

On Assessing Local Conditional Symmetric Instability from Atmospheric Soundings

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ABSTRACT

The standard parcel method of assessing the susceptibility of the atmosphere to moist convection using tephigrams is extended to account for the centrifugal as well as the gravitational potential energy of the displaced air parcel. This leads to a measure of the stability of the moist baroclinic atmosphere to finite slantwise reversible displacements of a two-dimensional air parcel; such a measure differs from previously derived measures of conditional symmetric instability which have considered only infinitesimal displacements in saturated atmospheres. It is demonstrated that the combined gravitational and centrifugal potential of a two-dimensional air parcel or "tube" can be assessed by displacing the tube slantwise along a surface of constant angular momentum, and that this combined potential energy can be estimated using a single atmospheric sounding. Several examples of the application of this technique are presented in the context of a case study of apparent slantwise convection. The results suggest that moist convection in a conditionally unstable baroclinic atmosphere proceeds in such a way as to render the atmosphere neutral to reversible slantwise displacements; such an atmosphere is characterized by moist adiabatic lapse rates along surfaces of constant angular momentum and may be considerably stable to purely vertical displacements.

The dynamics of slantwise moist convection are examined in a companion paper.

1. Introduction

The earliest detailed observational studies of extratropical cyclones (e.g., Bjerknes, 1919) revealed the presence of quasi-linear bands of clouds and precipitation which later came to be associated with fronts. With the advent of Doppler radar, however, it quickly became apparent that many of the banded structures which occur within extratropical cyclones are not directly or obviously associated with frontal circulations [see the review by Hobbs (1978)]. Some of the bands are convective in nature, while others occur in the absence of convective instability. Elliot and Hovind (1964) note that many precipitation bands are aligned with the mean thermal wind. This observation, together with the mesoscale character of the bands, led Bennetts and Hoskins (1979) to speculate that they are a result of conditional symmetric instability, while Emanuel (1979) demonstrated that circulations resulting from this instability are fundamentally mesoscale in character. Since then, there has been mounting evidence that conditional symmetric instability is responsible for some precipitation bands within large-scale storms (Bennetts and Sharp, 1982).

To date, the assessment of the degree of conditional symmetric instability in the atmosphere has relied upon theory governing infinitesimal displacements in an atmosphere which is everywhere just saturated (Bennetts and Hoskins, 1979; Emanuel, 1982). Such an assessment may be thought of as pertaining to the stability of an extensive layer when lifted to saturation. The

criterion for and nature of this "layer" instability differ, however, from the corresponding facets of "parcel" instability, which is assessed by lifting a highly localized air mass a finite distance through an ambient atmosphere, the latter of which is taken to be unaffected by the displacement. In general, the presence of layer instability is a necessary but insufficient condition for parcel instability. It is the purpose of this study to demonstrate that there exists for moist symmetric instability a natural analogue to the concept of parcel instability as it is applied to the study of moist convection.

It must first be pointed out that inertial, symmetric and convective instabilities are very closely related in a dynamical sense. In the simplest terms, each of these instabilities may be thought of as resulting from an unstable distribution of body forces acting on a fluid. In the case of convection, the responsible body forces are gravitational, while the body forces are centrifugal in the case of inertial instability. The two differ principally in direction, with gravity acting vertically and centrifugal forces acting radially. Symmetric instability simply represents motion resulting from a combination of these forces; thus, the motion is "slantwise." The similarity of these instabilities can also be demonstrated by writing the Lagrangian equations describing two-dimensional motion in a Boussinesq, inviscid, rotating stratified fluid:

$$\frac{dw}{dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} + \frac{g}{\theta_{v0}} (\theta'_v), \quad (1)$$

$$\frac{du}{dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} + fM', \quad (2)$$

where w and u are the vertical and eastward Cartesian velocities, ρ_0 and θ_{v0} are constant reference density and virtual potential temperature, respectively, p is the pressure, θ_v the virtual potential temperature, f the Coriolis parameter and

$$M \equiv v + fx, \quad (3)$$

where v is the meridional velocity. The primes denote differences between the quantity representing a perturbed state and that of a stationary state in geostrophic and hydrostatic balance. The last terms of (1) and (2) represent the deficit gravitational force and the surplus Coriolis force acting on a parcel, respectively. If the system in question is two-dimensional, with no variations in y , then the analogy between (1) and (2) is complete since both θ_v and M are conservative variables, i.e.,

$$\frac{dM}{dt} = \frac{dv}{dt} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} = 0.$$

We hereafter refer to M as the pseudo-angular momentum.

If the equilibrium flow is barotropic, then M is a function of x alone while θ_v varies only with z . If in a particular circumstance the perturbation pressure forces are small compared to buoyancy and Coriolis forces, then local sources of M and θ_v will produce horizontal and vertical plumes, respectively, and (1) and (2) are uncoupled. When the equilibrium flow is baroclinic, on the other hand, M and θ_v will both vary with z and x and a source of M or θ_v will produce a slantwise plume for which both Coriolis and buoyancy forces are active. Aside from the relative strength of Coriolis and buoyancy forces and the orientation of physical boundaries with respect to the force vectors, there can be no dynamical distinction between the action of these two forces on two-dimensional perturbations.

When condensation provides a source of θ_v , it may be that a finite slantwise displacement of a two-dimensional tube will result in instability. It is clear that in a conditionally unstable barotropic atmosphere with $\partial M/\partial x$ positive, the least stable displacement will be in the vertical, since horizontal displacements are stable and uncoupled from those in the vertical. When the atmosphere is baroclinic, however, the least stable displacement will, in general, be slantwise. If condensation is necessary to produce an instability, then the resulting motions will be localized in a slanting updraft and, as shown by Emanuel (1983), the relative strength of the perturbation pressure forces, as measured by the Froude number, will be small. If, in addition, turbulent mixing is neglected, then the forces operating on the displaced tube are conservative and a potential energy

created by the displacement can be defined. This energy represents a combined centrifugal and gravitational potential. It is our purpose here to determine the maximum potential that can be achieved by a reversible slantwise displacement in a moist baroclinic atmosphere, and to demonstrate that this potential may be readily used as a measure of the susceptibility of moist baroclinic flows to conditional symmetric instability. We emphasize that this measure of potential instability, which refers to local displacements of arbitrary magnitude, differs from and is more general than those previously considered by Bennetts and Hoskins (1979) and Emanuel (1982), which pertain to infinitesimal perturbations to a saturated atmosphere. In a companion paper (Emanuel, 1983) we examine the motion of two-dimensional air parcels subject to conditional symmetric instability.

2. The total potential energy generated by reversible slantwise displacements of two-dimensional air parcels

Consider a purely meridional, steady, moist baroclinic flow in thermal wind balance. By definition, the virtual potential temperature θ_v must in this case be a function of x and z alone. Now consider the potential energy generated by a finite, reversible, slantwise displacement of a small "tube" of air which extends indefinitely in the y direction and which is initially at rest at $z = 0$ (Fig. 1). In order to help visualize the stability of such a displacement, we define two surfaces which intersect the air tube and which together completely specify the dynamically relevant properties of the tube. The first is simply a surface along which M , defined by (3), is constant and equal to the M of the tube. The second surface is defined such that the air tube, when lifted reversibly along this surface, has the same virtual potential temperature as its environment. Its x and z coordinates may be obtained from this definition:

$$\theta_v(x, z) = \theta_{vt}(x_0, z_0, z), \quad (4)$$

where $\theta_v(x, z)$ is the distribution of virtual potential temperature of the environment and $\theta_{vt}(x_0, z_0, z)$ is the virtual potential temperature of the air tube located initially at $x = x_0, z = z_0$. When phase changes of water are permitted, θ_{vt} is considered to be a function of the tube's initial position and z alone, neglecting the effect of horizontal variations of pressure on the saturation mixing ratio. A hypothetical configuration of M and S surfaces is shown in Fig. 1. It should be remarked that unlike M surfaces, S surfaces can in general be constructed only with respect to a particular air sample. When the atmosphere is dry, S surfaces are surfaces of constant potential temperature, while when the atmosphere is saturated, they are surfaces of constant virtual equivalent potential temperature. If the parcel is saturated and its environment is not, S

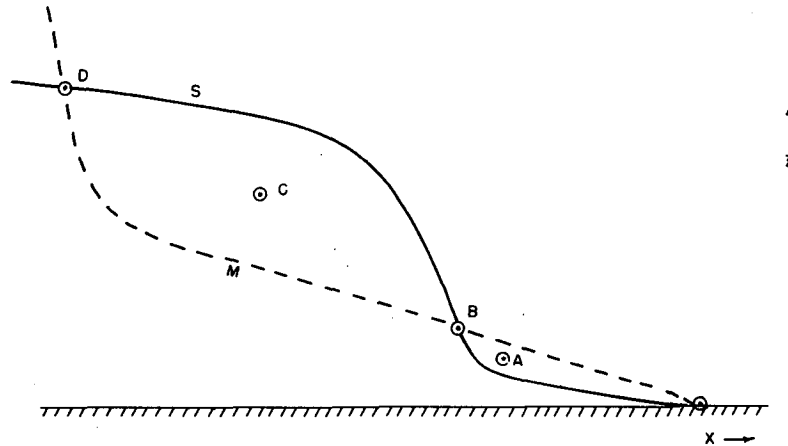


FIG. 1. Hypothetical configuration of *M* (dashed) and *S* (solid) surfaces as viewed in a plane which is orthogonal to the mean vertical shear vector. The *S* surface is defined with respect to the encircled surface tube. The position of this tube and point *D* are stable equilibrium positions for reversible displacements of the tube, while point *B* is an unstable equilibrium position. The tube, when reversibly displaced to *A*, will return to its initial position; if it is displaced to *C* it will accelerate toward *D*.

