Basics of convection and radiative-convective equilibrium

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Program

- Elements of radiative transfer in earth's atmosphere
- Dry convection and dry radiative-convective equilibrium (RCE)
- Effect of phase changes of water (no big deal)
- Effect of irreversible fall-out of condensed water (big deal)
- Meta-stability
- Wet RCE
- Spontaneous aggregation of wet convection

Elements of Thermal Balance: Solar Radiation

- Luminosity: 3.9×10^{26} J s^{-1} = 6.4×10^{7} Wm^{-2} at top of photosphere

- Mean distance from earth: 1.5×10^{11} m
- Flux density at mean radius of earth

$$S_0 \equiv \frac{L_0}{4\pi d^2} = 1370\, Wm^{-2}$$

Stefan-Boltzmann Equation: $F = \sigma T^4$

$\sigma = 5.67 \times 10^{-8}\ Wm^{-2} K^{-4}$

Sun: $\sigma T^4 = 6.4 \times 10^7\ Wm^{-2}$

$\rightarrow \quad T \simeq 6{,}000\, K$

Disposition of Solar Radiation:

$$\text{Total absorbed solar radiation} = S_0\left(1-a_p\right)\pi r_p^{\,2}$$

$$a_p \equiv \text{planetary albedo}\left(\simeq 30\%\right)$$

$$\text{Total surface area} = 4\pi r_p^{\,2}$$

$$\text{Absorption per unit area} = \frac{S_0}{4}\left(1-a_p\right)$$

Absorption by clouds, atmosphere, and surface

Terrestrial Radiation:

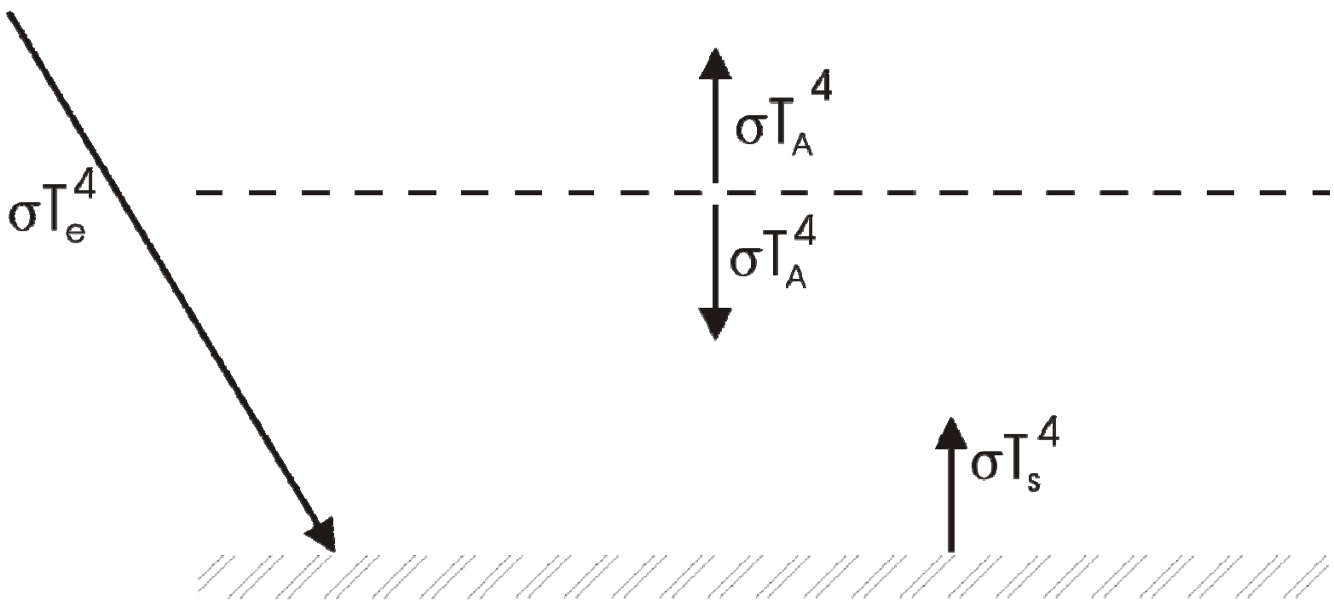
Effective emission temperature:

$$\sigma T_e^{\,4} \equiv \frac{S_0}{4}\left(1 - a_p\right)$$

Earth: $T_e = 255K = -18°C$

Observed average surface temperature $= 288K = 15°C$

Highly Reduced Model



- Transparent to solar radiation
- Opaque to infrared radiation
- Blackbody emission from surface and each layer

Radiative Equilibrium:

Top of Atmosphere:

$$\sigma T_A^{\ 4} = \frac{S_0}{4}\left(1 - a_p\right) = \sigma T_e^{\ 4}$$

$$\rightarrow \quad \boxed{T_A = T_e}$$

Surface:

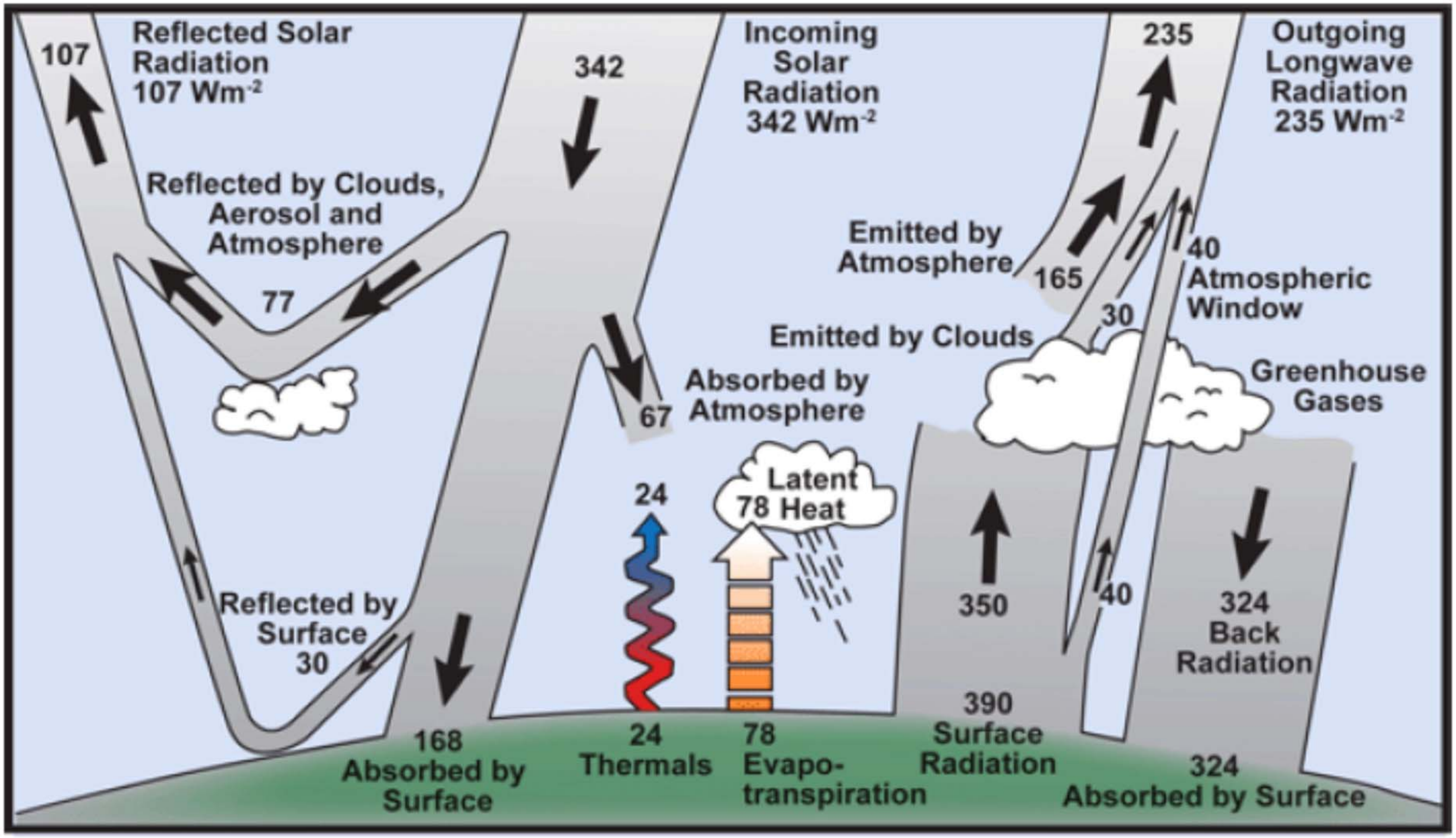
$$\sigma T_s^{\ 4} = \sigma T_A^{\ 4} + \frac{S_0}{4}\left(1 - a_p\right) = 2\sigma T_e^{\ 4}$$

$$\rightarrow \boxed{T_s = 2^{1/4} T_e} = 303\ K$$

Surface temperature too large because:

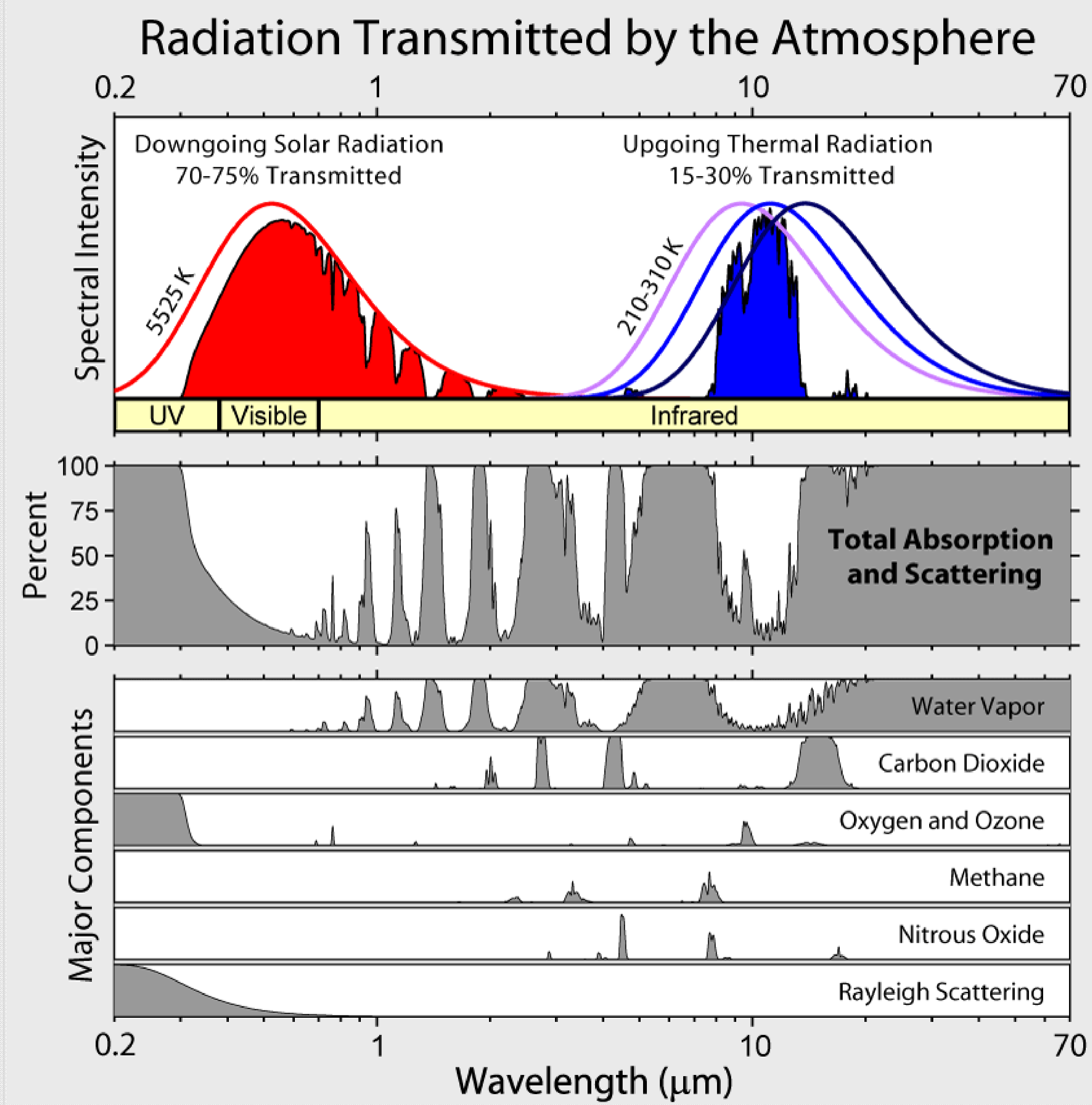
- Real atmosphere is not opaque
- Heat transported by convection as well as by radiation

Elements of the Greenhouse Effect



Principal Atmospheric Absorbers

- H_2O: Bent triatomic, with permanent dipole moment and pure rotational bands as well as rotation-vibration transitions
- O_3: Like water, but also involved in photodissociation
- CO_2: No permanent dipole moment, so no pure rotational transitions, but temporary dipole during vibrational transitions
- Other gases: N_2O, CH_4



Radiation Transmitted by the Atmosphere
0.2
1
10
70
Spectral Intensity
Downgoing Solar Radiation
70-75% Transmitted
Upgoing Thermal Radiation
15-30% Transmitted
5525 K
210-310 K
UV
Visible
Infrared
Percent
100
75
50
25
0
Total Absorption and Scattering
Major Components
Water Vapor
Carbon Dioxide
Oxygen and Ozone
Methane
Nitrous Oxide
Rayleigh Scattering
Wavelength (μm)

