**Revised PBL Model with Soil Physics**

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This version allows for time dependence of net surface radiative flux (approximately the solar flux) and includes the simple soil model of Noilhan and Planton (1989).

# Atmospheric model

We consider the growth of a shallow boundary layer into a deep, dry adiabatic layer with no moisture, characterized by equal dry and moist static energies of magnitude  The equations for dry and moist static energy are



and



where  and  are the dry and moist static energies,  is the air density,  is the (variable) depth of the boundary layer,  is the entrainment velocity at the top of the boundary layer,  is the surface sensible heat flux, and  is the surface latent heat flux.

We use Lilly’s formulation for the entrainment velocity:



where  is a dimensionless constant, which we usually take to be 0.2.

We use aerodynamic flux formulae for the surface sensible and latent fluxes:



and



where  is the turbulent exchange coefficient for sensible and latent heat (assumed equal),  is the near-surface wind speed,  is the dry and static energy of air at the temperature and pressure of the surface soil. The moist static energy of the surface is defined as



where  is a number between 0 and 1 that describes the degree of saturation of the soil surface and will be discussed in the soil section of this document.

# Soil Model

We apply the simple soil model of Noilhan and Planton (1989). This substitutes a force-response formulation for a truly one-dimensional soil model. Besides the soil model parameters, the coupled model will also depend on the initial soil conditions chosen.

The two new variables we must predict are the surface temperature,  and the degree of surface saturation  The latter is parameterized as



where  is the volumetric soil water content, and  is the field capacity, parameterized by



where  is the saturation water content, which equals the porosity, which we get from <http://www.geotesting.info/parameter/soil-porosity.html> .

Now we need to know  and the surface soil temperature  which we obtain from 4 time-dependent equations:



where



with  the ground cover of vegetation (0 to 1),  and  is parameterized as



 and  are given in Table 2 of Noilhan and Planton (1989) as a function of soil type.

In (9),  is the soil layer temperature and  is the soil depth, while  is a normalization depth of *0.1 m*;  is the density of liquid water. The quantity  is given by



with  and  given in in Table 2 of Noilhan and Planton (1989). Finally, the constants  and  are given by



and



where  is a small numerical value that caps  The constants  and  are provided as a function of soil type in Noilhan and Planton (1989).

This completes the soil model.

# Normalization

The system is made a bit simpler by substituting new, nondimensional variables:



where 

With these substitutions, and combining (1) and (3) give



which can be directly integrated to yield



where  is the initial nondimensional dry static energy deficit of the boundary layer.

Likewise, (2) – (5) can be combined to give



where  and



We can now combine (3) and (4) to give



where .

The variables  and  are related by the Clausius-Clapeyron equation:



We choose not to normalize the water content variables  and  which are already nondimensional. But we re-write (9) in terms of normalized time and atmospheric variables:



These are supplemented by (10)-(14) in their unaltered form, and using the tables of soil properties described previously.

A MATLAB code for solving this system is available, in the script *PBL\_soil\_evol.m* .