surfaces are neither surfaces of constant θ nor surfaces of constant θ_{ev} , but continue to represent the locus of points along which the parcel buoyancy vanishes. In this last case, moist convection will be localized in comparatively narrow updrafts and, as shown by Emanuel (1983), the Froude number is small and perturbation pressure forces may be neglected in the first approximation. Then (1) and (2) may be rewritten as

$$\frac{dw}{dt} = \frac{g}{\theta_{v0}} (\theta_{vt} - \theta_{vg}), \tag{5}$$

$$\frac{du}{dt} = f(M_t - M_g), \tag{6}$$

where the subscripts *t* and *g* refer, respectively, to the tube and the geostrophically balanced environment of the tube, the latter of which obeys the thermal wind relation

$$f \frac{\partial M_g}{\partial z} = \frac{g}{\theta_{v0}} \frac{\partial \theta_{vg}}{\partial x}. \tag{7}$$

The expressions (5) and (6) simply show that the *x* and *z* accelerations experienced by the tube will be proportional to its *M* and θ_v surpluses, respectively. We assume that the flow is inertially and statically stable, i.e., that $\partial M/\partial x$ and $\partial \theta_v/\partial z$ are both positive. Referring to these expressions and Fig. 1, it is evident that if the air tube initially at rest at $z = 0$ is displaced to point *A*, its *M* and θ_v will be greater than and less than those of its environment, respectively, and the net acceleration of the tube will act to return it to its initial position. By similar reasoning, a displacement to point *C* will result in an upward and westward unstable acceleration. Points *B* and *D* as well as the initial position are equilibrium positions of the tube; point

D and the initial point are stable equilibria, while point *B* represents an unstable equilibrium and may be regarded as the “level of free slantwise convection” (LFS).

The potential energy of the reversibly displaced tube may be found from (5) and (6):

$$\begin{aligned} PE &= \int_1^2 \mathbf{F} \cdot d\mathbf{l} \\ &= \int_1^2 [f(M_t - M_g)\mathbf{i} + \frac{g}{\theta_{v0}} (\theta_{vt} - \theta_{vg})\mathbf{k}] \cdot d\mathbf{l}, \end{aligned} \tag{8}$$

where *F* is the vector force per unit mass, *i* and *k* are unit vectors in the *x* and *z* directions, respectively, and 1 and 2 represent the initial and final positions of the air tube.

With the use of the thermal wind relation (7), and since *M_t* is a constant and θ_{vt} is a function of *z* alone, it may easily be verified that

$$\nabla \times \mathbf{F} = 0,$$

so that the forces acting on the tube are conservative and the potential energy calculated from (8) does not depend on the path of integration, but only on the end points. From Fig. 1, it is obvious that the net potential energy available to the air tube up to point *D* may be found by displacing the tube from its initial position to the final stable equilibrium point *D*, which points may be taken to represent the end points in (8). If, as is often the case, we wish to evaluate the accumulated potential energy at intermediate levels, then it is not immediately clear what one should take as the intermediate end point in evaluating (8). It is easily demonstrated, however, that for fixed vertical displacement, a maximum potential energy can be found. We first write (8) in the differential form

$$\delta PE = f(M_t - M_g)dx + \frac{g}{\theta_{v0}}(\theta_{vt} - \theta_{vg})dz,$$

where δPE is the incremental potential energy gained from a small displacement in x and z . The above may alternatively be written

$$\delta PE = [f(M_t - M_g)x' + \frac{g}{\theta_{v0}}(\theta_{vt} - \theta_{vg})]dz, \quad (9)$$

where x' ($\equiv dx/dz$) is the inverse slope of the displacement. For a given increment in z , then, the change in potential energy may be regarded as a function of x , z and x' . At a given altitude, maxima of the potential energy with respect to x and x' are given by the equations

$$\begin{aligned} \frac{\partial PE}{\partial x} &= -\left[fx' \frac{\partial M_g}{\partial x} + \frac{g}{\theta_{v0}} \frac{\partial \theta_{vg}}{\partial x}\right] \\ &= -f\left[x' \frac{\partial M_g}{\partial x} + \frac{\partial M_g}{\partial z}\right] = 0, \quad (10) \end{aligned}$$

$$\frac{\partial PE}{\partial x'} = f(M_t - M_g) = 0, \quad (11)$$

where we have made use of the thermal wind relation (7) in (10). From (10) and (11) it may be seen that for a given displacement in z , the change in potential energy will be maximized by lifting the tube directly along an M surface.¹ Thus the form of (8) which, for a given vertical displacement, maximizes the potential energy [which we call the Slantwise Convective Available Potential Energy (SCAPE)] is

$$SCAPE(z) = \int_{M_1}^z \frac{g}{\theta_{v0}}(\theta_{vt} - \theta_{vg})dz, \quad (12)$$

where the subscript M denotes that the tube is displaced along an M surface. The total potential of the tube can be assessed by taking z to be the equilibrium point (D in Fig. 1). The important result of this analysis, then, is that *the susceptibility of moist baroclinic flows to slantwise convection may be assessed by reversibly lifting a (two-dimensional) parcel along a surface of constant M and comparing its density, as represented by θ_v , to that of its environment*. It should be noted that, provided the displaced parcel is two-dimensional and that the displacement is reversible, the above is a completely general statement about the stability of the flow to local displacements. If the flow is barotropic, then M surfaces are vertical and (12) represents the standard analysis of positive area on a tephigram. If the flow is baroclinic, then (12) implicitly includes the centrifugal contribution to the total potential energy. Also note that although the potential energy is max-

imized for displacements along M surfaces, the free motion of unstable air tubes, as shown by Emanuel (1983), will be more nearly along S surfaces provided that the atmosphere is stable to purely vertical displacements.

3. The assessment of total potential energy from atmospheric soundings

The preceding discussion suggests several simple methods of assessing the stability to two-dimensional displacements of an atmosphere with uni-directional shear. A very direct method which would, however, be useful only in analyzing experimental data, is to measure temperature and moisture along an M surface using instrumented aircraft. The aircraft would follow a gently ascending (or descending) path transverse to the shear in such a way that the quantity $v + fx$ is approximately constant, where v is the component of flow in the direction of the shear and x is the distance orthogonal to the shear vector. Small departures from the M surface can be easily corrected for in analyzing the data, using methods to be discussed presently. The temperature and moisture sounding along the M surface is then plotted on a tephigram and analyzed in the usual manner, with positive area indicating parcel instability. It should be noted that under conditions typically associated with moist symmetric instability, the ascent (or descent) rate of the aircraft necessary to maintain constant M is

$$w = v_a \bar{\eta} \bar{v}_z^{-1},$$

where v_a is the airspeed, $\bar{\eta}$ the vertical component of absolute vorticity, and \bar{v}_z the vertical shear. Typically, $\bar{\eta} \sim 10^{-4} \text{ s}^{-1}$ and $\bar{v}_z \sim 5 \times 10^{-3} \text{ s}^{-1}$. An aircraft flying at 130 m s^{-1} would need to ascend or descend at about 2.5 m s^{-1} in order to maintain constant M .

Standard rawinsoundings may also be used to estimate the total potential energy available for slantwise convection. It should first be pointed out that the typical slope of an M surface is on the order of 100 km in the horizontal for every 10 km in the vertical; this is comparable to the typical slope of a trajectory followed by a weather balloon. Since M surfaces are material surfaces for two-dimensional flow, however, the component of flow which advects M also transports the balloon so that the sounding will not generally pass through M surfaces due to its horizontal motion. Large horizontal displacements may be expected in planes which are tangent to M surfaces, but these will not affect the measurement of quantities which do not vary in the long-shear direction. It would appear then that, to a first approximation, horizontal drift of rawinsoundings need not be accounted for in assessing conditional symmetric instability.

The most straightforward method of assessing moist symmetric instability from soundings involves constructing vertical cross sections from two or more

¹ The potential energy thus derived will be defined only when $\partial M/\partial x > 0$, which is just the condition for the absolute inertial stability of purely horizontal displacements.

soundings which are aligned across the geostrophic shear. Provided the horizontal distance between soundings is known, M may easily be estimated from the component of geostrophic flow normal to the cross section and surfaces of constant M may then be constructed. After plotting the distributions of potential temperature and mixing ratio, it is a straightforward matter to construct temperature and moisture soundings along M surfaces. When these soundings are then plotted on tephigrams, stability may be assessed in the usual way. Each of the original soundings should be examined first to determine the presence or absence of classical conditional instability which, if it is present, should override any consideration of the potential for slantwise convection. Examples of this kind of analysis will be presented in the next section.

It is also possible to use single soundings to estimate the stability of the atmosphere to slantwise displacements of those parcels which are sampled by the sounding. In order to make such an estimate, however, some information about the vertical component of shear vorticity must be obtained from constant pressure analyses or forecasts.

The first step in this analysis is to determine the direction of the geostrophic vertical shear vector in the layer in which potential instability is suspected. This may be accomplished by noting the direction of the horizontal temperature gradient on constant pressure analyses. Define x as the direction normal to the geostrophic vertical shear vector, and v as the scalar horizontal wind component in the direction of the geostrophic shear. Also define $\eta(z)$ as $f + \partial v_g / \partial x$ and estimate this quantity from constant pressure analyses or forecasts.