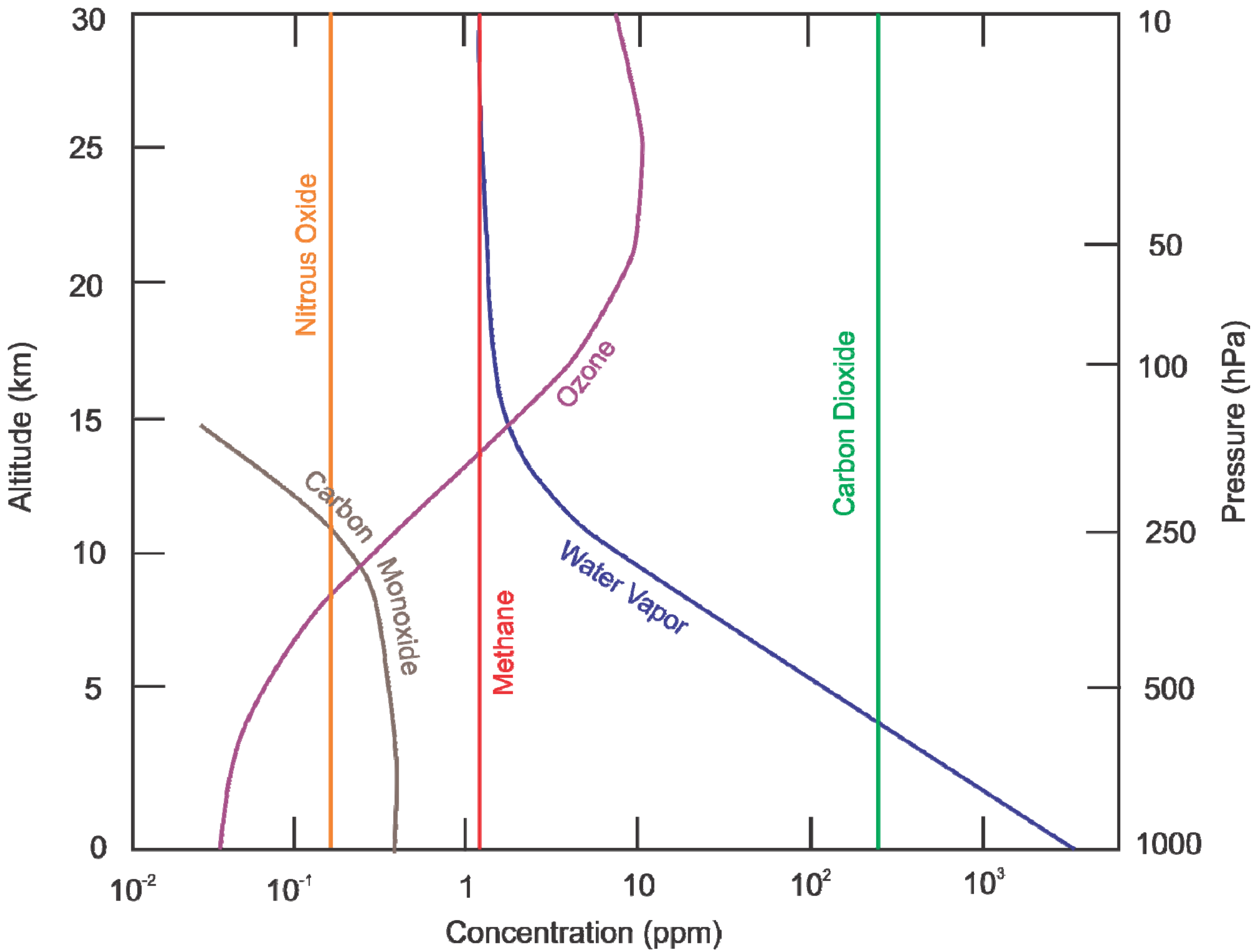
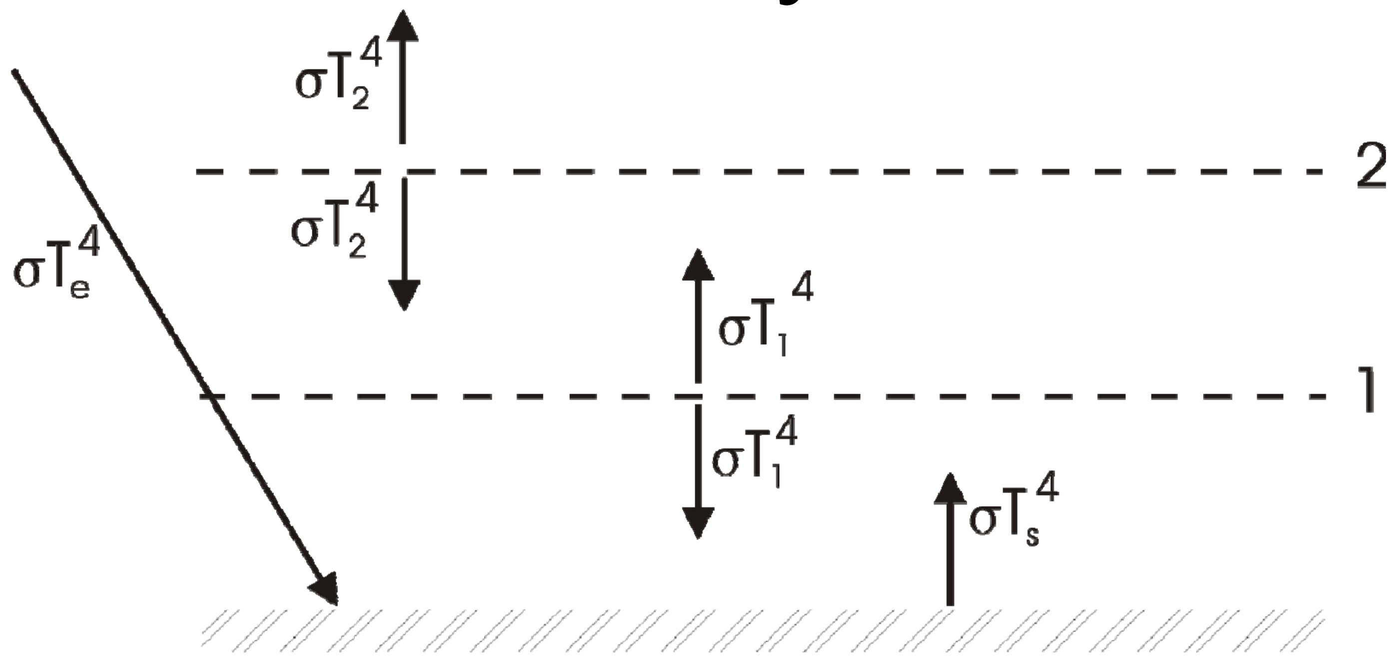


Image credit: *Kerry Emanuel*

Radiative Equilibrium

- Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
- Radiative heating drives actual state toward state of radiative equilibrium

Extended Layer Models



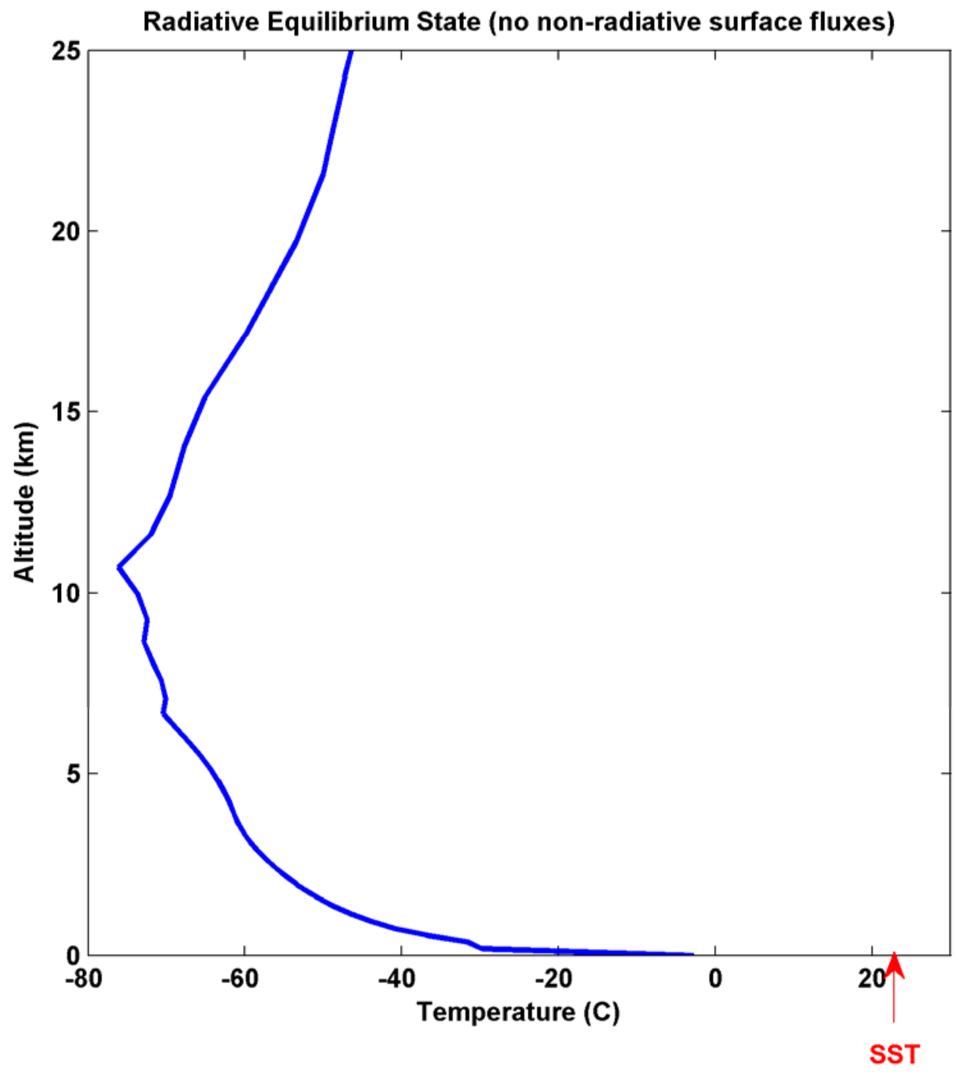
$$TOA: \quad \sigma T_2{}^4 = \sigma T_e{}^4 \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1{}^4 = \sigma T_2{}^4 + \sigma T_s{}^4 = \sigma T_e{}^4 + \sigma T_s{}^4$$

$$Surface: \quad \sigma T_s{}^4 = \sigma T_e{}^4 + \sigma T_1{}^4$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

Full calculation of radiative equilibrium: (no non-radiative fluxes, fixed relative humidity)



Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer

When is a fluid unstable to convection?

- Buoyancy
- Stability

Buoyancy

$$B \equiv g \frac{\alpha'}{\bar{\alpha}},$$

$\alpha' =$ fluctuation of specific volume on isobaric surface

$\bar{\alpha} =$ mean specific volume on isobaric surface

Buoyancy and Entropy

Specific Volume: $\alpha = 1/\rho$

Specific Entropy: s

$$\alpha = \alpha(p, s)$$

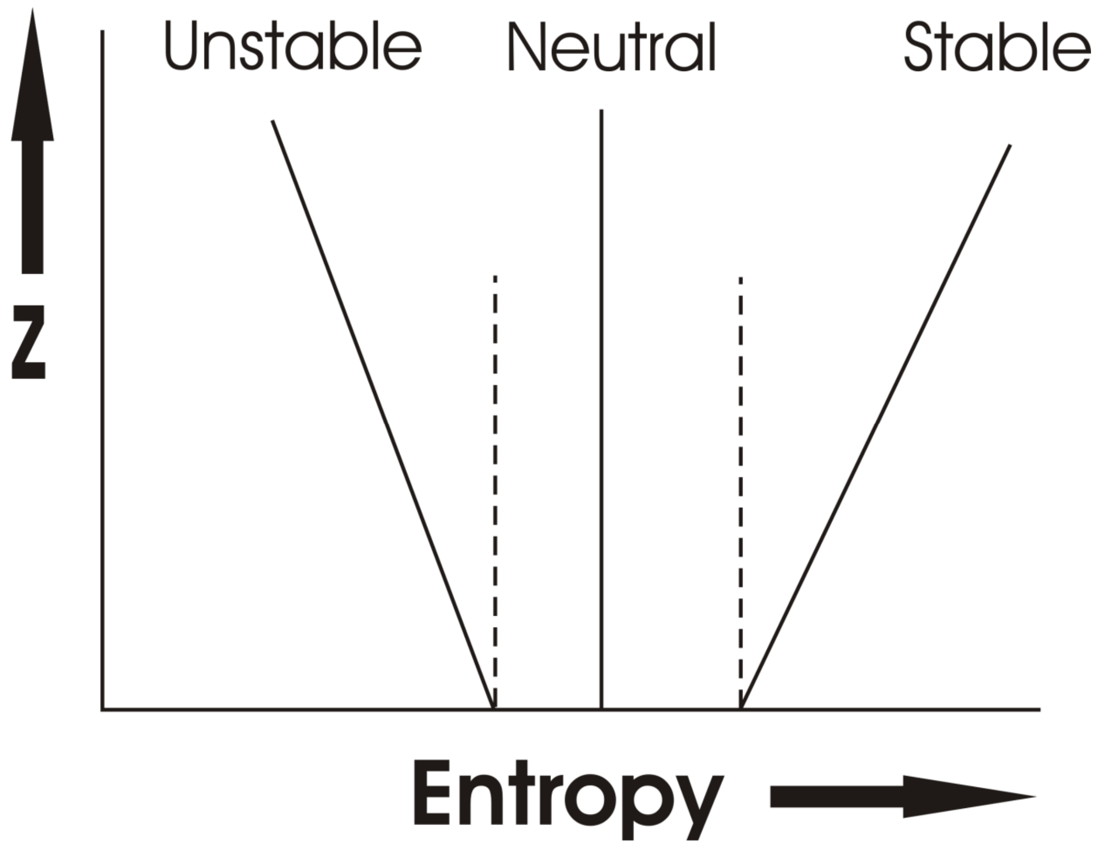
$$(\delta\alpha)_p = \left(\frac{\partial \alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{(\delta\alpha)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma \delta s$$

