations, the procedure for computation is not uniquely defined. The integrals in the formulae must be estimated from observations at discrete points. The heating and friction needed to compute G_z , G_E , D_z , and D_E , and the values of ω needed for C_z and C_E , must themselves be estimated from observable quantities, while the deriva-

tives needed for C_A and C_K must be replaced by finite differences. Partly because different means have been used to accomplish these tasks, separate estimates often disagree.

In addition, different investigators have used different samples of atmospheric data, many of them less than a year in length. Energy conversion processes, like other weather elements, do not maintain identical values from one month or one year to the next, as is evident from the studies of Winston and Krueger (1961) and Krueger and Haines (1964). Indeed, it is remarkable that separate estimates agree for the most part to within a factor of two or three.

There is some disagreement as to the sign of C_z , but general agreement that C_z is not an important fraction of $C_z + C_E$ the total conversion from APE to KE. Suomi and Shen (1963) obtained a positive value of G_E by using values of heating measured from artificial satellites, but could not include the effects of heating by processes other than infra-red radiation. Wiin-Nielsen, Brown, and Drake (1964) found a large positive value of C_K for the single month of January 1963, in contrast to the apparently normal negative values which they and other investigators obtained for other periods. Obasi (1963) also obtained negative values of C_K from southern hemisphere data.

One might suppose that so basic a quantity as the total KE of the atmosphere would be well known, but even here the estimates of Wiin-Nielsen, Brown, and Drake (1964) and Krueger and Haines (1964) exceed those of Oort (1964a) by a factor of two, and imply a mean atmospheric wind speed in excess of 20 metres per second, as opposed to Oort's 15 metres per second.

Except for Obasi (1963), all investigators have used data from the northern hemisphere only, and have encountered strong seasonal variations. In general the energy cycle is most intense in winter, although the total solar heating is greatest in summer. Apparently the atmospheric engine operates most efficiently in winter, when the cross-latitude temperature contrast is at its greatest.

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