The object is to estimate the distribution of θ_v along M surfaces which intersect the vertical path of the sounding. Consider a hypothetical sounding located at $x = 0$ and an M surface which intersects the surface parcel, as represented in Fig. 2. Suppose the surface parcel is reversibly lifted to point 1, while its M is conserved. Let ΔM be the difference between the M of the parcel and that of its environment (since the parcel has been lifted vertically, $\Delta M = \Delta v$). Now provided that $\eta(z)$ is not a function of x in the region between the lifted parcel and the M surface, we can estimate the horizontal distance L between the parcel and its "home" M surface as

$$L = -\frac{\Delta M}{\partial M / \partial x} = -\frac{\Delta M}{\eta(z)}. \quad (13)$$

If, in addition, the thermal wind is not a function of x over the interval L , then the virtual potential temperature along the M surface can be estimated from the thermal wind equation

$$\theta_{vM} = \theta_v + L \frac{\partial \theta_v}{\partial x} = \theta_v + L \frac{\theta_{v0}}{g} f \bar{v}_z, \quad (14)$$

where θ_{vM} is the virtual potential temperature along the M surface, θ_v is the ambient temperature at $x = 0$, and \bar{v}_z is the local geostrophic vertical shear of v . Using (13) and (14), then, the total slantwise convective available potential energy from (12) is

$$\text{SCAPE} = \int^1 \left(\frac{g}{\theta_{v0}} \Delta \theta_v + \frac{f}{\eta(z)} \bar{v}_z \Delta M \right) dz, \quad (15)$$

where $\Delta \theta_v$ is just the difference between the virtual potential temperature of a reversibly lifted parcel and that of its environment. Since ΔM is just the difference between the velocity v of the parcel and that of its environment, Eq. (15) may be rewritten as

$$\text{SCAPE} = \int^1 \left\{ \frac{g}{\theta_{v0}} \Delta \theta_v + \frac{1}{2} \frac{f}{\eta(z)} \frac{d}{dz} [(v - v_0)^2] \right\} dz, \quad (16)$$

where v_0 is the parcel's v .

The first term in (16) is the standard positive area on a tephigram and represents the gravitational contribution to the potential energy, while the second term is the contribution from centrifugal forces. A straightforward way of evaluating stability, then, is simply to add the quantity

$$\frac{1}{2} \frac{T_{v0}}{g} \frac{f}{\eta(z)} \frac{d}{dz} [(v - v_0)^2] \quad (17)$$

to the reversibly lifted parcel temperature (T_{v0}) at each level and evaluate the sounding in the usual way. Examples of this simple calculation will be given in the next section.

It is worth noting that if $\eta(z)$ is constant in (16), the second term can be integrated exactly, resulting in

$$\text{SCAPE} = \frac{1}{2} \frac{f}{\eta} (v_1 - v_0)^2 + \int_0^1 \frac{g}{\theta_{v0}} \Delta \theta_v dz, \quad (18)$$

where 1 and 0 represent the top and bottom of the layer, respectively. The first term of (18) is positive

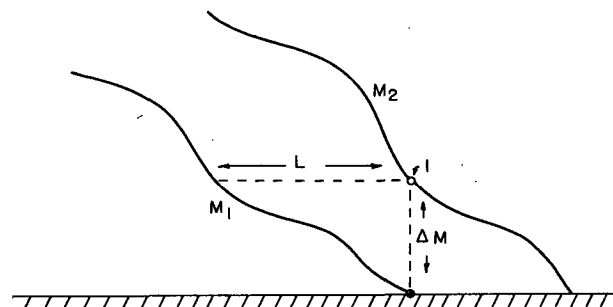


FIG. 2. Computation of the distribution of θ_v along an M surface using a single sounding. The horizontal distance L from the sounding to the M surface of the parcel is proportional to the difference between the parcel and environment M , and inversely proportional to the absolute vorticity η .

definite and may be thought of as the "available kinetic energy." For large but observable shears, this quantity can make a substantial contribution to the total potential energy. For example, when $\eta = f$, a shear of about 14 m s^{-1} over 1 km would be sufficient to compensate for the negative potential energy contained in a "cap" defined by a 3°C lifted parcel temperature deficit over the same distance.

Finally, we reiterate that the estimate of stability made using a single sounding by adding (17) to the reversibly lifted parcel temperature will be valid provided that the flow in the direction of the shear is geostrophic and that the vertical and horizontal gradients of M are constant over the horizontal interval L between the sounding location and the M surface. This horizontal distance will typically be less than several hundred kilometers.

4. A case study of an apparent example of moist symmetric instability

a. Cross-sectional analyses

On 2–3 December 1982, a diffuse cold front moved slowly eastward across portions of the southern and central plains (Fig. 3). Above and to the west of the frontal zone the flow was from the south to southwest at virtually all levels in the troposphere, as a high-amplitude trough of considerable meridional extent advanced slowly eastward (Fig. 4). The frontal zone was characterized by strong vertical wind shear from the south–southwest as indicated by the large gradient of 1000–500 mb thickness shown in Fig. 3.

To the east of the slowly advancing trough and frontal system, several convective squall lines developed in the warm humid air and moved eastward during the 24 h period beginning at 2000 GMT 2 December 1982, as illustrated by the sequence of GOES imagery presented in Fig. 5. To the west of the squall lines, banded high clouds were visible during the first part of the period. Later on, a large shield of stratiform clouds extended westward from the convective towers which comprised the main squall line and banded structures were visible in Oklahoma and northern and western Texas.

In order to assess the susceptibility of the atmosphere to moist slantwise convection, two cross sections in planes nearly orthogonal to the mean shear were constructed. The first of these (Fig. 6a) is constructed from rawinsoundings taken at 0000 GMT 3 December, while the second (Fig. 6b) uses observations made at 1200 GMT 3 December. Both depict the distributions of equivalent potential temperature $[\text{K}]$ (dashed lines) and pseudo-angular momentum M $[(\text{m s}^{-1})]$ (solid lines). The latter quantity was calculated from the horizontal distance from Amarillo in the plane of the cross section and the component of wind from 200°T (0000 GMT) and 190° (1200 GMT), assuming that the measured wind is geostrophic.

The westward deflection of the M surfaces in the upper troposphere reveals the presence of a strong jet flow in the western portion of the domain, near 300 mb, while contours of θ_e show upward and westward perturbations roughly in the direction along M surfaces at 0000 GMT. Twelve hours later, the distributions of M and θ_e are very similar in the lower troposphere. The atmosphere east of Little Rock is nearly barotropic in both cross sections.

Figure 7 shows the thermodynamic properties of the rawinsoundings taken at Little Rock and Oklahoma City at 0000 GMT. The Little Rock sounding (Fig. 7a) is saturated between 670 and 740 mb, and also near 400 mb; the satellite infrared photograph taken 4 h earlier (Fig. 5) suggests that the sounding rose through high-level outflow from active convective towers to the west. The lifted surface parcel indicates moderate conditional instability, amounting to about 4°C lifted parcel temperature surplus at 400 mb.

The 0000 GMT Oklahoma City sounding (Fig. 7b) exhibits a small saturated region near 750 mb and a nearly moist adiabatic temperature lapse rate above 680 mb. No conditional instability is indicated.

Inspection of Fig. 6a reveals that in the baroclinic zone centered near Oklahoma City, a parcel lifted along an M surface will experience a considerably smaller environmental potential temperature than will the same parcel lifted vertically. In this region, then, the centripetal contribution to the total available potential energy is substantial. This is nicely illustrated in Fig. 8a which shows a thermodynamic sounding along the $M = 50$ surface at 0000 GMT, constructed by subjectively interpolating M , dew point and temperature from cross sections similar to those shown in Fig. 6. The sounding is almost perfectly moist adiabatic above 750 mb, and exhibits neutral buoyancy for parcels lifted from near the surface or from the saturated region near 750 mb. According to the criterion developed in Section 2, then, these parcels are neutral to moist slantwise displacements. This is in sharp contrast to the vertical sounding at Oklahoma City, which is clearly stable.

Figure 8b shows the sounding along the $M = 70$ surface which intersects the surface near Little Rock (Fig. 5). A parcel from anywhere between the surface and 850 mb, when lifted reversibly along the M surface, develops almost twice as much temperature surplus (8°C) at 400 mb as the surface parcel lifted vertically (Fig. 7a). Although in this case the atmosphere is conditionally unstable to a reversible vertical displacement, the additional contribution to the available potential energy from centrifugal accelerations is not insubstantial.