Buoyancy is produced by fluctuations of entropy on isobaric surfaces

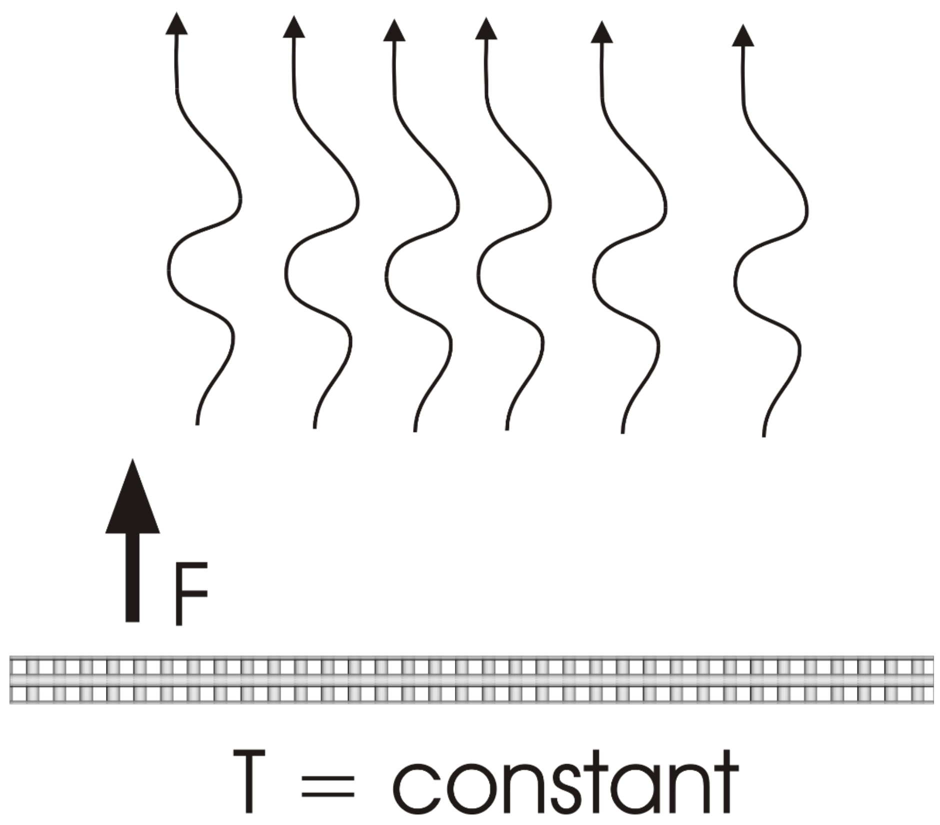
$$\Gamma = g/c_p$$

Earth's atmosphere: $\Gamma = 1\,K/100\,m$



Prototypical Convective Problem: The Prandtl Problem

Prandtl L 1925 Z. Angew. Math. Mech. 5 136



$$F \equiv \overline{w' B'}$$

$$q \sim \left(zF \right)^{1/3}$$

$$g\beta \,| T' | \sim F^{2/3} z^{-1/3}$$

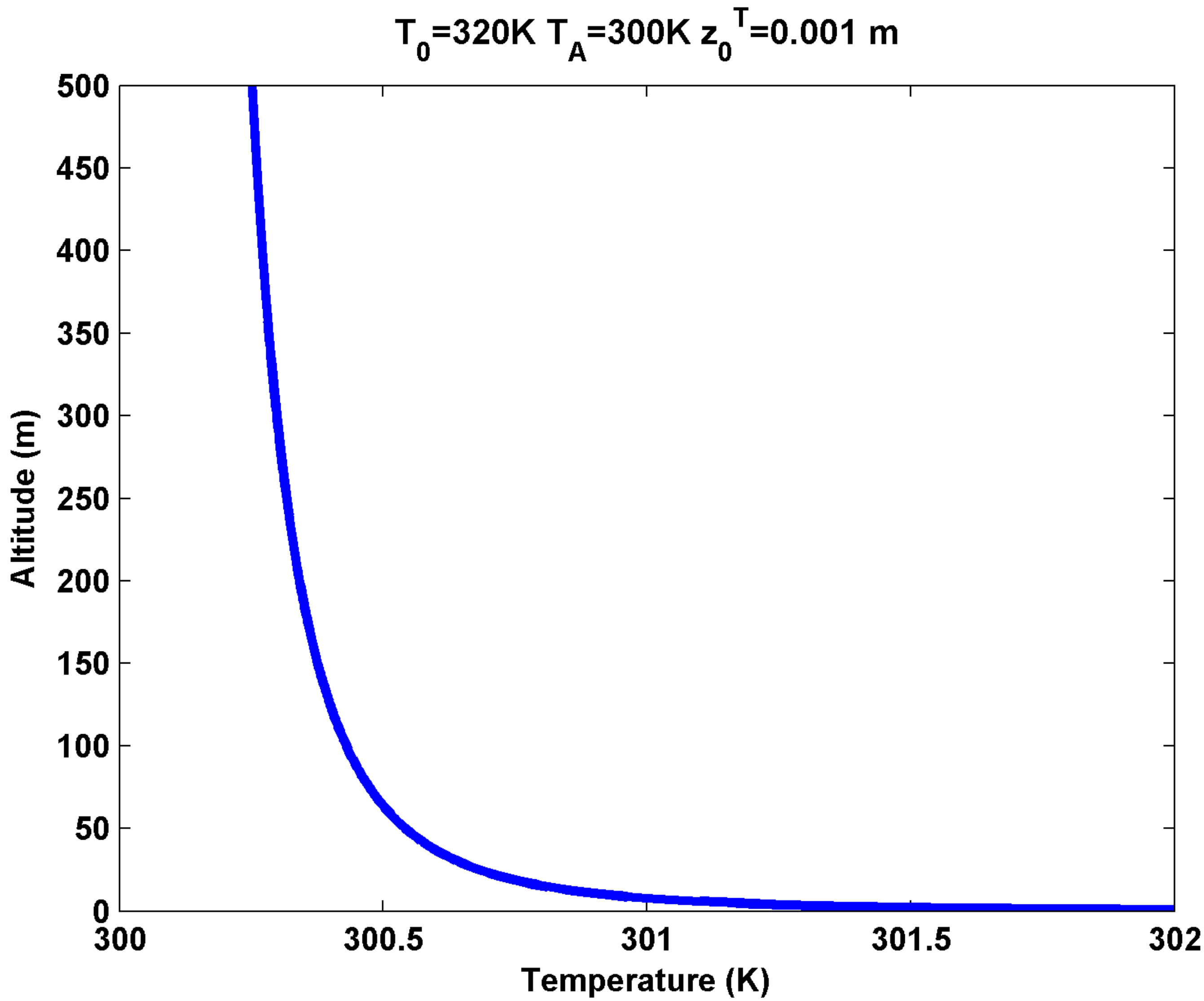
$$q_0 \sim \left(z_0^T F \right)^{1/3}$$

$$g\beta \,| T_0' | \sim F^{2/3} \left(z_0^T \right)^{-1/3}$$

$$g\beta \frac{d\overline{T}}{dz} = -c_1 F^{2/3} z^{-4/3}$$

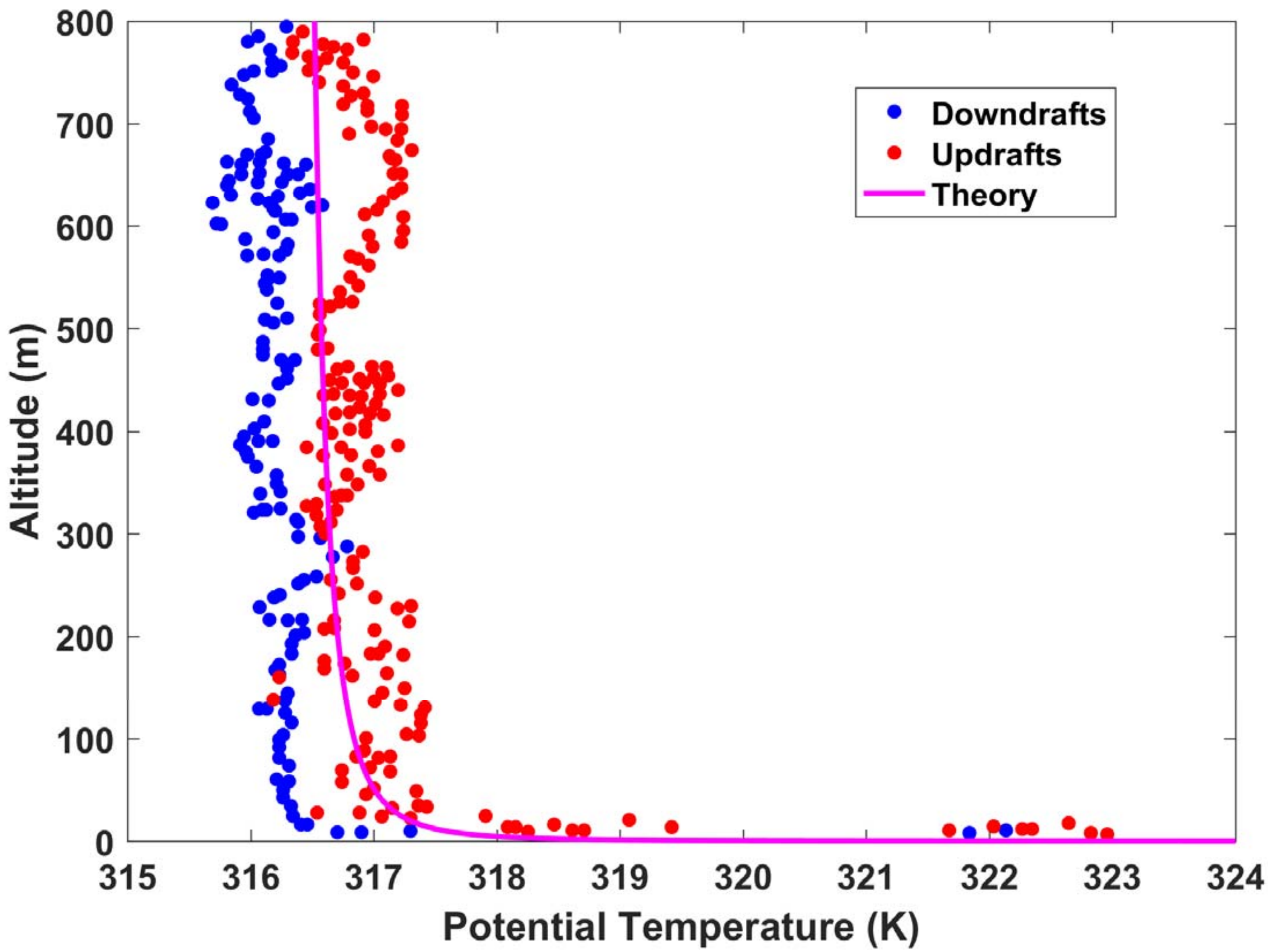
$$\rightarrow \overline{T} = \overline{T}_0 - \frac{3c_1}{g\beta} F^{2/3} \left[\left(z_0^T \right)^{-1/3} - z^{-1/3} \right]$$

$$\overline{T}_A^{\infty} = \overline{T}_0 - \frac{3c_1}{g\beta} F^{2/3} \left(z_0^T \right)^{-1/3}$$

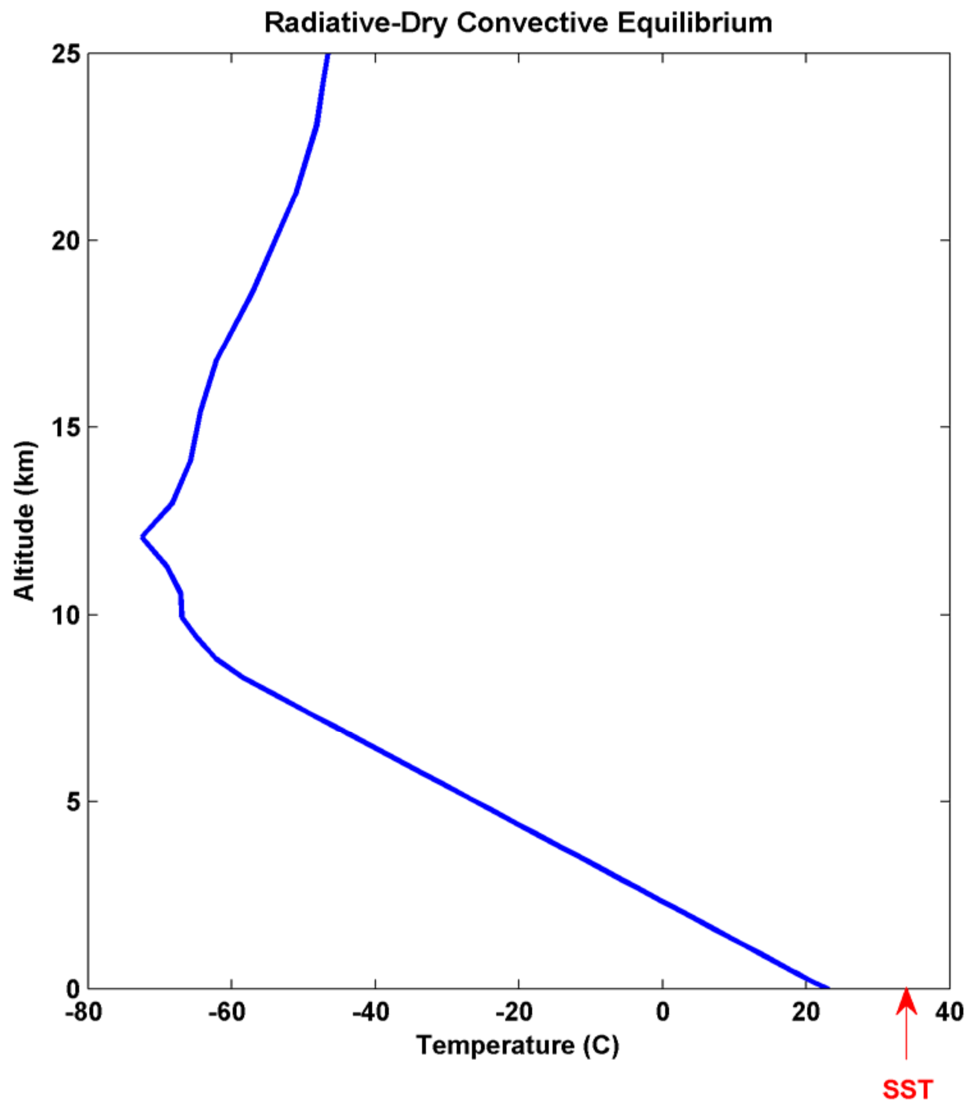


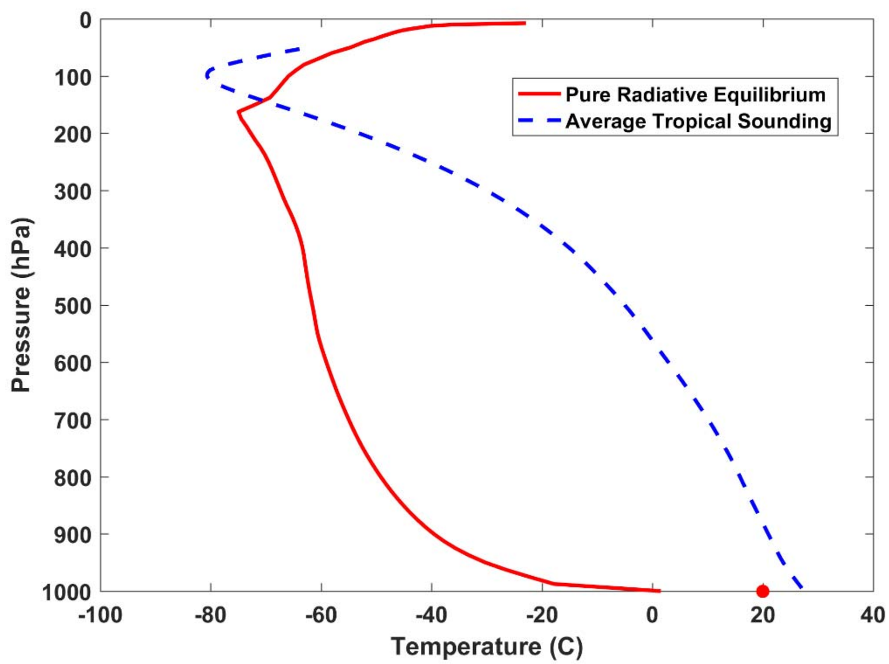
T_0=320K T_A=300K $z_0{}^T$=0.001 m
Altitude (m)
500
450
400
350
300
250
200
150
100
50
0
300
300.5
301
301.5
302
Temperature (K)

Model Aircraft Measurements
(Renno and Williams, 1995)

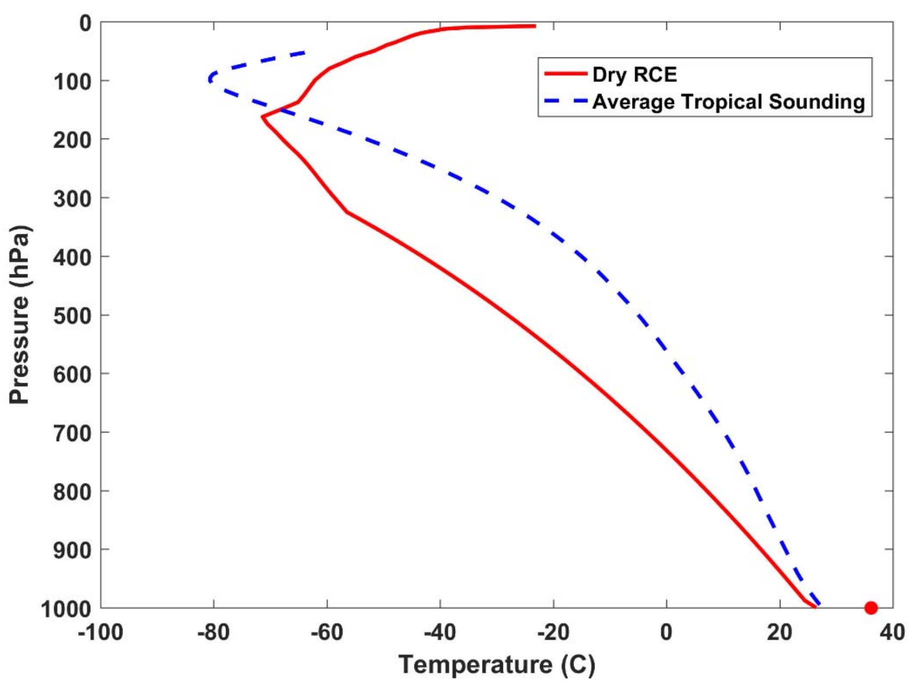


Full calculation of radiative-dry convective equilibrium
(Surface dry turbulent enthalpy flux, fixed relative humidity)



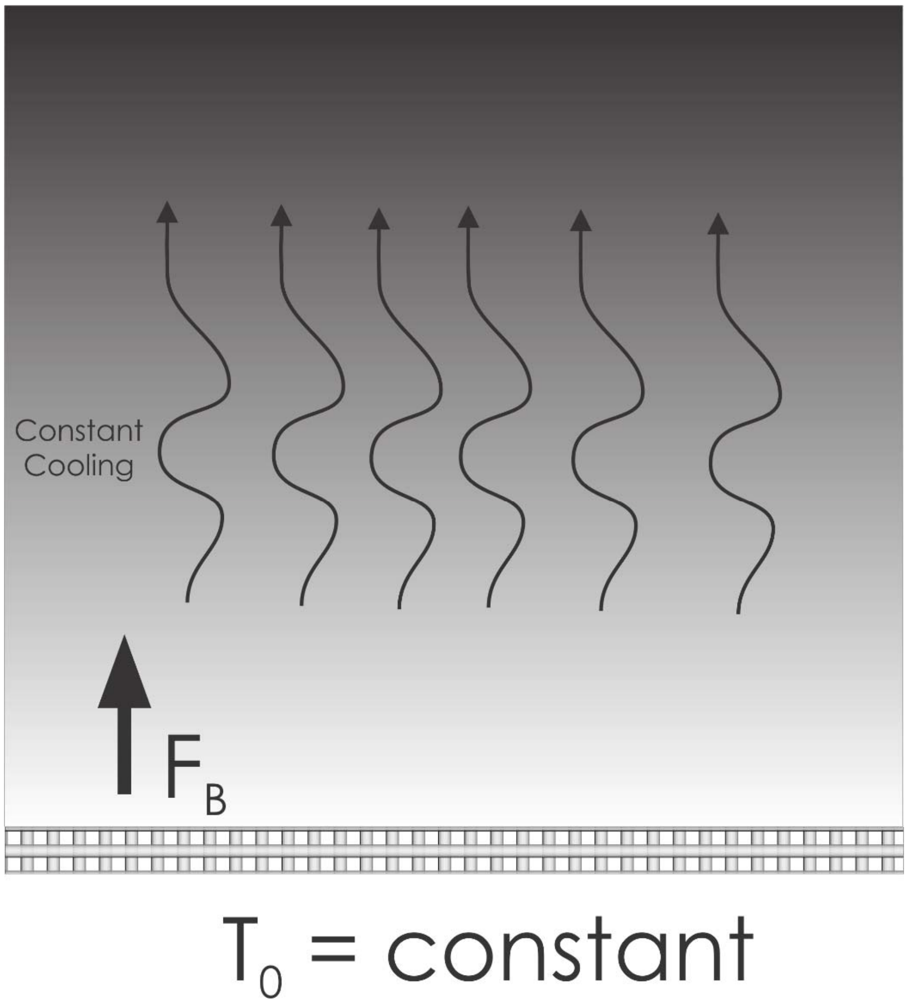


Pure radiative equilibrium



Radiative-dry convective equilibrium

Moist Prandtl Problem



Air is everywhere saturated and filled with cloud

Phase change is fast!

Problem isomorphic to classical Prandtl problem with some transformations:

$$\text{Entropy:}\;\; s = \left(c_{pd} + r_t c_l\right)\ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p_d}{p_0}\right) + \frac{L_v r^*}{T},$$

where c_{pd} is the heat capacity at constant pressure of dry air, c_l is the heat capacity of liquid water, r is the mixing ratio (mass of water vapor per unit mass of dry air), r_t is the total water mixing ratio (mass per unit mass of dry air), R_d is the gas constant for dry air, p_d is the partial pressure of dry air, L_v is the latent heat of vaporization

$$B = \frac{\Gamma_m}{1+r_t} s', \quad \Gamma_m = -\left(\frac{\partial T}{\partial z}\right)_{s,r_t} = \text{moist adiabatic lapse rate}$$

TKE Budget: $\frac{\partial}{\partial z}\left[\overline{w'^3}+\alpha_0\overline{p'w'}\right]=\frac{\Gamma_m F}{\left(1+r_t\right)T_0}.$

New vertical coordinate: $\mu\equiv\frac{T_0-\overline{T}}{\Gamma_d}$

TKE: $\frac{\partial}{\partial\mu}\left[\overline{w'^3}+\alpha_0\overline{p'w'}\right]=\frac{\Gamma_d F}{\left(1+r_t\right)T_0}\equiv F_B.$

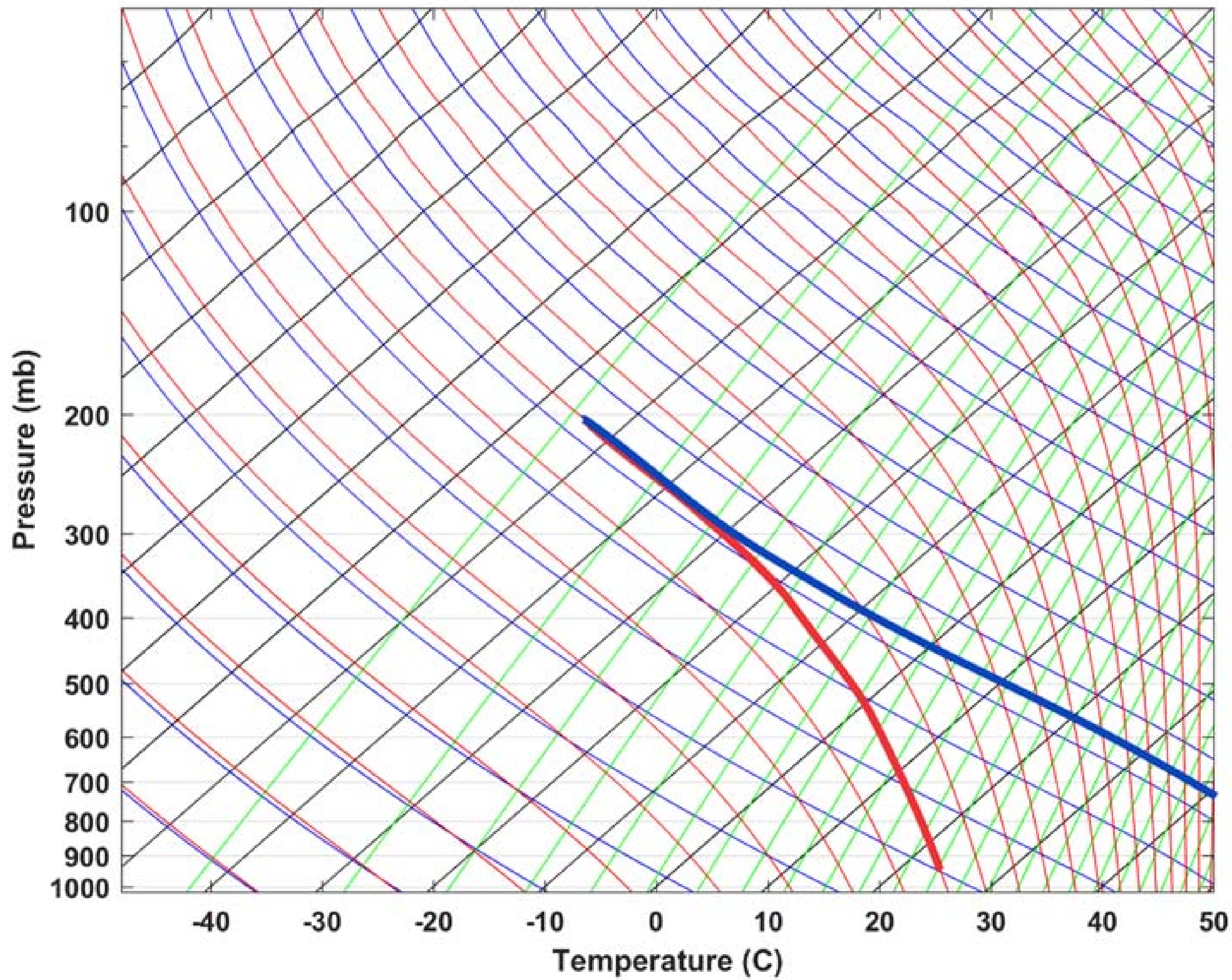
Back to one-parameter problem; all dry scalings are valid, e.g. :

$$q\sim\left(\mu F_B\right)^{1/3}$$

Irreversible fall-out of precipitation

"Pseudo-adiabatic" limit: A demon removes all condensed water as soon as it forms

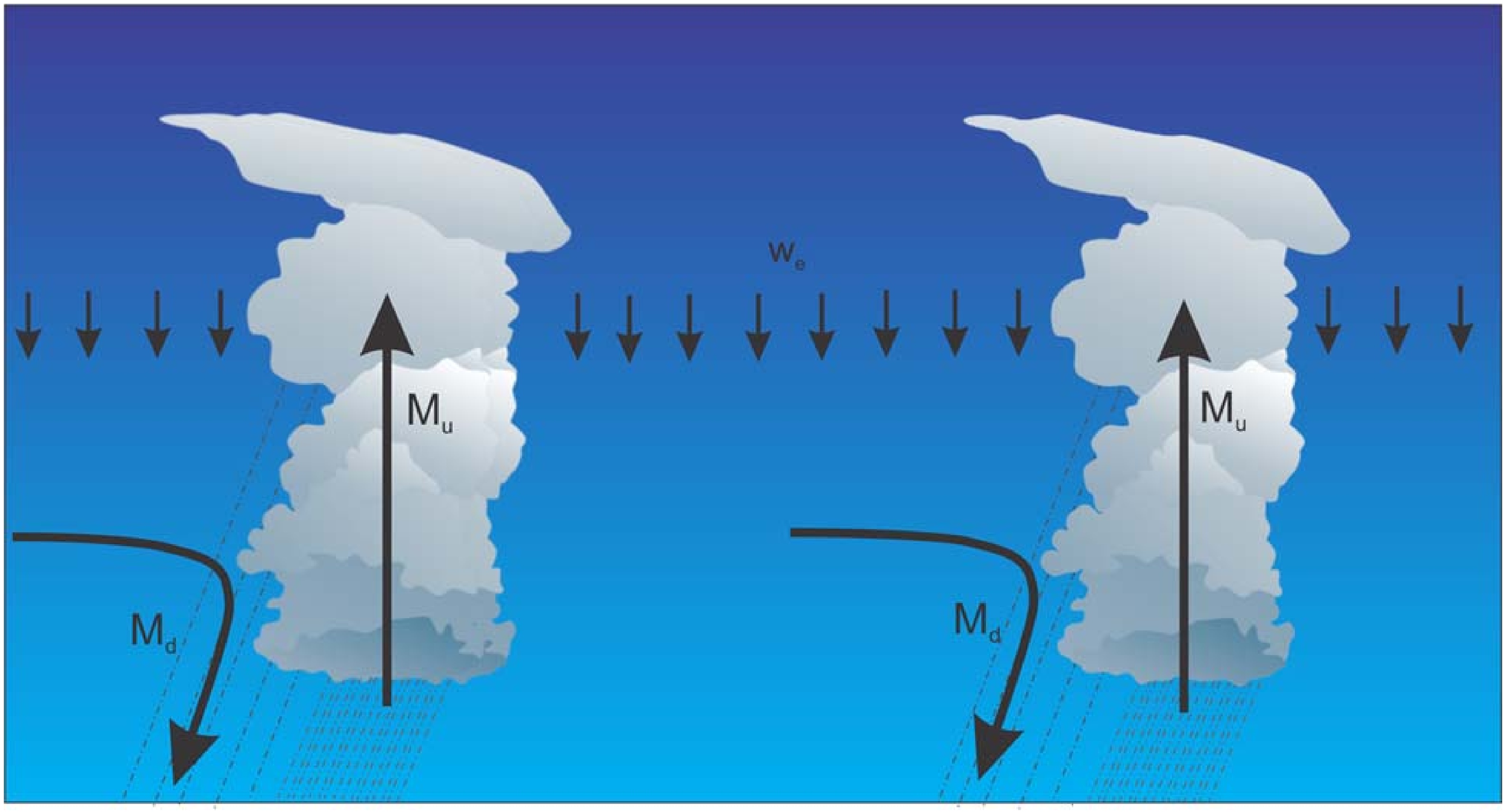
Pseudo-adiabatic process on a tephigram



Precipitation Formation:

- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes

Moist RCE with precipitation



Narrow updrafts “entrain” environmental air!
Ascent not adiabatic.

Properties of Moist RCE

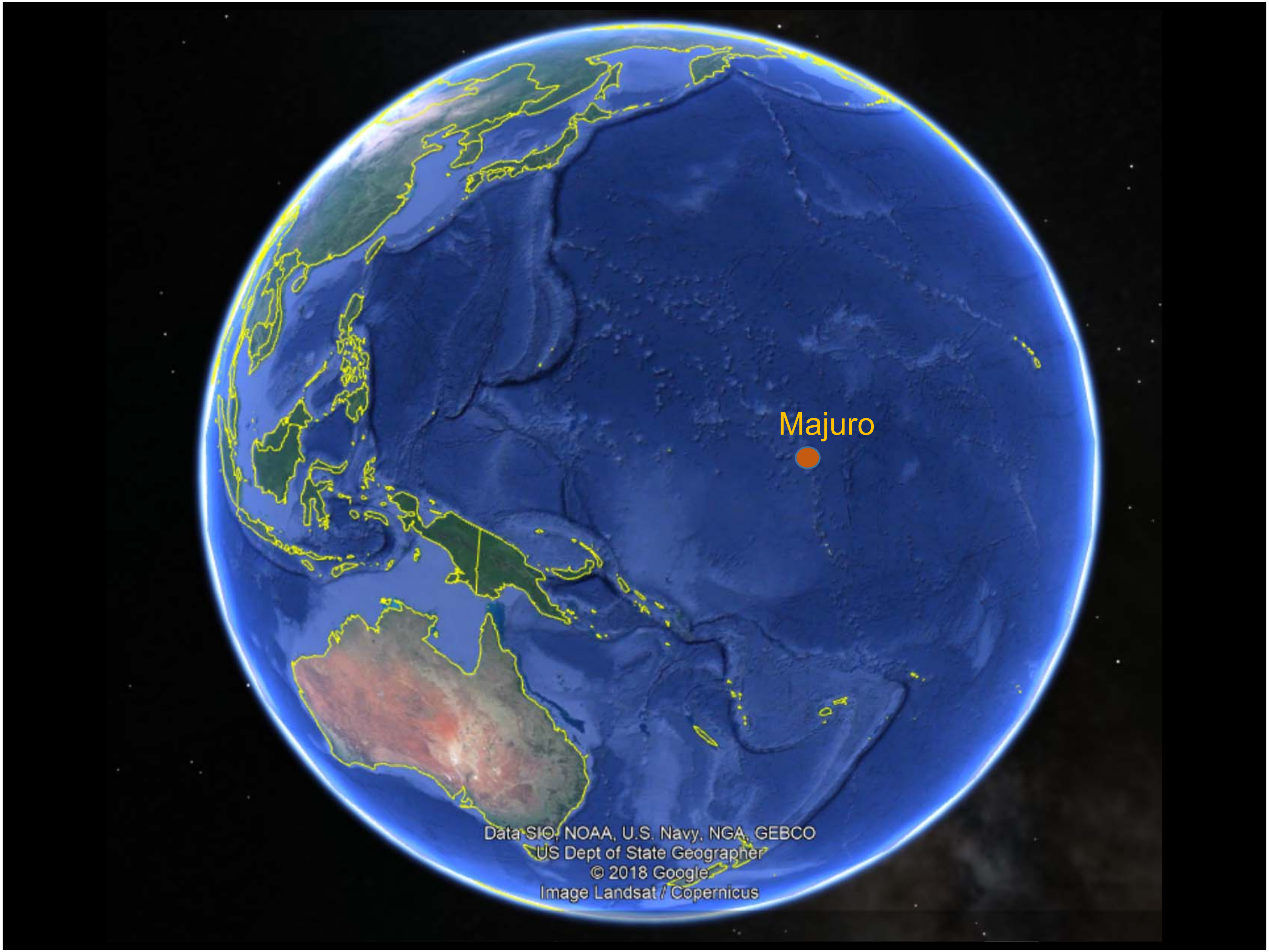
- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
- $$MT\frac{\partial s_d}{\partial z} = -\dot{Q}$$
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive

Unsolved Problems in Moist RCE

- What determines fractional area covered by updrafts? (Observed to be ~1%)