Twelve hours later, the rawinsounding at Little Rock (Fig. 9a) shows a saturated and nearly neutrally stable atmosphere up to 400 mb, while the Oklahoma City sounding (Fig. 9b) exhibits strong stability between the surface and 500 mb and nearly neutral stability between

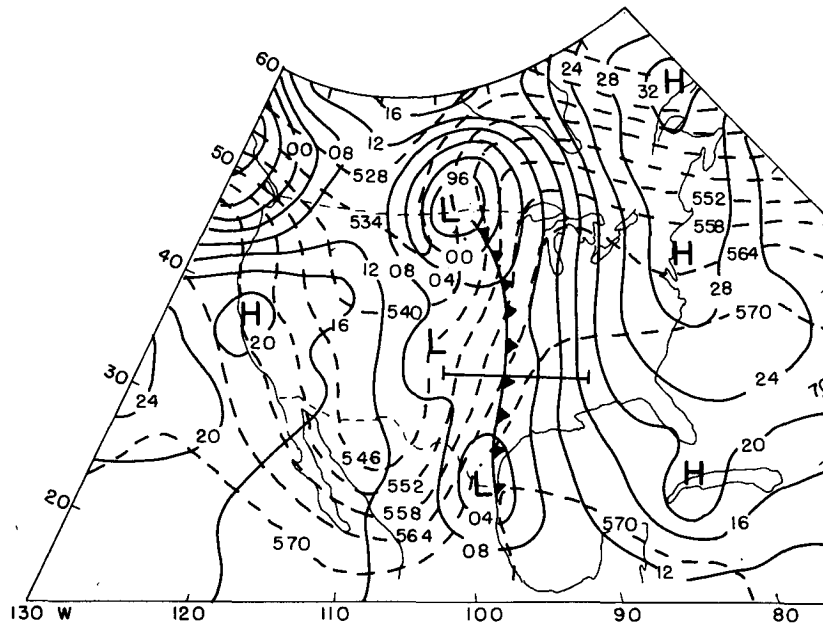


FIG. 3. (a) Surface pressure (solid lines, in millibars with hundreds and thousands digits missing) and 1000–500 mb thickness (dashed lines, in decameters) for North America at 0000 GMT 3 December 1982. Major frontal positions and pressure centers are indicated. Solid bar denotes location of cross sections shown in Fig. 6.

500 mb and 400 mb. Saturation is also evident in the latter sounding between 500 and 660 mb, while the air below 800 mb is considerably subsaturated.

Soundings constructed along three M surfaces at 1200 GMT are shown in Fig. 10. The temperature soundings in all cases are remarkably close to being moist adiabatic through their entire depth (up to 400 mb where the analysis was discontinued). Parcels lifted

reversibly from nearly any level (except near the surface in the $M = 30$ and $M = 40$ soundings) are almost neutrally buoyant.

It would appear in this case that the conditionally unstable moist baroclinic flow underwent a form of slantwise moist adiabatic adjustment which rendered the atmosphere neutral to slantwise reversible displacements and which used up the total slantwise con-

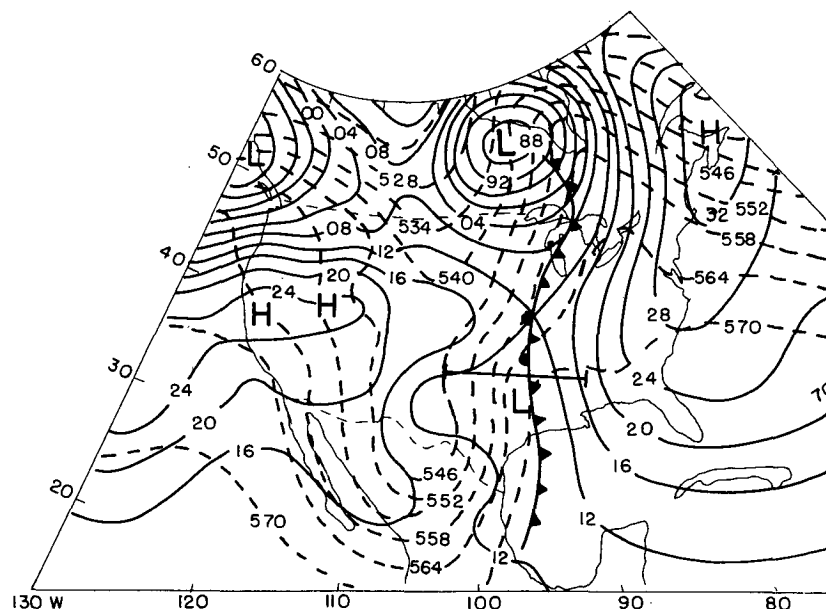


FIG. 3. (b) As in Fig. 3(a) but for 1200 GMT 3 December 1982.

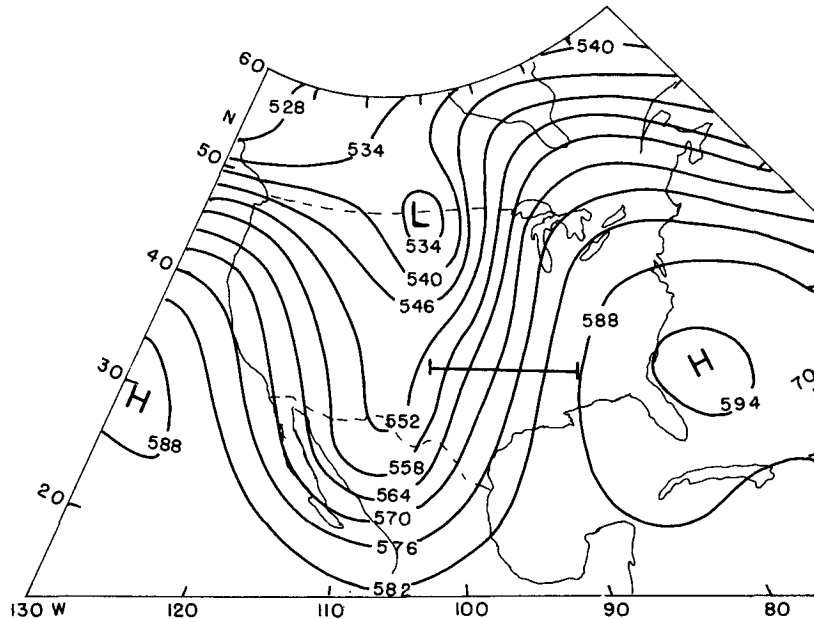


FIG. 4. (a) 500 mb geopotential height (decameters) over North America at 0000 GMT on 3 December 1982.

vective available potential energy, as given by (12), rather than just the gravitational part of the potential energy, which has been called the Convective Available Potential Energy (CAPE). After such an adjustment, the baroclinic atmosphere will have considerable stability to *vertical* reversible displacements. Thus the adjustment will render the SCAPE of the atmosphere very close to zero, while the CAPE actually becomes negative.

Aside from obvious geometrical differences, there is an important respect in which slantwise moist adiabatic adjustment differs from purely vertical adjustment: while the latter involves principally thermodynamic alterations to the atmosphere, the former may be accomplished equally well by irreversible adjustments of the angular momentum distribution. Clearly, further investigation is needed to determine the basic nature of the slantwise adjustment process.

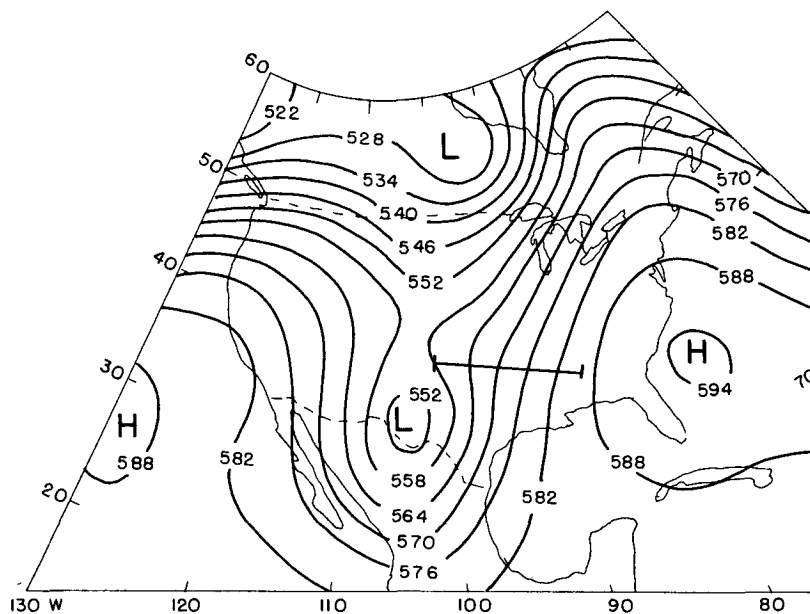


FIG. 4. (b) As in Fig. 4(a) but for 1200 GMT 3 December 1982.