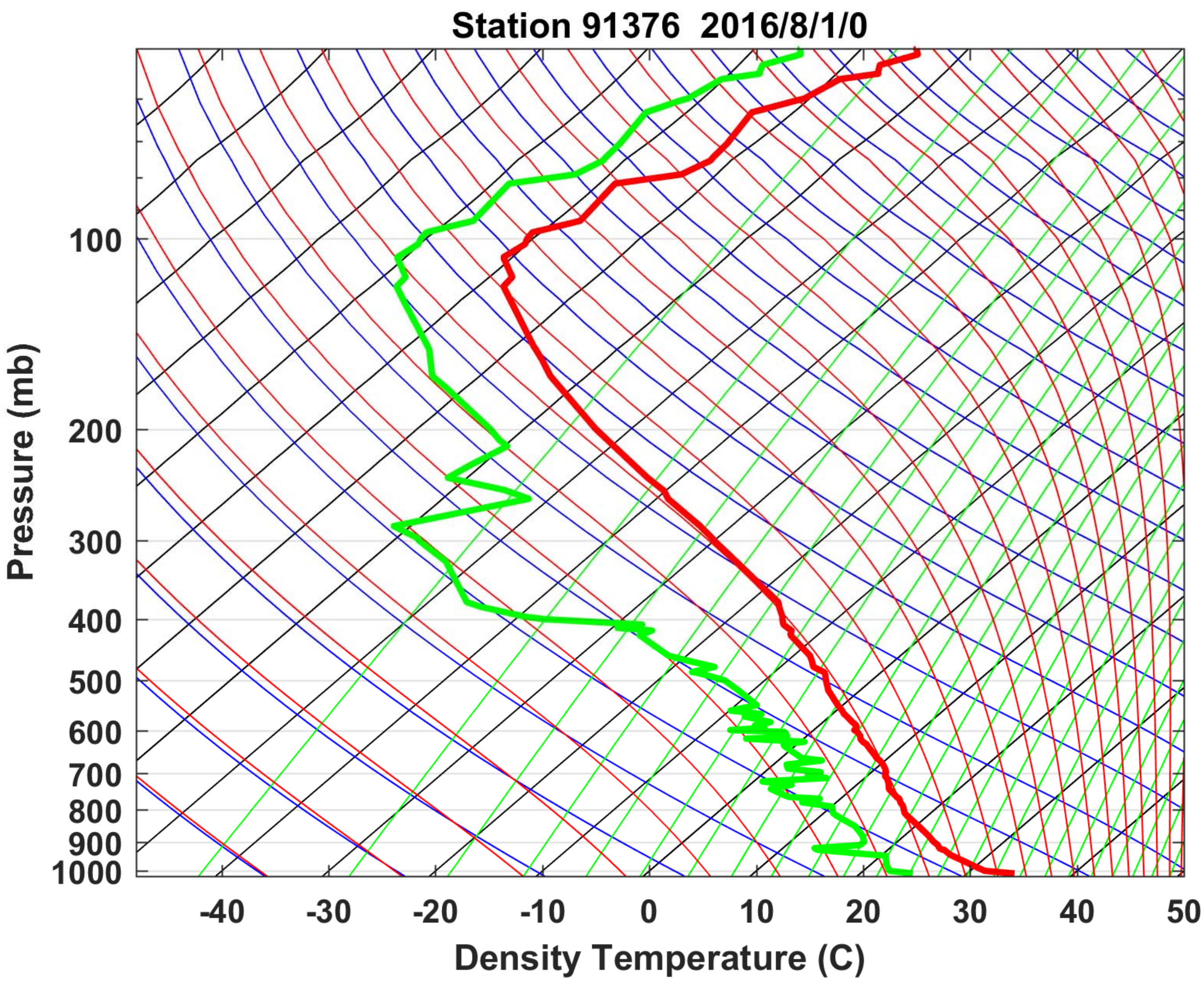
- What determines characteristic updraft velocity? (Really same as first problem)

- What sets the average vertical temperature profile?

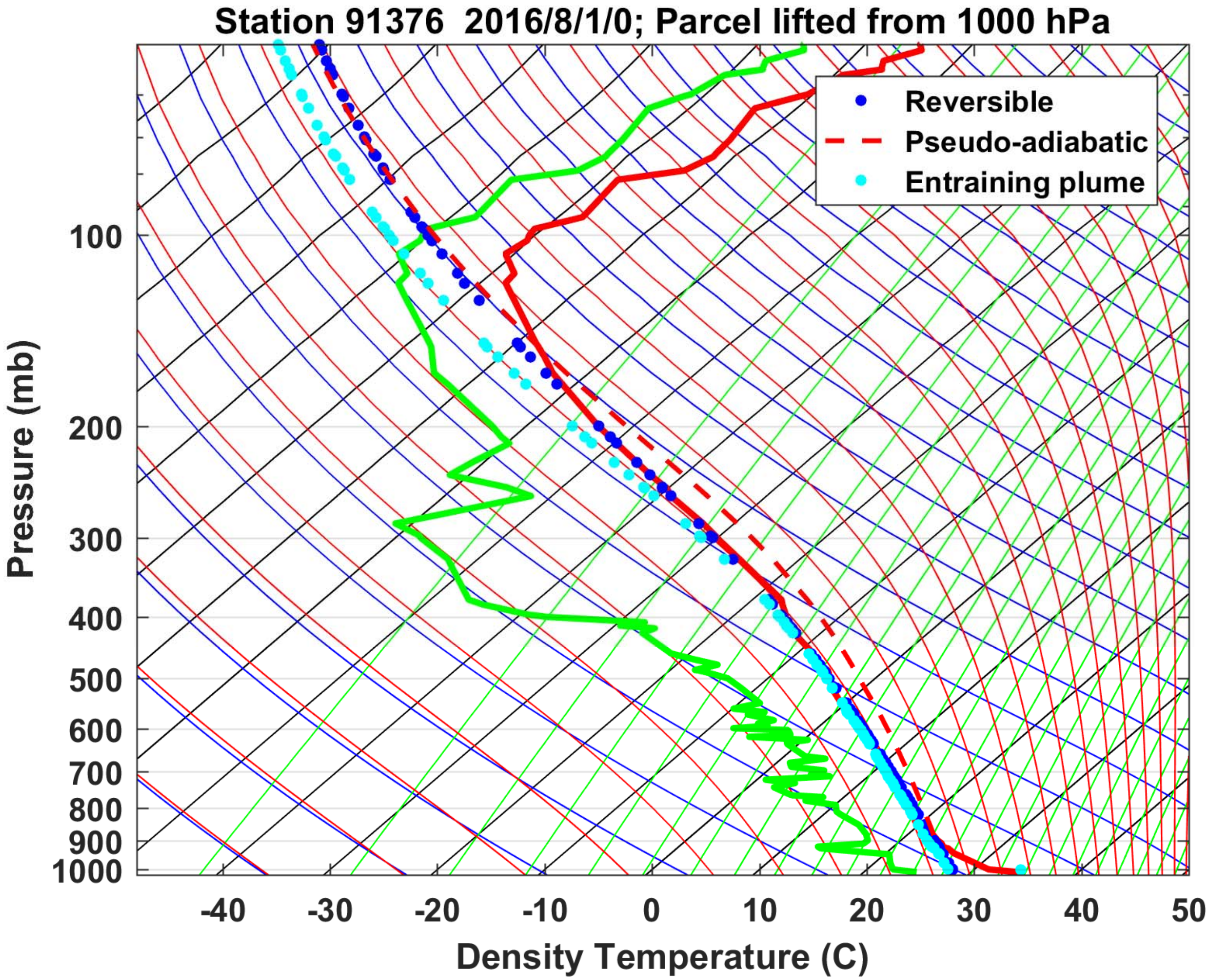

Majuro
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
US Dept of State Geographer
© 2018 Google
Image Landsat / Copernicus

Rawinsondes

Sounding at Majuro, August 1 2016 0 GMT



Sounding at Majuro, August 1 2016 0 GMT



Plot soundings from anywhere in the world, from any time back to 1973

and lift parcels from any level, adiabatically or with entrainment

ftp://texmex.mit.edu/pub/emanuel/CLASS/Tropical/Plotsoundpak.zip

Requires MATLAB

Coincidence of saturated and unsaturated air allows for metastable states: ***Conditional Instability***

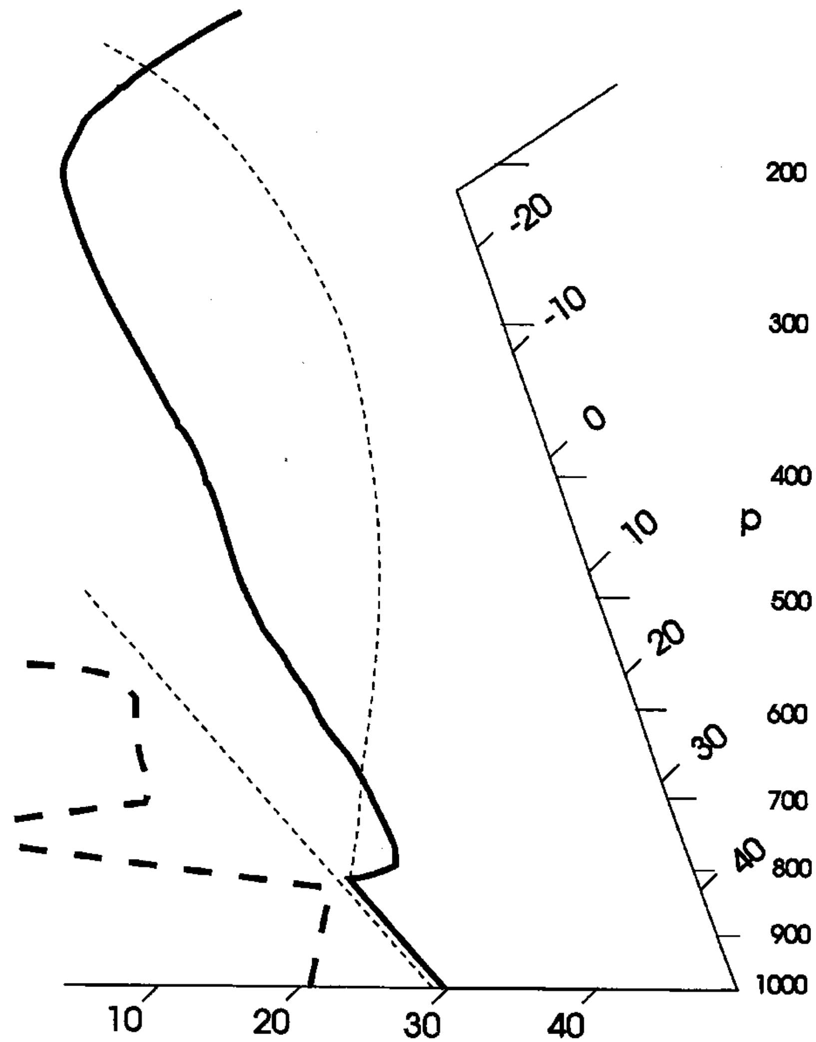
- Relatively unusual
- Mostly over some mid-latitude continents in spring
- Important ingredient in severe convective storms (including large hail, strong winds, tornadoes)

Stability Assessment using Tephigrams:

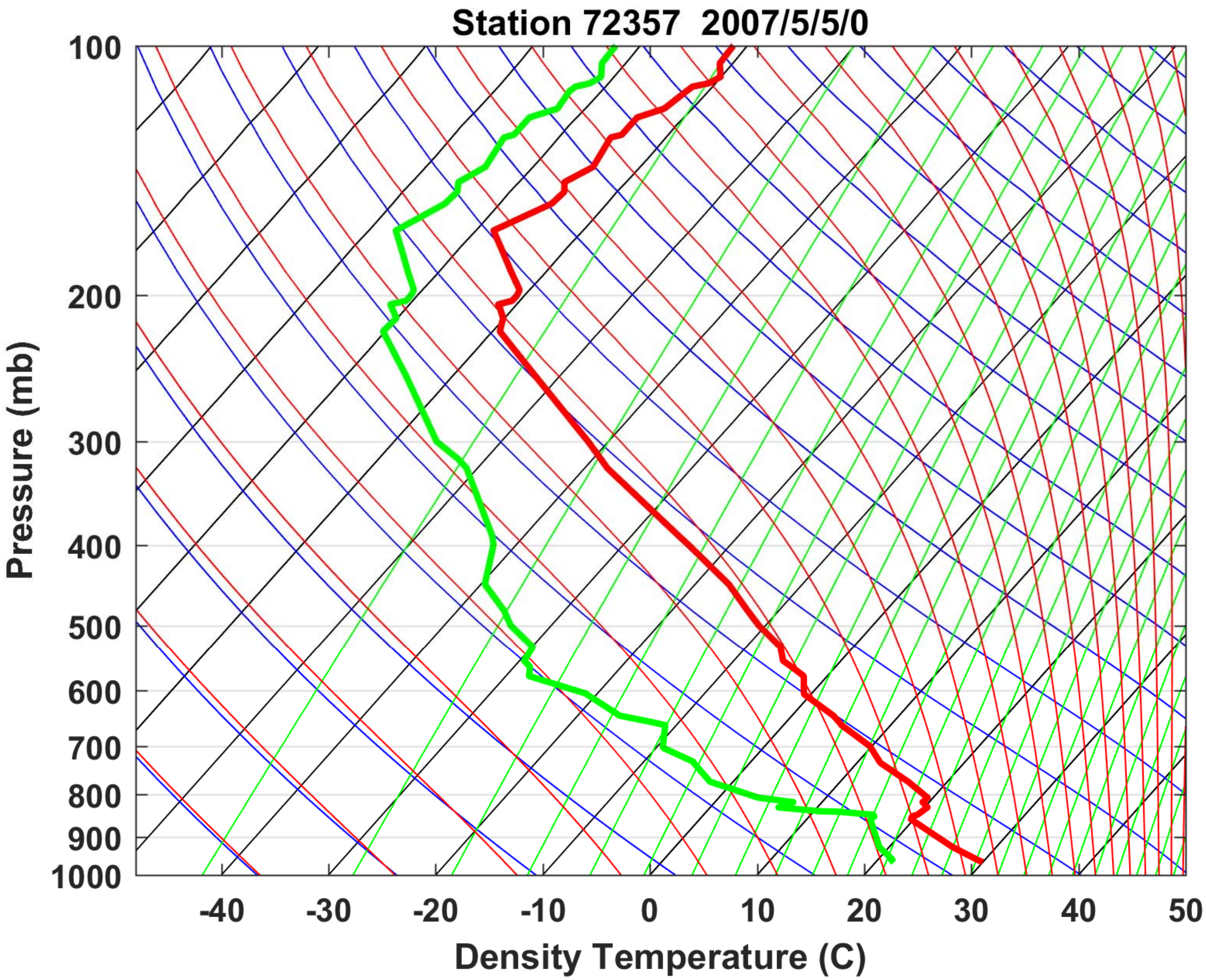
Convective Available Potential Energy
(CAPE):

$$CAPE_i \equiv \int_{p_n}^{p_i} \left(\alpha_p - \alpha_e\right) dp$$

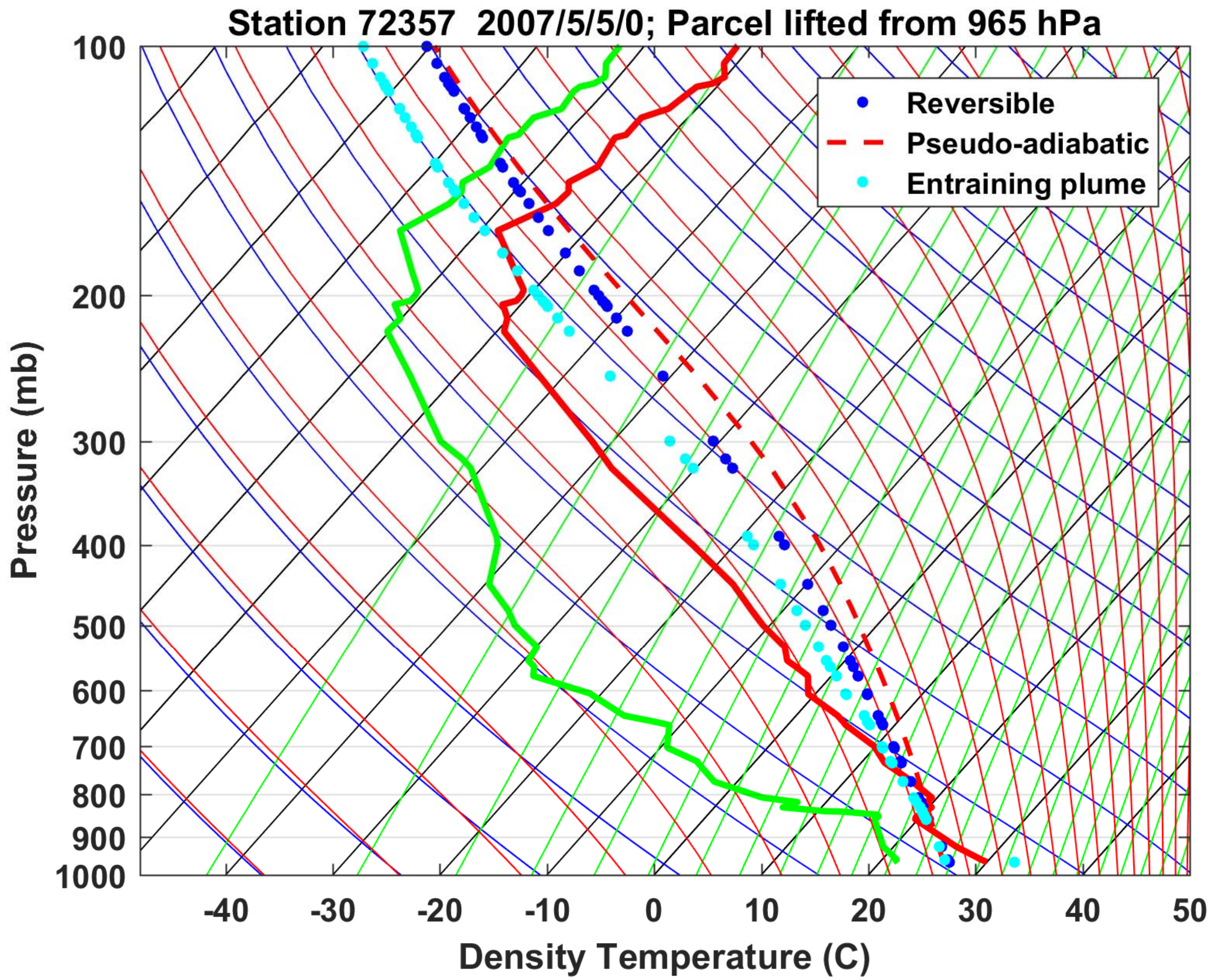
$$= \int_{p}^{p_i} R_d \left(T_{\rho_p} - T_{\rho_e}\right) d\ln\left(p\right)$$



Sounding at Norman, Oklahoma, May 5th 2007

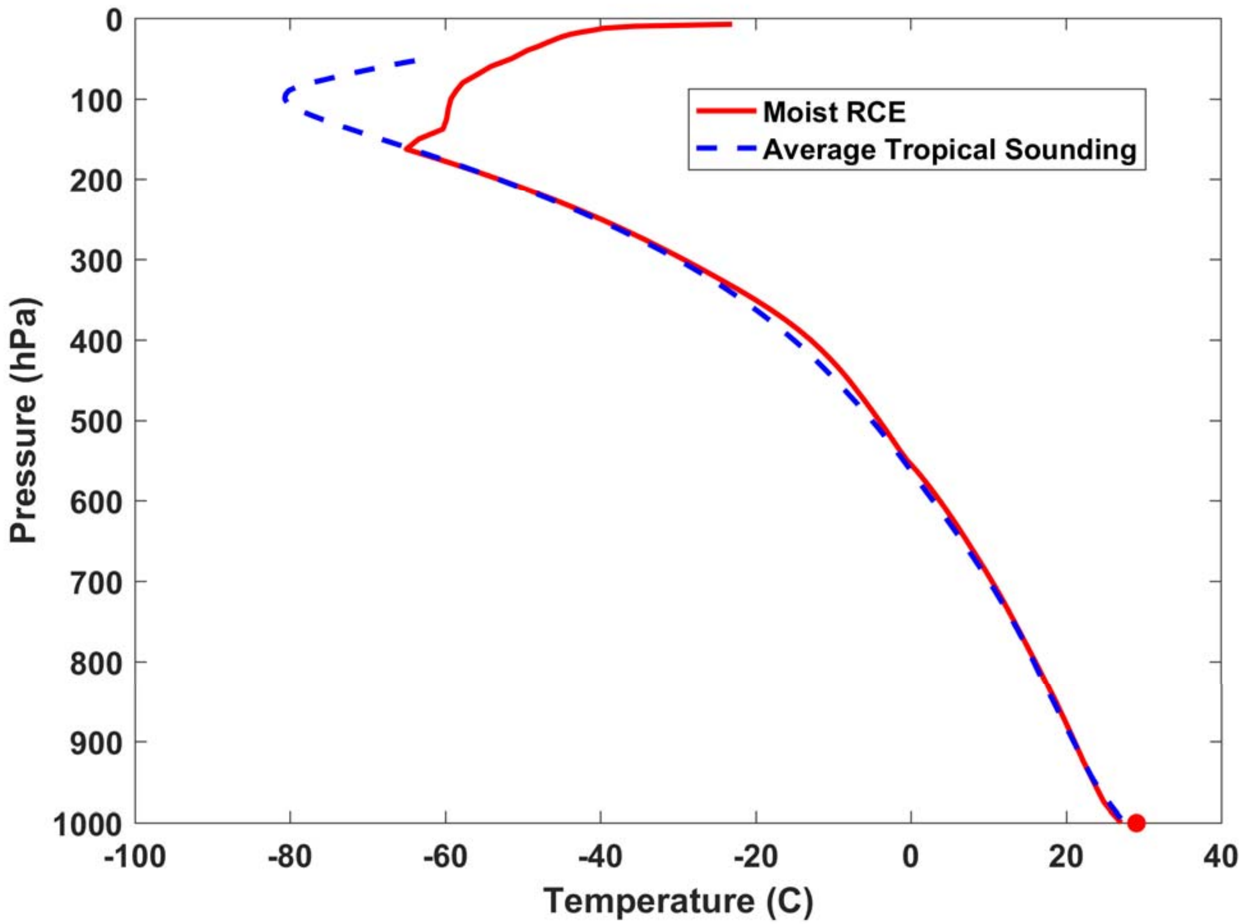


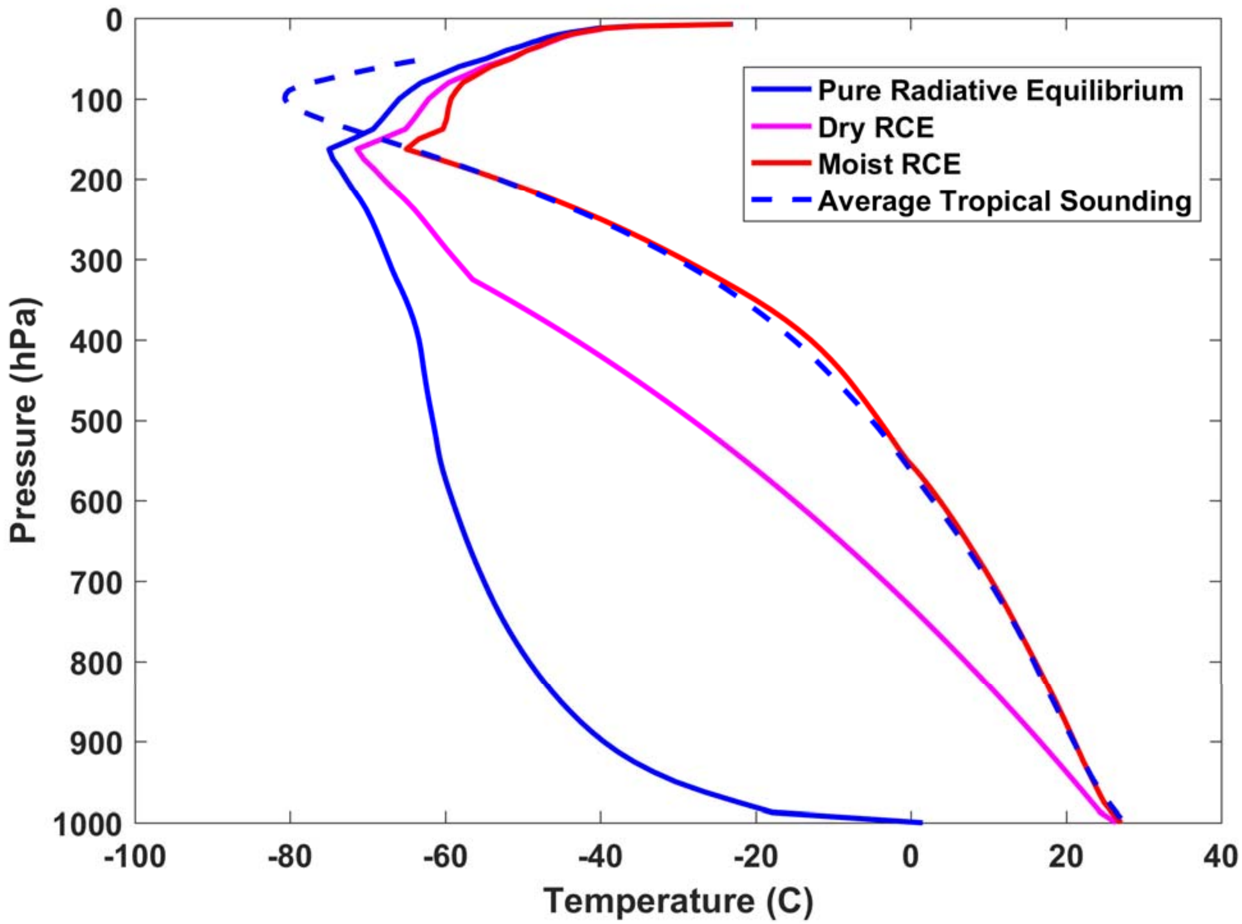
Sounding at Norman, Oklahoma, May 5th 2007



Back to Moist RCE

Radiative-Moist Convective Equilibrium

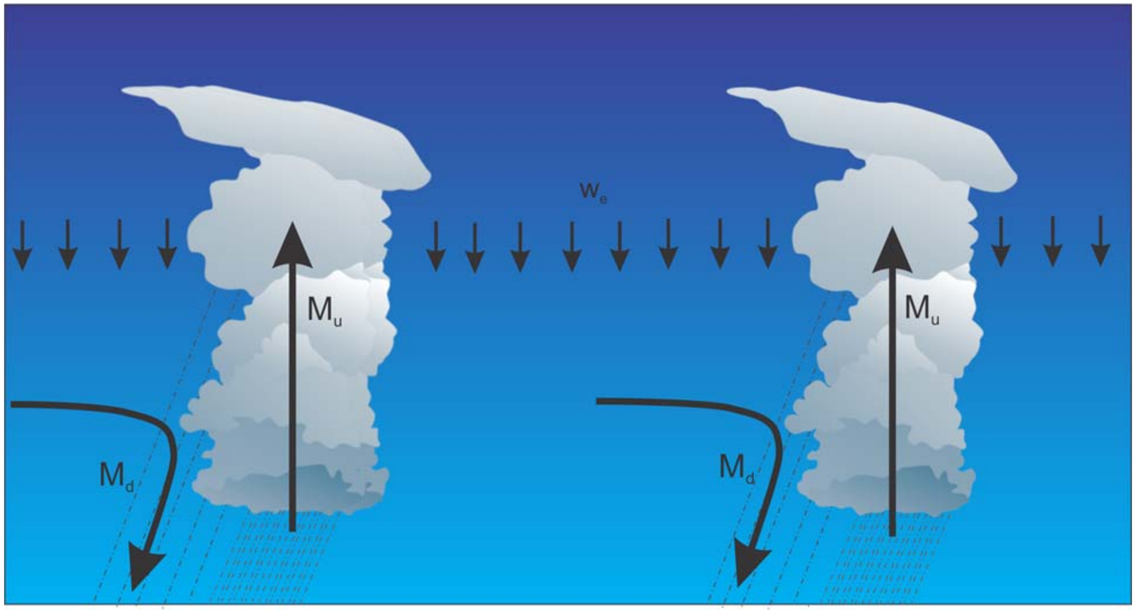




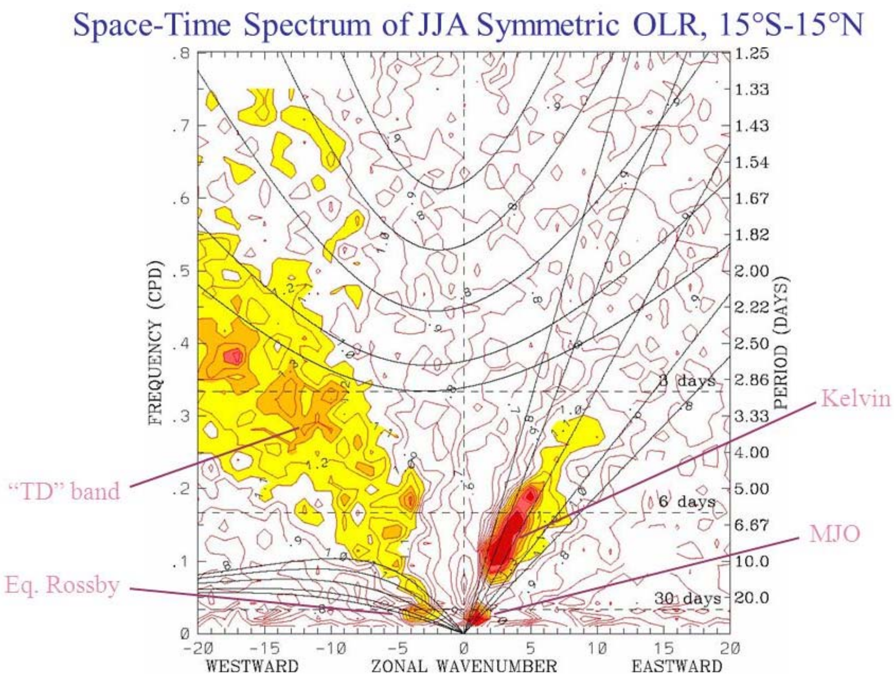
Pure Radiative Equilibrium
Dry RCE
Moist RCE
Average Tropical Sounding
Pressure (hPa)
0
100
200
300
400
500
600
700
800
900
1000
-100
-80
-60
-40
-20
0
20
40
Temperature (C)