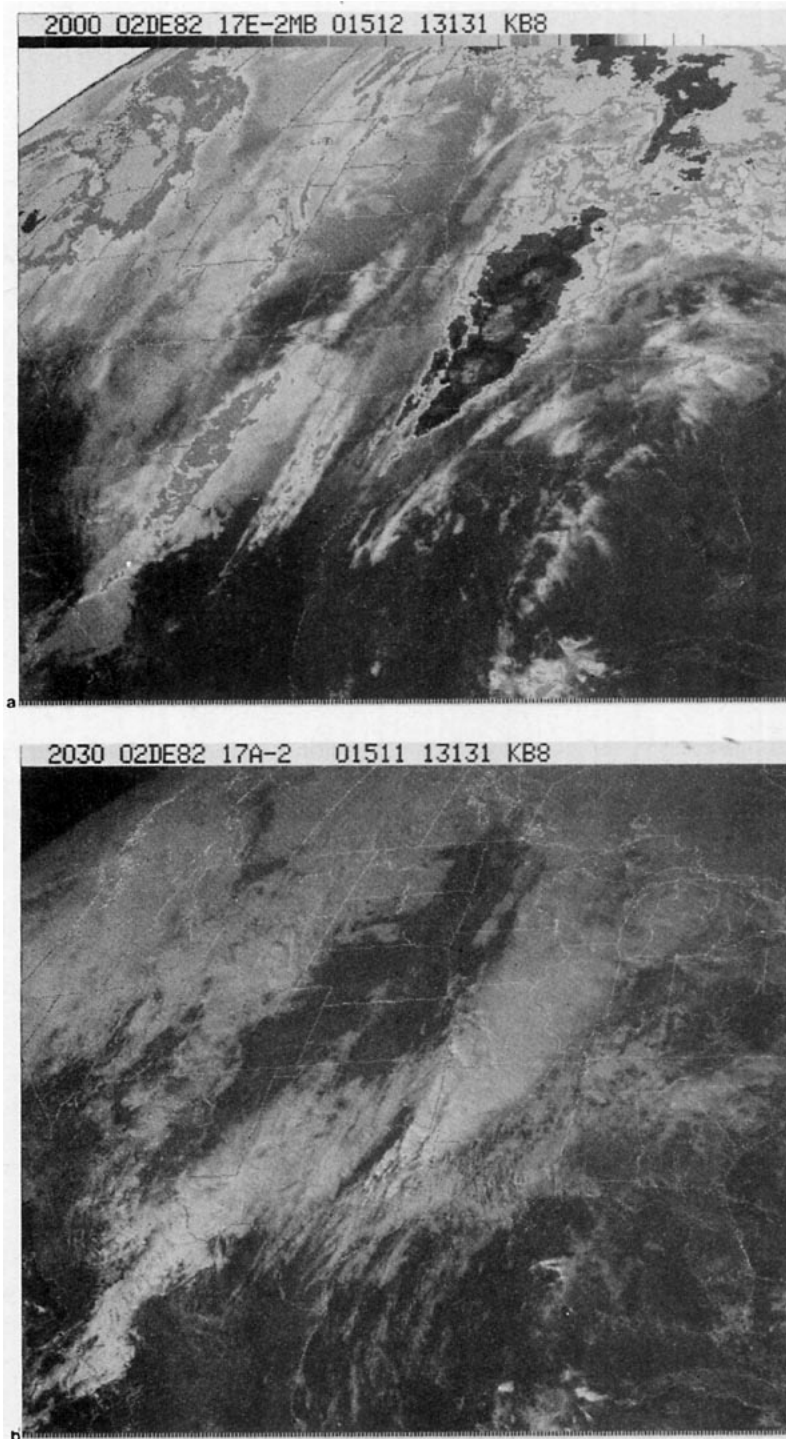


FIG. 5. Sequence of imagery over the United States from the Geostationary Operational Environmental Satellite (GOES): (a) infrared at 2000 GMT 2 December 1982; (b) visible at 2030 GMT, 2 December; (c) infrared at 1500 GMT, 3 December; (d) visible at 1530 GMT 3 December.

b. Single sounding analysis

As indicated in Section 3, it is possible to estimate the stability of the atmosphere to slantwise reversible

displacements from a single sounding provided that some information about the vertical component of vorticity is available. In order to demonstrate the application of this technique, we perform a single sound-

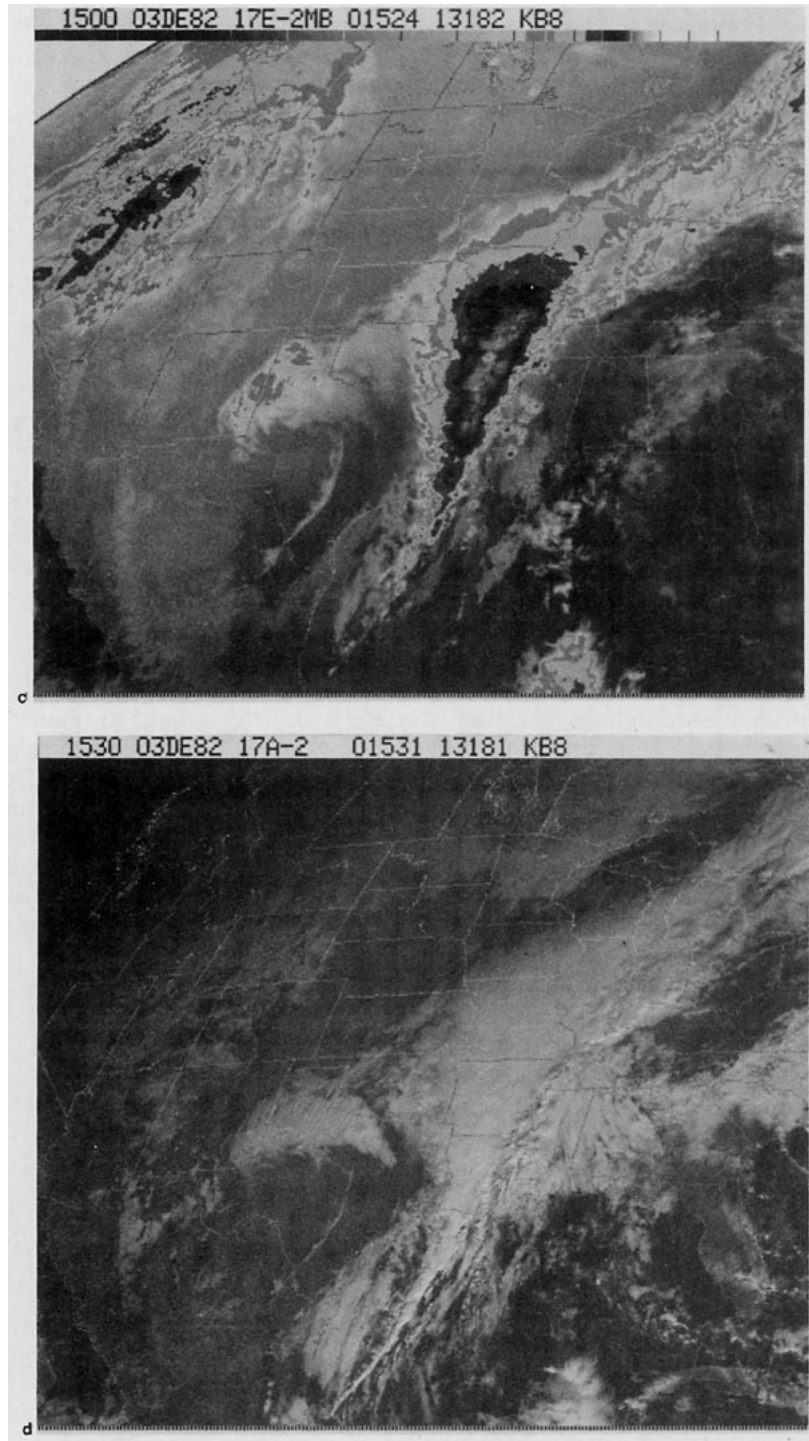


FIG. 5. (Continued)

ing analysis using the Oklahoma City rawinsounding at 1200 GMT 3 December 1981. The vertical component of absolute vorticity is estimated from the horizontal gradients of M in Fig. 6 and is displayed in Fig. 11. Using these vorticities and the component of wind from 190° derived from the sounding, the quan-

tity expressed by (17) is added at each level to the temperature of a parcel lifted reversibly from 690 mb (which is between the $M = 30$ and the $M = 40$ surfaces at Oklahoma City). The result is denoted by the small circles in Fig. 9b, which show that the modified parcel temperature is close to the environmental temperature

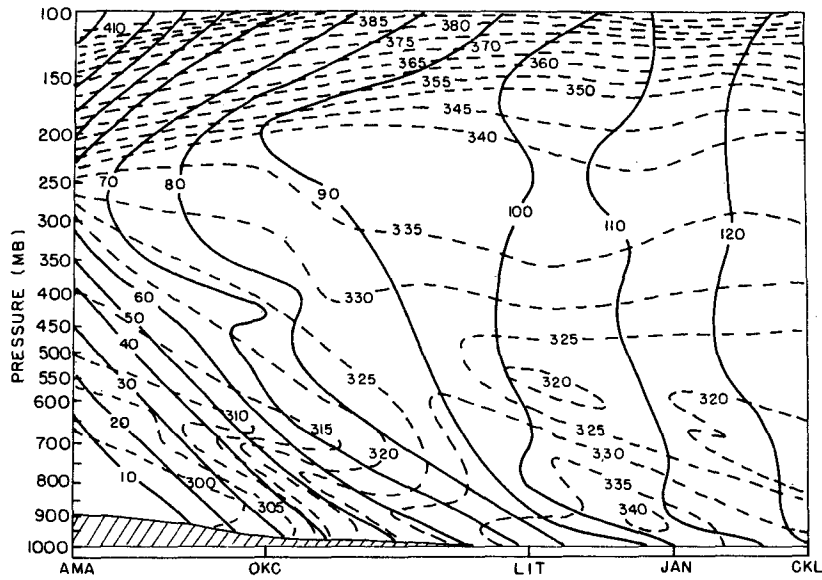


FIG. 6. (a) Cross section from Amarillo, Texas to Centreville, Alabama at 0000 GMT 3 December 1982. Solid lines denote M ($m s^{-1}$), dashed are θ_e (K).

through most of the troposphere above 690 mb. Thus the neutral stability evident in soundings along the $M = 30$ and $M = 40$ surfaces made from the 1200 GMT cross section is fairly well reproduced in the single sounding analysis.

5. Discussion

Since the case examined here was unusually two-dimensional and since the direction of the vertical shear

was nearly independent of height above a shallow surface layer, it is natural to inquire about the stability of flows which do not have these properties. The two-dimensional assumption is applied by supposing that the slopes of virtual potential temperature and pseudo-angular momentum surfaces associated with the large-scale flow do not vary substantially in the long-shear direction. A loose criterion for this condition is that the flow does not vary substantially over a down-shear

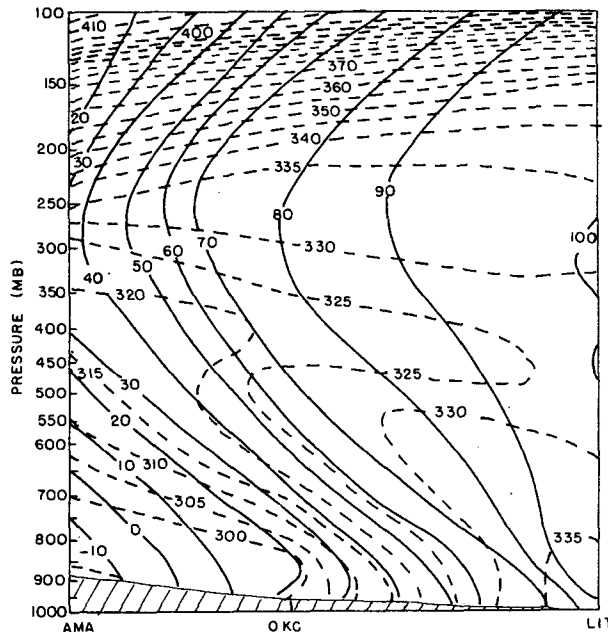


FIG. 6. (b) As in Fig. 6(a) but from Amarillo to Little Rock, Arkansas at 1200 GMT 3 December 1982.

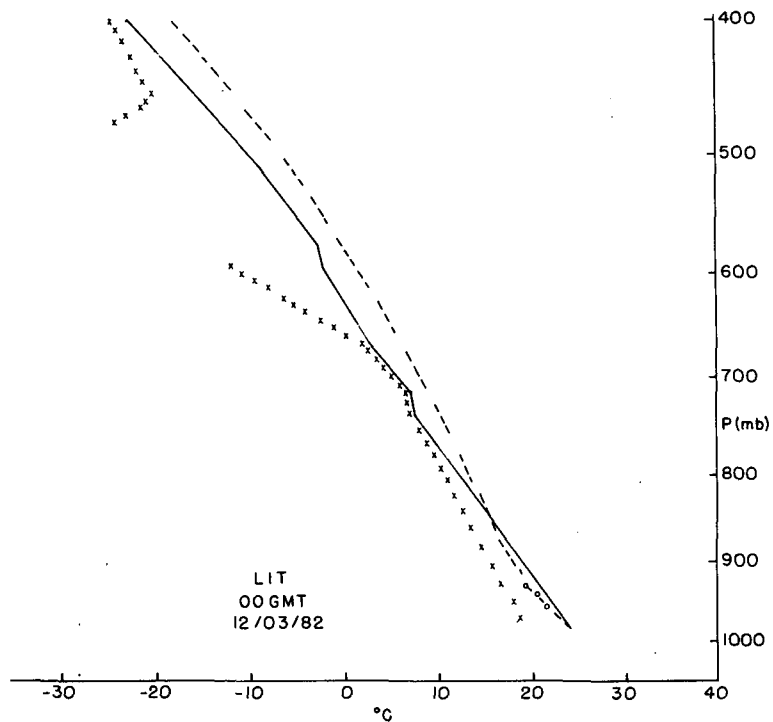


FIG. 7. (a) Little Rock, Arkansas sounding at 0000 GMT 3 December 1982. Solid line shows temperature while crosses depict dew-point temperature. Dashed lines are pseudo-moist adiabats and dashed lines with circles show dry adiabats.

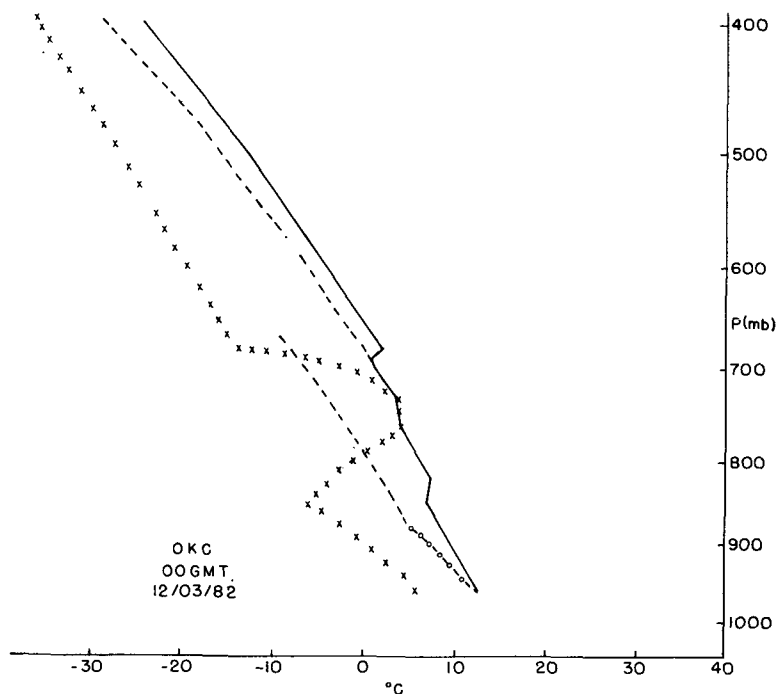


FIG. 7. (b) As in Fig. 7(a) but for Oklahoma City at 0000 GMT 3 December 1982.

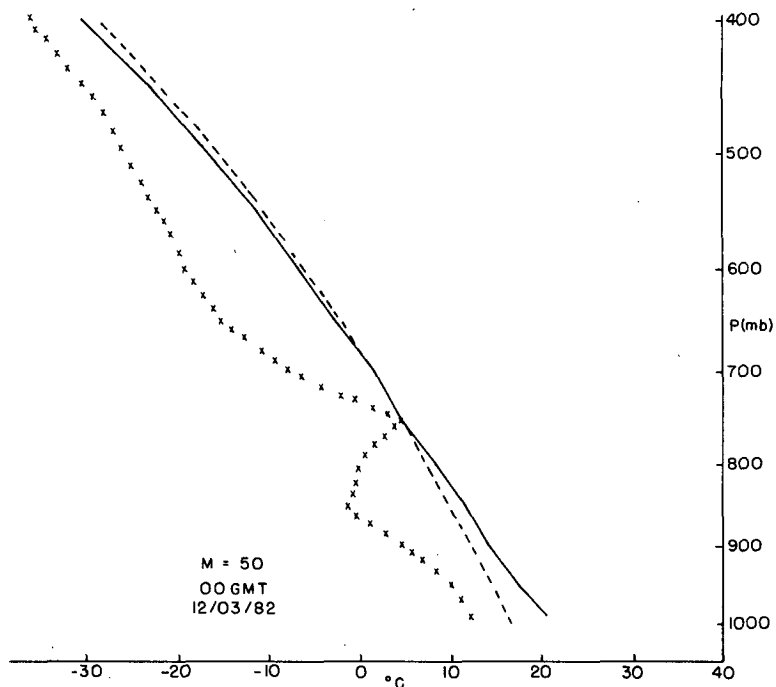


FIG. 8. (a) As in Fig. 7(a) but for sounding along $M = 50$ surface [Fig. 6(a)].

distance much larger than the typical width of the disturbance. As shown by Emanuel (1983), this length will be of the order u_0/f , where u_0 is a typical velocity of the high-level jet. Thus the length scale over which

the large-scale flow varies must satisfy

$$L \gg \frac{u_0}{f},$$

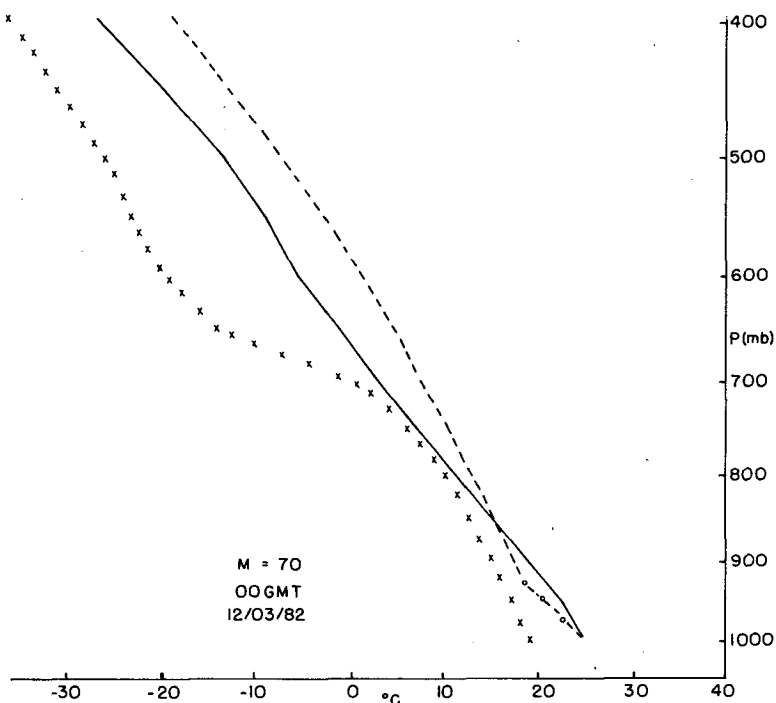


FIG. 8. (b) As in Fig. 8(a) but for $M = 70$ surface.