Response of Moist RCE State to Perturbations

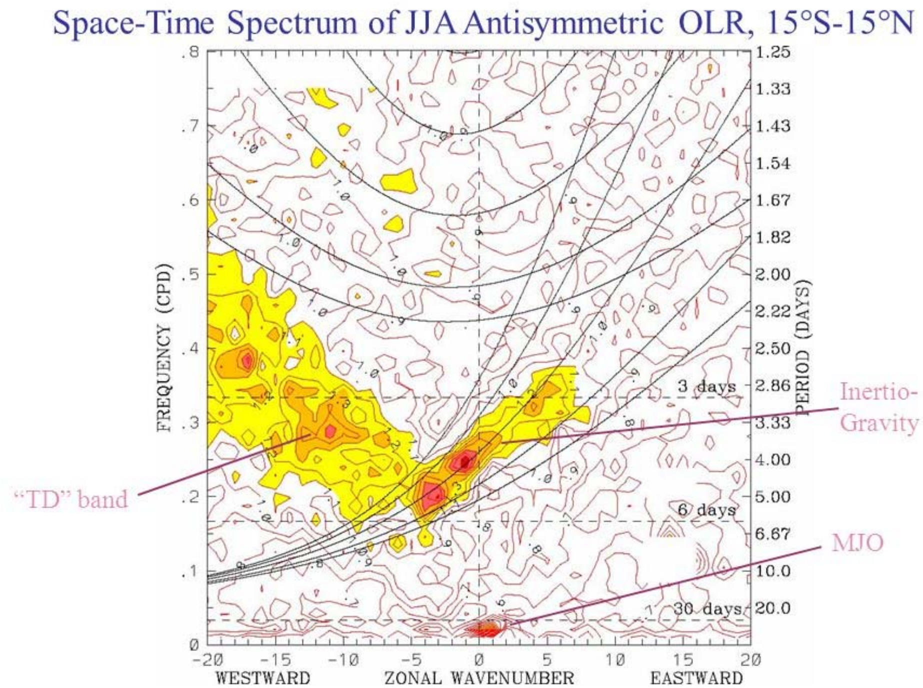
Response to Large-Scale Vertical Motion



- Free tropospheric temperature tied to boundary layer moist entropy
- Large-scale ascent requires more convection
- More convection causes more downdrafts
- Boundary layer entropy decreases
- Free tropospheric temperature decreases
- STABLE



Wheeler and Kiladis, 1999



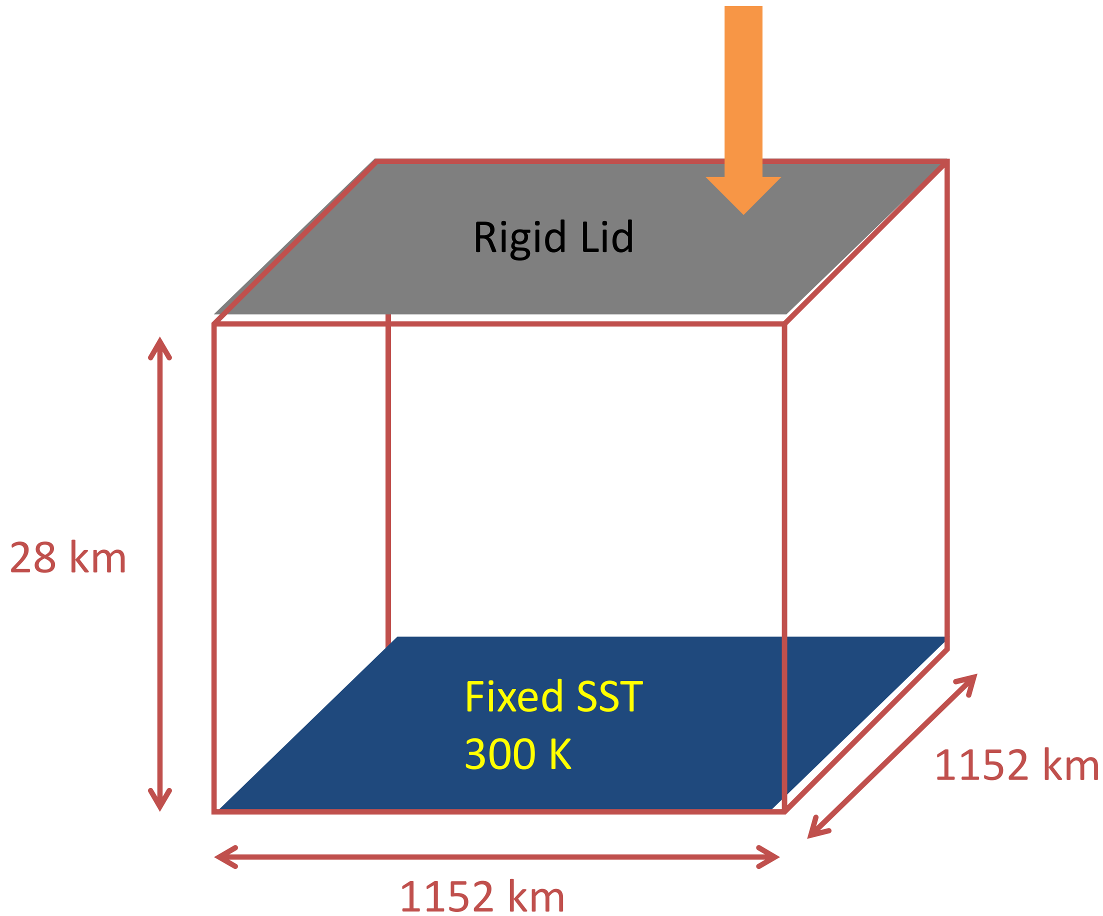
Wheeler and Kiladis, 1999

- Little evidence that perturbations to moist RCE state are unstable in the absence of external diabatic processes
- RCE state can be destabilized by
  - Interactions between perturbation surface winds and surface enthalpy fluxes (WISHE)
  - Interactions between radiation and clouds and/or water vapor

If moistening of the free troposphere results in positive perturbation radiative heating, the moist RCE state can be unstable and lead to self-aggregation of moist convection

Cloud-permitting models of radiative-convective equilibrium run in moderately large domains

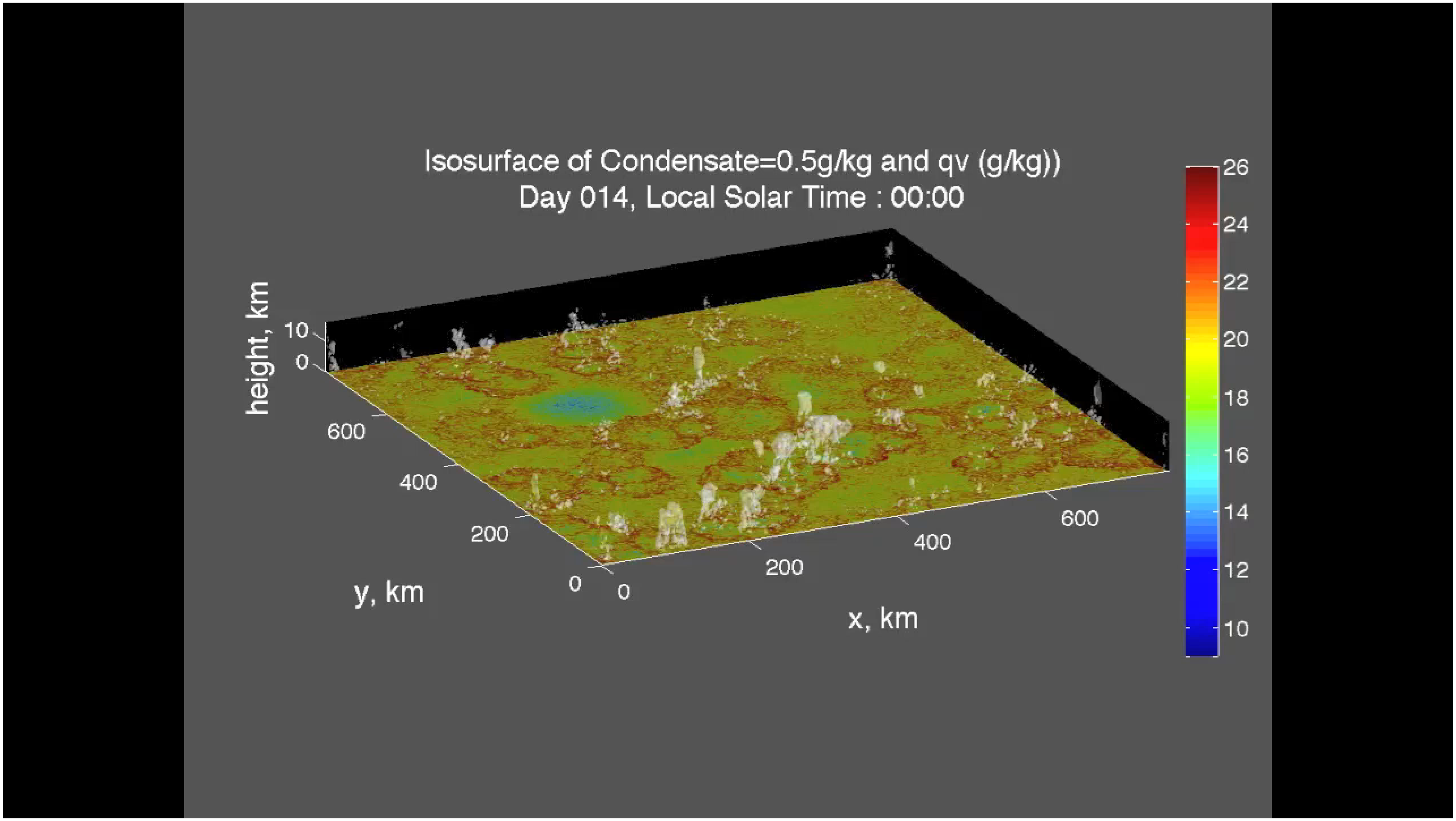
Example: System for Atmospheric Modeling (SAM) of Khairoutdinov and Randall (2003)

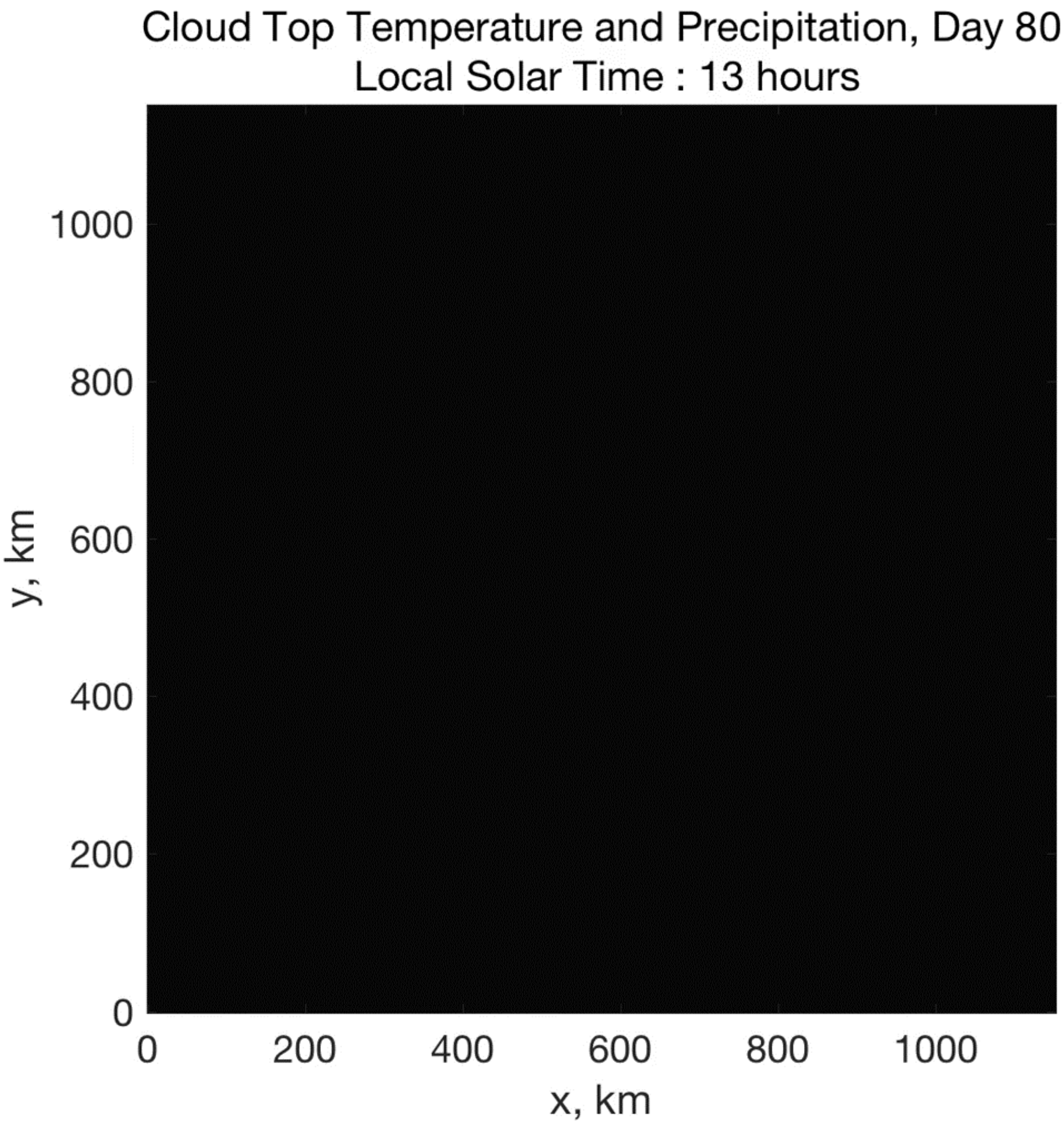


- Horizontal Resolution: 2km
- Vertical Resolution: 64 levels
- Periodic lateral boundaries
- Initial sounding from domain average of smaller domain run in RCE
- No solar fluxes; specified radiative cooling rate.

Courtesy Tim Cronin, MIT

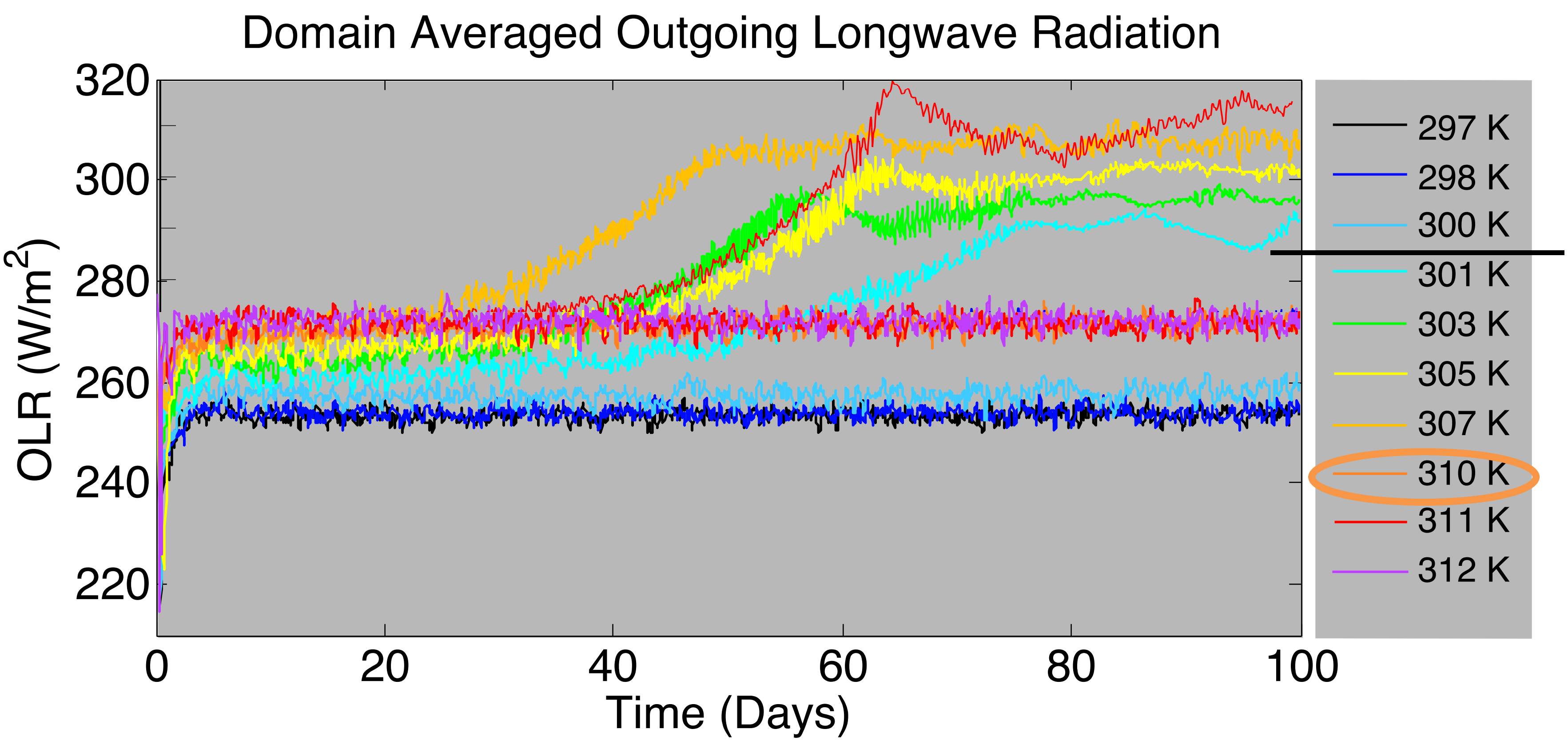
Explicit numerical simulations of radiative-moist convective equilibrium