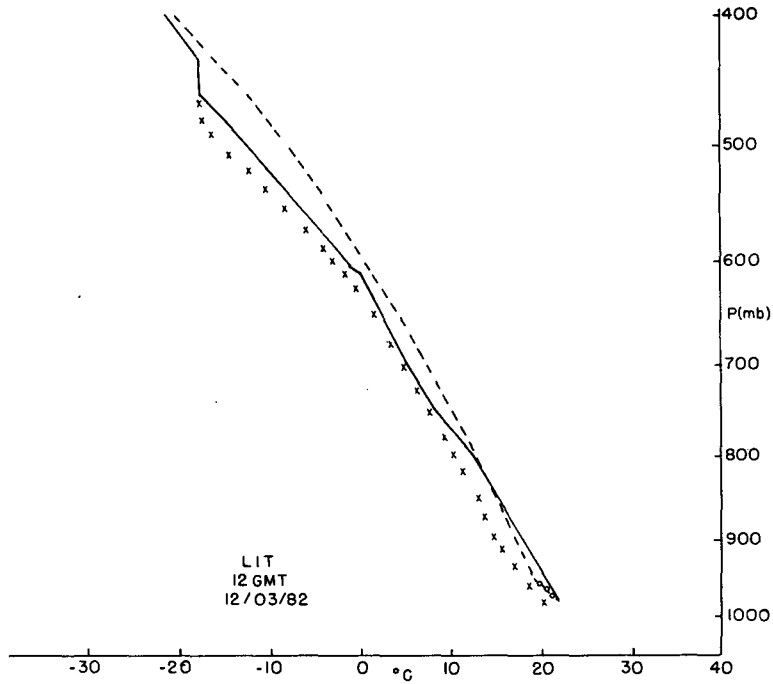


FIG. 9. (a) Little Rock, Arkansas sounding at 1200 GMT 3 December 1982. Notation as in Fig. 7.

which may be simply restated as

$$Ro \ll 1,$$

where Ro is the Rossby number, u_0/fL . Since the

Rossby number typical of baroclinic waves is small, this condition will frequently be satisfied and thus slantwise moist convection may be regarded as *local* in the context of the synoptic-scale flow.

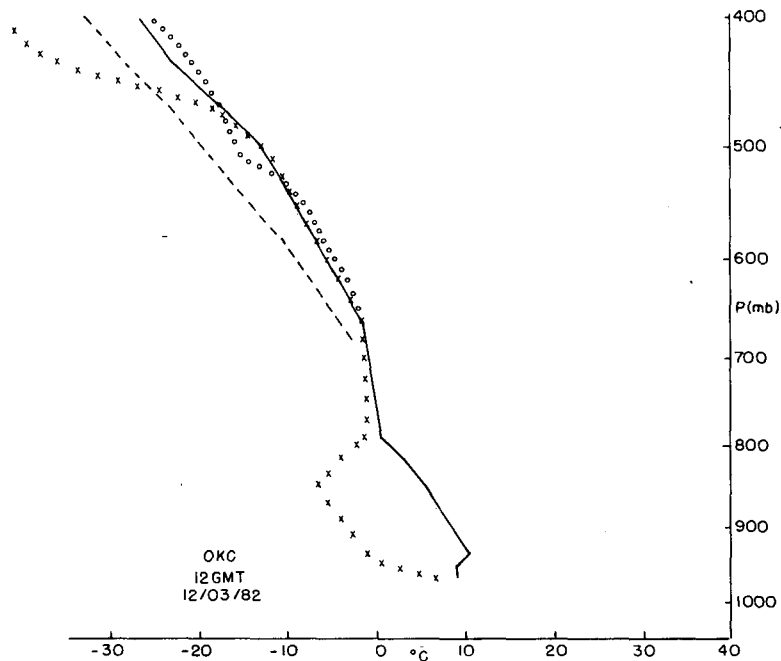


FIG. 9. (b) As in Fig. 9(a) but for Oklahoma City at 1200 GMT 3 December 1982. Circles denote modified lifted parcel temperature for parcel at 690 mb (see text).

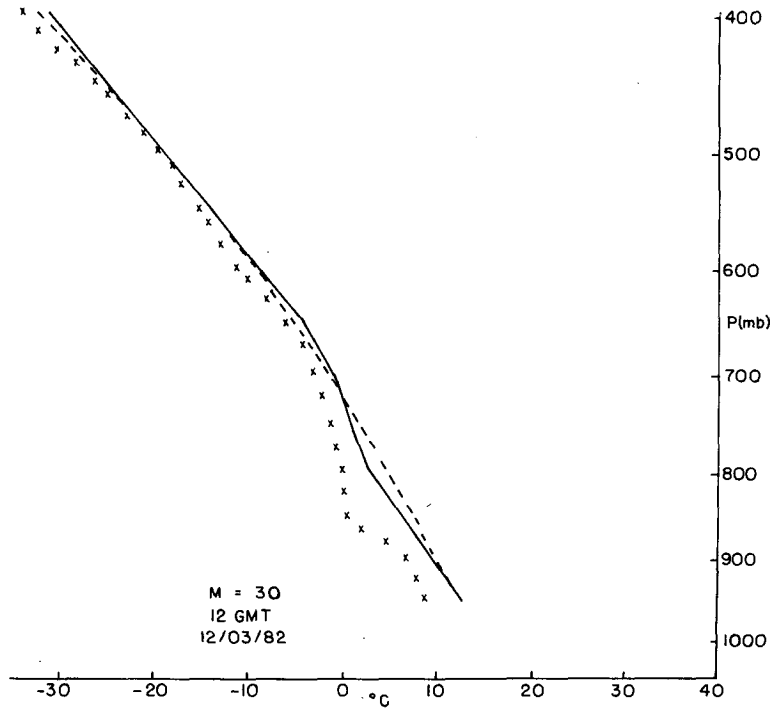


FIG. 10. (a) Sounding along $M = 30$ surface at 1200 GMT 3 December 1982 [from Fig. 6(b)]. Notation as in Fig. 7.

The question of the constancy of the direction of the vertical shear (or horizontal temperature gradient) with height is a somewhat more subtle one, and de-

serves more attention than can be devoted to it here. The present analysis, as well as the one presented in Emanuel (1983), must be regarded for the time being

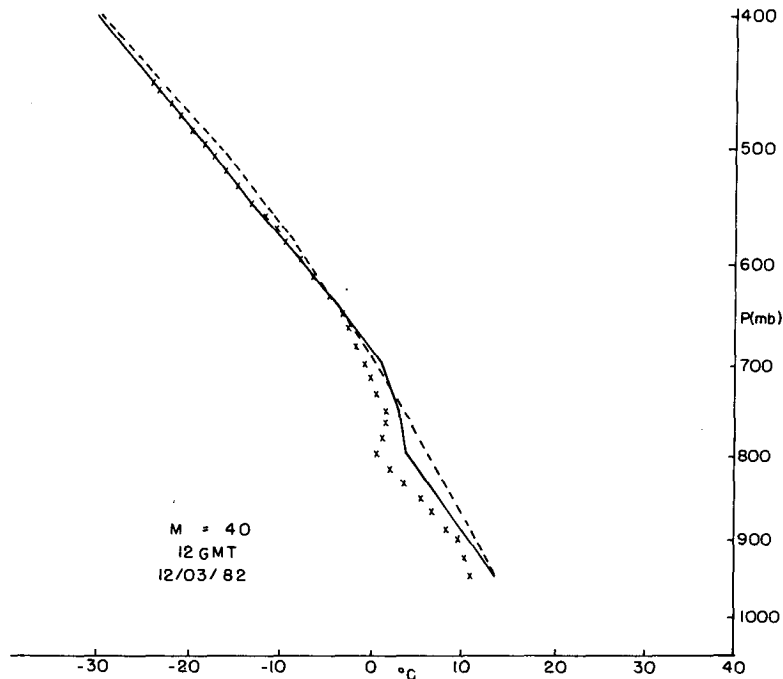


FIG. 10. (b) As in Fig. 10(a) but for $M = 40$ surface.

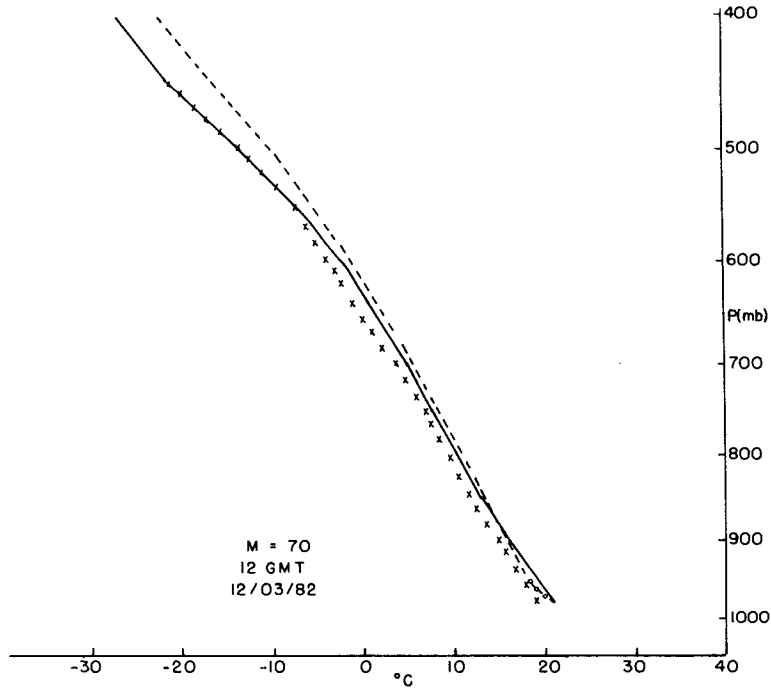


FIG. 10. (c) As in Fig. 10(a) but for $M = 70$ surface.

as applicable only to flows for which the direction of the vertical shear does not vary significantly through the depth of the layer in which instability is being considered.

The preceding discussion suggests that for large-scale flows for which the Rossby number is small and within which the direction of the geostrophic vertical shear vector (or the horizontal temperature gradient) is not locally a strong function of height, the stability to grav-

itational and/or slantwise convection may be assessed by evaluating vertical profiles of temperature and moisture in *geostrophic coordinates*, rather than physical coordinates. The latter are defined (e.g., Hoskins, 1982) by

$$\left. \begin{aligned} X &= x + v_g/f \\ Y &= y - u_g/f \end{aligned} \right\}$$

where the upper case letters represent the new coord-

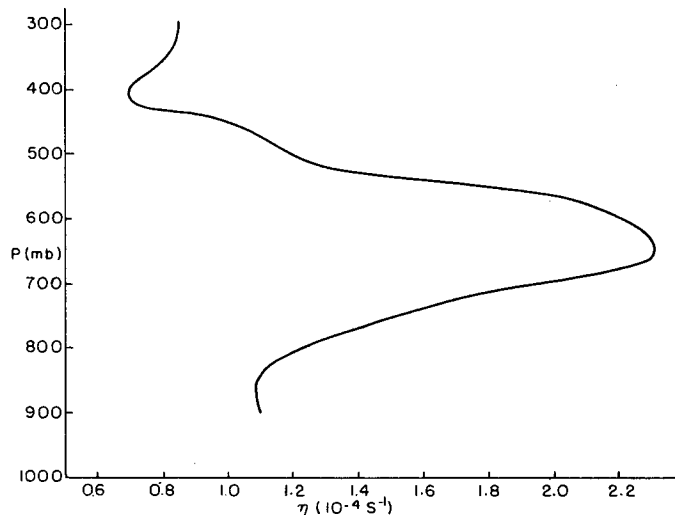


FIG. 11. Absolute vorticity (10^{-4} s^{-1}) as a function of pressure at Oklahoma City at 1200 GMT 3 December 1982.

dinates and the velocities are geostrophic. In geostrophic coordinates, M surfaces are by definition vertical, so that vertical profiles of temperature and moisture are along M surfaces. This provides a very simple means of evaluating combined gravitational/centrifugal instability in numerical models, such as semi-geostrophic models, which are run in geostrophic coordinates. It also suggests that general moist convection, which as indicated by the case study of Section 4 tends to restore moist adiabatic lapse rates along M surfaces, may be at least crudely parameterized in large-scale models by performing moist adiabatic adjustment in geostrophic coordinates rather than in physical coordinates. It should be remarked that such adjustment schemes are carried out under the philosophy that the convective time scale is short compared to the time scale characterizing the large-scale flow; this separation of scales should work for slantwise convection, although not as well as it works for upright convection. In fact, the time scales characterizing baroclinic instability, slantwise convection and upright convection are, respectively, $Ri^{1/2}/f$, $1/f$ and $1/N$ where Ri is the Richardson number and N the Brunt-Väisälä frequency. (These time scales are associated with horizontal length scales of NH/f , u_0/f and H , respectively, where H is a typical scale height.) This highlights the three-scale nature of moist convection in baroclinic flows.

It is useful to inquire about processes which act to change the slantwise convective available potential energy. We first note that since M surfaces are material surfaces for two-dimensional flow, differential (ageostrophic) horizontal temperature advection cannot change the distribution of potential temperature or mixing ratio along the M surfaces, as indicated in Fig. 12. All that can happen is that the M surfaces change their slope, so that the SCAPE is distributed over greater or smaller path lengths, depending on whether the

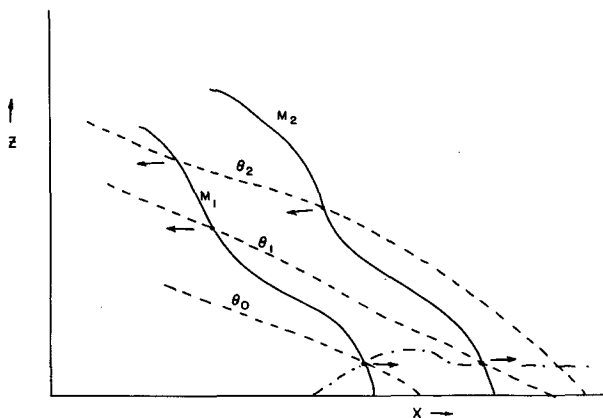


FIG. 12. The effect of horizontal ageostrophic velocities (arrows) on vertical distribution of θ (dashed) along M surfaces (solid). Dash-dot line represents surface of constant water vapor mixing ratio.

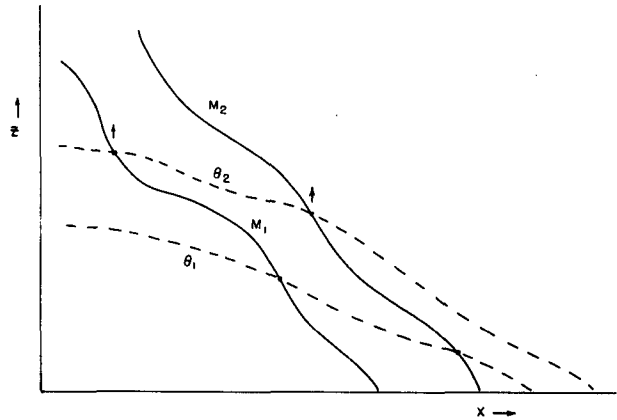


FIG. 13. As in Fig. 12 but for vertical motions.

slope is large or small. By such a process, instability to slantwise convection may be converted to instability to upright convection. Also, the path length may be important if irreversible processes such as mixing depend upon it.

Differential vertical potential temperature advection along M surfaces can, however, alter the stability. An obvious special case is barotropic flow, in which M surfaces are vertical; here, upward motion will decrease the positive lapse rate of potential temperature below the level of maximum upward motion. An example of this process in a baroclinic flow is illustrated in Fig. 13.

Diabatic heating at the base of M surfaces, or cooling at high levels, will generally act to increase instability, as with upright convection. We note here, however, that since M surfaces are generally sloped, horizontal gradients of diabatic heating which act to intensify the existing horizontal temperature gradient will also decrease the stability.

Finally, surface friction, by irreversibly changing the distribution of M , will generally alter the stability. Stability will be decreased if irreversible processes increase the slope of the M surfaces. If the surface wind has a component in the same direction as the surface thermal wind, then friction will act to move M surfaces toward the warmer air near the ground, as illustrated in Fig. 14. Viewed another way, surface friction will cause an Ekman drift across fixed M surfaces, thus advecting warmer air across them.

6. Conclusions

The main result of this analysis is that the stability of a moist baroclinic flow to finite slantwise reversible displacements of a two-dimensional air parcel may be assessed by reversibly lifting the parcel along a surface of pseudo-angular momentum (M) and comparing its virtual temperature to that of its environment in the usual way. This may be done by constructing cross

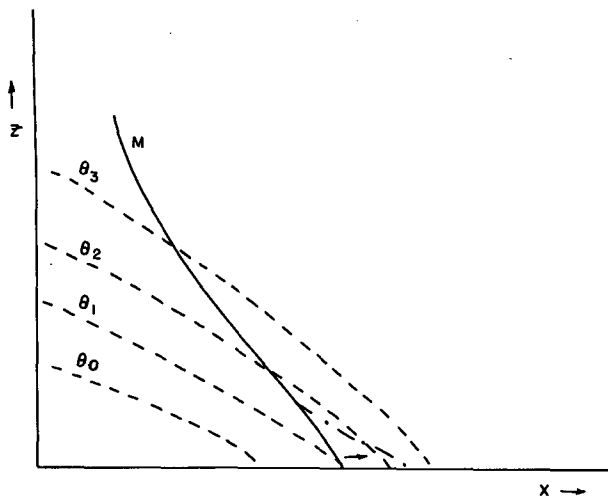


FIG. 14. The effect of surface friction on distribution of M (solid) when the surface wind has a component in the same direction as the surface thermal wind. The θ surfaces are shown by dashed lines. The dash-dot line shows the movement of the M surface with respect to θ surfaces caused by surface friction.

sections orthogonal to the mean vertical geostrophic shear and plotting on them potential temperature, pseudo-angular momentum and a water vapor variable; alternatively, the stability may be approximately assessed using a single sounding if some independent information about the geostrophic shear and the vertical distribution of the vertical component of vorticity is available.

A case study of a strongly baroclinic flow with observed linear bands of clouds and precipitation reveals that this atmosphere, which was initially conditionally unstable to upright convection, tended toward a state which was neutral to reversible slantwise displacements. Such an atmosphere exhibits moist adiabatic lapse rates of temperature along M surfaces, even though it may be substantially stable to vertical displacements.

The dynamics of motions resulting from slantwise moist convective instability have not been addressed here, but are discussed in Emanuel (1983). In that paper, it is shown that the instability results in slantwise motion along surfaces on which the buoyancy of the ascending air vanishes; this motion may reach magnitudes comparable to those characterizing the unperturbed mean flow. These results, taken together with those presented here, suggest that the centrifugal component of convective instability plays an important role in the moist baroclinic atmosphere.

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