Cloud Top Temperature and Precipitation, Day 80
Local Solar Time : 13 hours
1000
800
600
400
200
0
y, km
0
200
400
600
800
1000
x, km

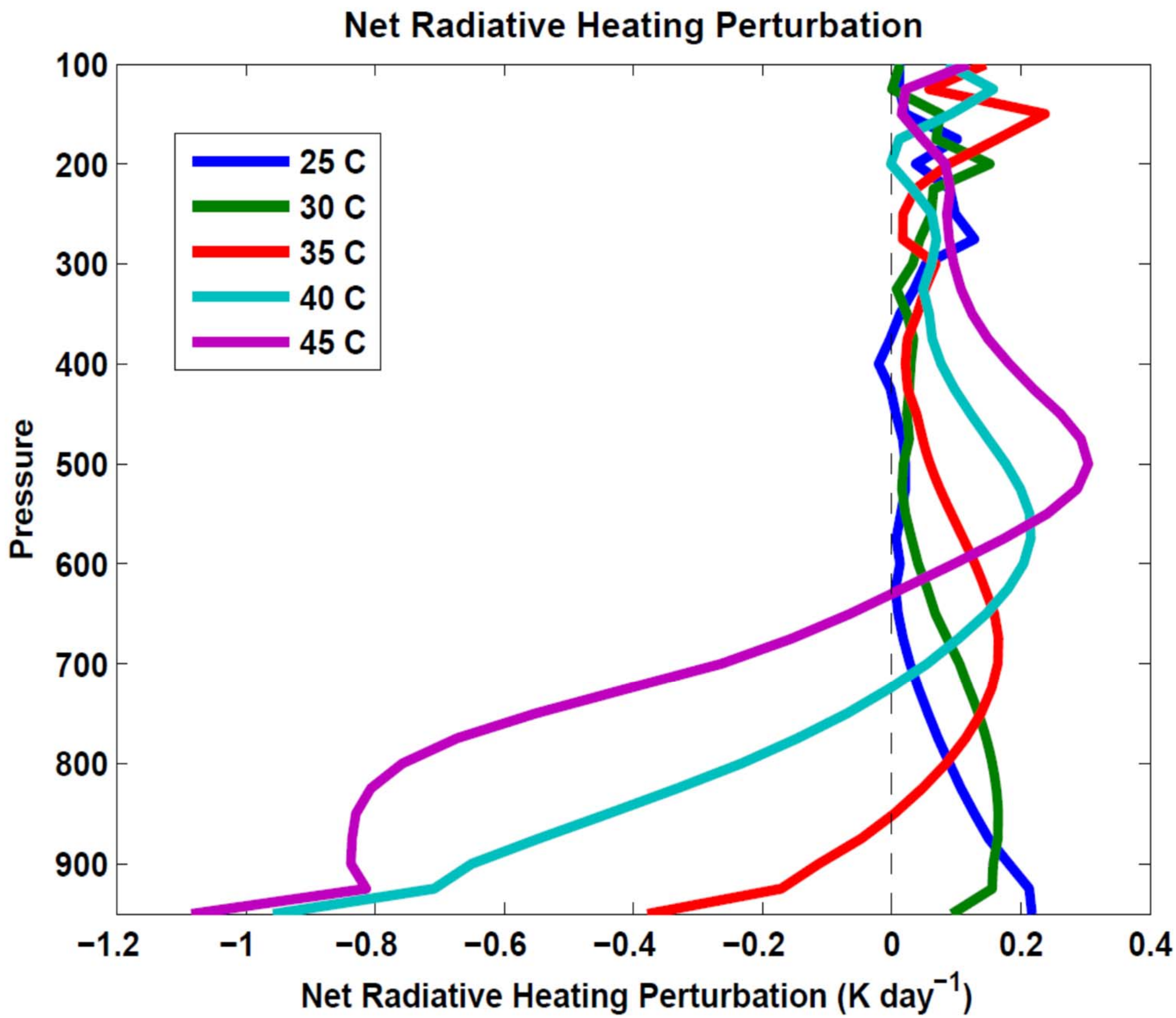
Physics of Self-Aggregation

Aggregation Depends on Surface Temperature (in some models and boundary conditions)



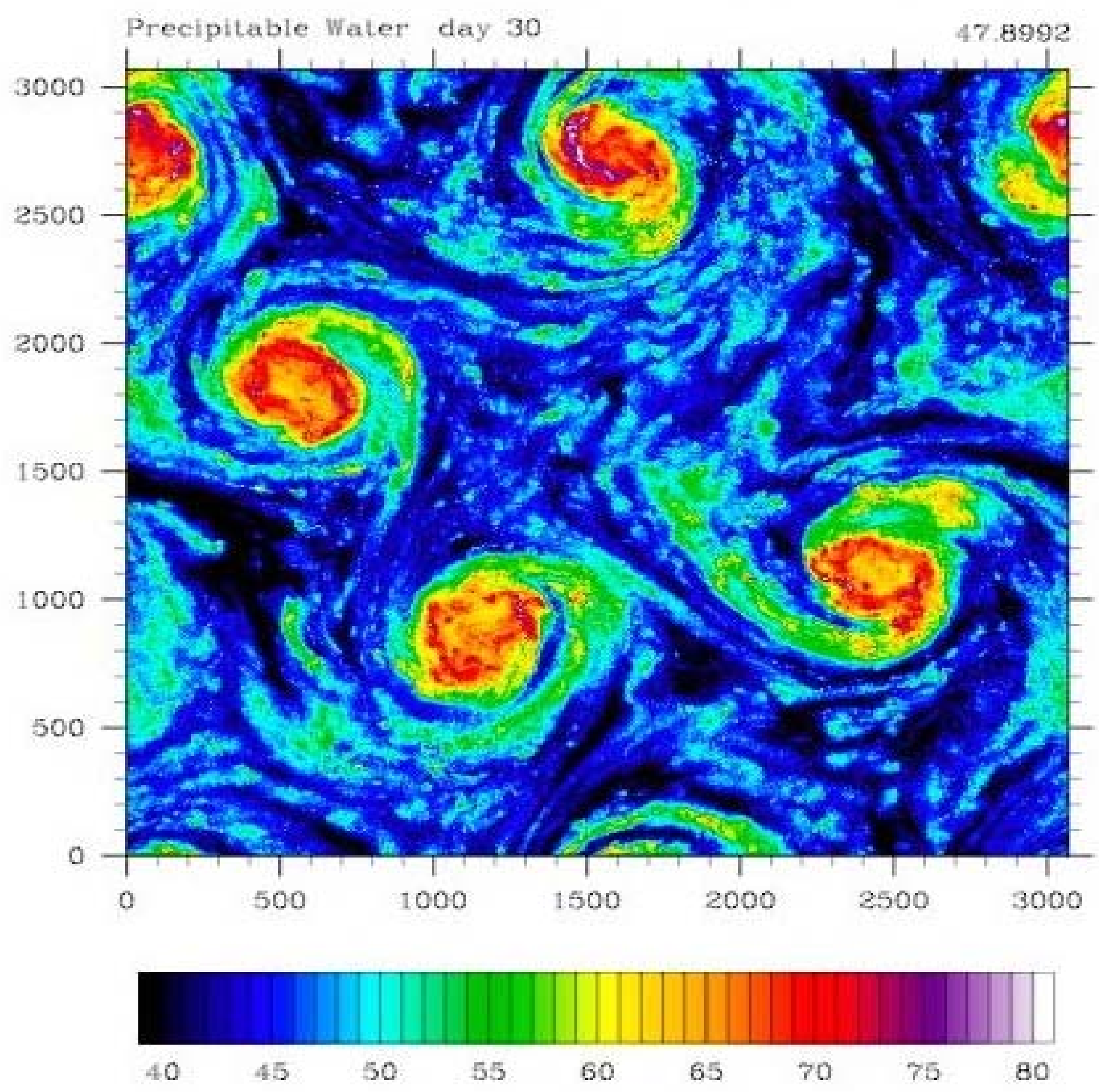
Larger domain needed for high SSTs to aggregate

Allison Wing, PhD thesis, 2014



Perturbation net radiative heating rates in response to an instantaneous reduction of specific humidity of 20% from the RCE states for SSTs ranging from 25 to 45C.

Self-Aggregation on an f-plane

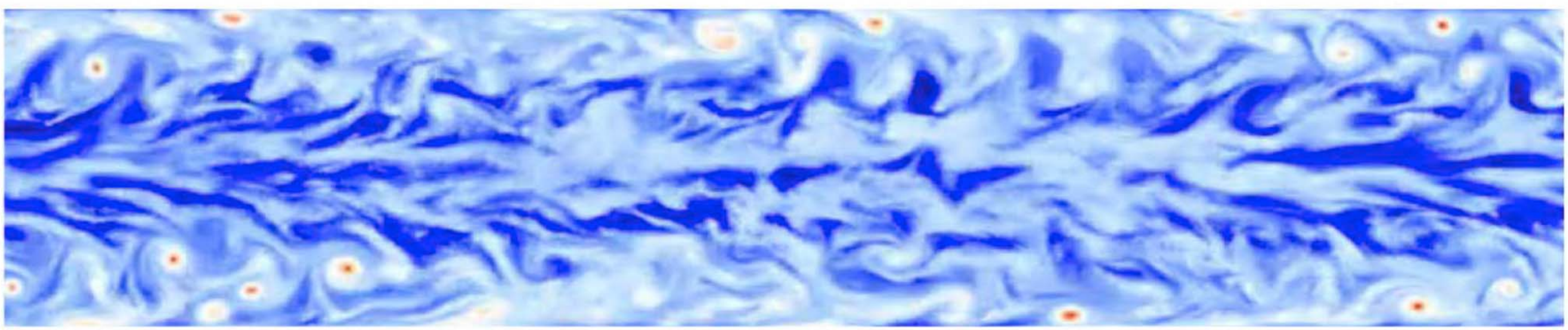


Ongoing work with Marat Khairoutdinov

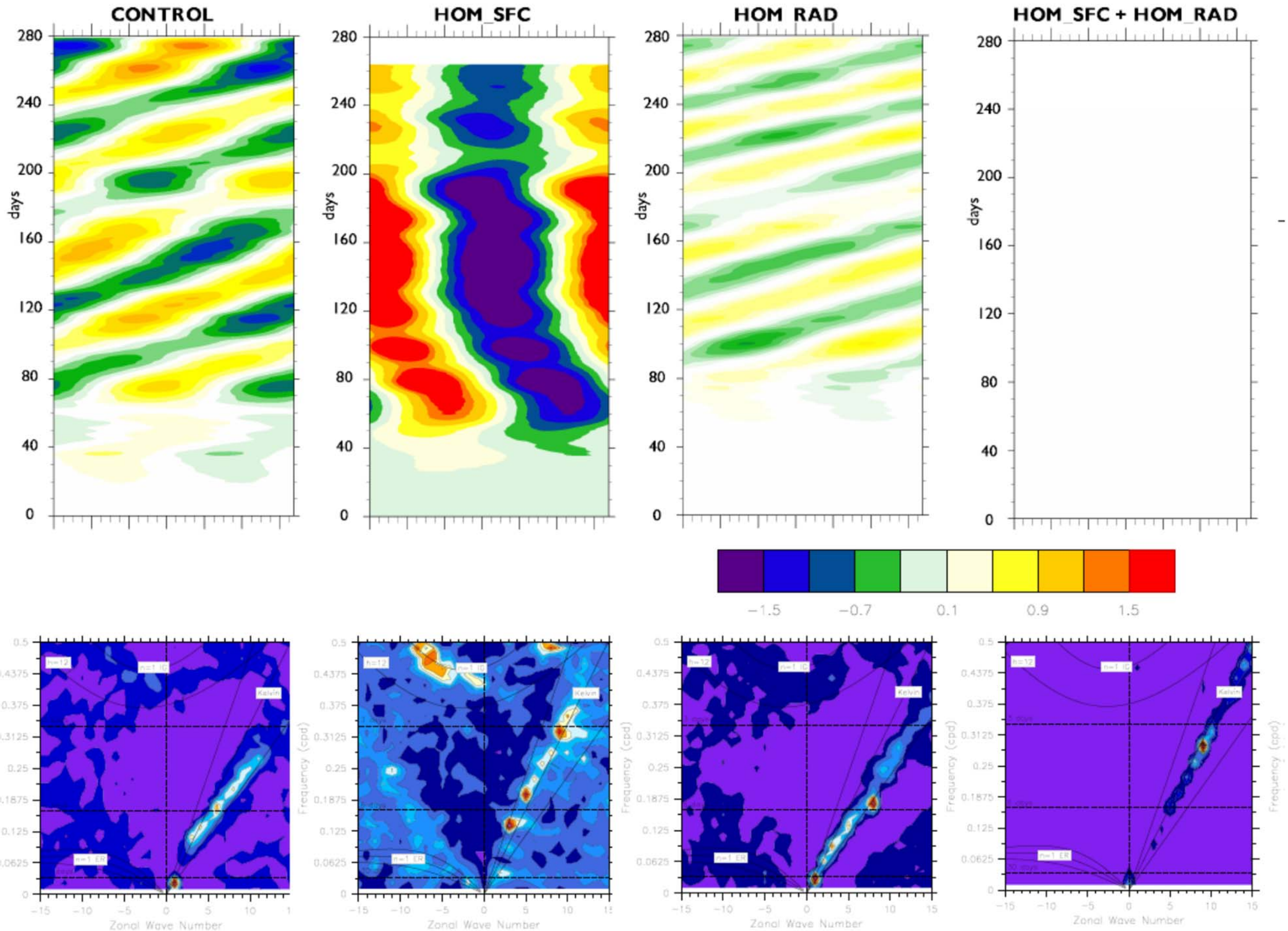
Set-up

- **Model: System for Atmospheric Modeling (SAM; Khairoutdinov and Randall 2003)**
- **Domain: 40,000 km x 10,000 km; top at 30 km**
- **Zonally periodic; solid walls at 46oN/S; equator at the center**
- **Grid spacing 20 km**
- **Aqua-planet with *uniform* SST=300K**
- **Constant insolation: 650 W/m2with constant zenith angle of 52o**
- **Realistic Coriolis parameter**

Precipitable Water



Snapshot of precipitable water in CONTROL simulation



CONTROL
HOM_SFC
HOM_RAD
HOM_SFC + HOM_RAD
days
−1.5
−0.7
0.1
0.9
1.5
Frequency (cpd)
Zonal Wave Number
Kelvin
n=1 ER

Summary

- Atmospheric convection is mostly a quasi-equilibrium phenomenon, though there are important exceptions
- Phase change of water, because it occurs near thermodynamic equilibrium, does not fundamentally change the character of convection
- Irreversible fall-out of precipitation dramatically changes the character of convection

- Moist RCE itself presents ongoing challenges, such as fundamental scale of velocities in clouds
- Moist RCE appears stable to perturbations that do not excite external enthalpy fluxes
- RCE state can be destabilized by interactions with surface enthalpy fluxes and radiation, giving rise to a rich variety of large-scale phenomena