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COMMENT

Comment on Makarieva *et al.* 'A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist'

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Makarieva *et al.* (2010) assert that a dissipative heat engine is impossible and criticize earlier published work that they claim violates the laws of thermodynamics. Here we show that the earlier work does not violate fundamental physical laws and suggest that Makarieva *et al.* (2010) were misinterpreting expressions for wind speed as ones for work done on external objects. Moreover, we dispute their assertion that dissipative heating is necessarily compensated by a reduction of external heating.

Keywords: hurricane; Carnot; heat engine; tropical cyclone

1. Introduction

We welcome the paper of Makarieva *et al.* (2010; hereafter MGLN) and the opportunity it affords us to clarify some terminology that we used in our previously published work on hurricanes. While we hold that the equations we presented in the earlier work are correct, there are several issues with some of the terms we used, and it would be of benefit to clarify these. But we take issue with one of the assumptions made by MGLN.

First, we agree with MGLN's fundamental point that dissipation cannot increase the work performed by a heat engine. In our published work, we have argued only that dissipation can increase wind speeds in a hurricane; indeed, we considered a limit in which the hurricane heat engine does *no* work

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on its environment. On the most fundamental level, the equation derived in Bister & Emanuel (1998),

$$V^2 = \frac{T_s - T_o}{T_o} \frac{C_k}{C_D} (k^* - k), \tag{1.1}$$

is an equation for wind speed V, not one for work done on any external system. Here T_s is the surface temperature (at which heat flows into the system), T_o the mean temperature at which heat is exported, k^* the specific enthalpy of air saturated at the sea surface temperature, k the actual enthalpy of air next to the sea, and C_k and C_D are dimensionless exchange coefficients for enthalpy and momentum, respectively. In fact, as we will here reiterate, the derivation of equation (1.1) assumes that the hurricane does no work on its environment, thus there is no violation of Carnot's theorem or any of the known principles of thermodynamics. But it may not be correct to call such a hurricane an engine (of any kind), as it does not perform useful work. (Note that Bister & Emanuel (1998) do not refer to the hurricane as a heat engine, whereas the other reference cited by MGLN, Emanuel (2003), states that 'The mature tropical cyclone may be idealized as a steady, axisymmetric flow whose energy cycle is very similar to that of an ideal Carnot engine'.) If an engine is defined as a mechanism that operates, regardless of whether it does useful work, then we assert that a 'dissipative heat engine' is clearly possible and that a hurricane is an example of such an engine.

2. Dissipation versus work

It may be helpful to consider a system very much like the idealized steady, axisymmetric flow discussed in Bister & Emanuel (1998), Emanuel (2003) and related papers, but replacing the lower boundary by a stress-free boundary condition (so that no work is done on the surface) and at the same time placing in the atmospheric boundary layer an array of idealized wind turbines attached to electric generators. For the purposes of discussion, we will take these windmills and generators to be perfectly efficient in converting wind energy into electricity. The rate of power generation scales as the cube of the wind velocity of the air passing through the windmills, and this is also the rate of loss of mechanical energy from the atmospheric flow (figure 1*a*).

Clearly, this system functions nearly as an ideal Carnot heat engine and, because the conversion of wind energy to electricity is assumed to be 100 per cent efficient and, if we assume that there is no turbulent dissipation in the boundary layer, then there is no dissipative heating in the atmospheric flow. Under these conditions, the rate of work (here used to generate an electrical current) is

$$W = Q_{in} \frac{T_s - T_o}{T_s}, \tag{2.1}$$

where Q_{in} is the rate of heat input from the sea. This is the classical Carnot expression. At the same time, the wind speed is that derived by Emanuel (1986), which is identical to that given by equation (1.1), except that the temperature in the denominator is T_s, assuming that the windmills extract energy at a rate per unit surface area given by $\rho C_D V^3$.

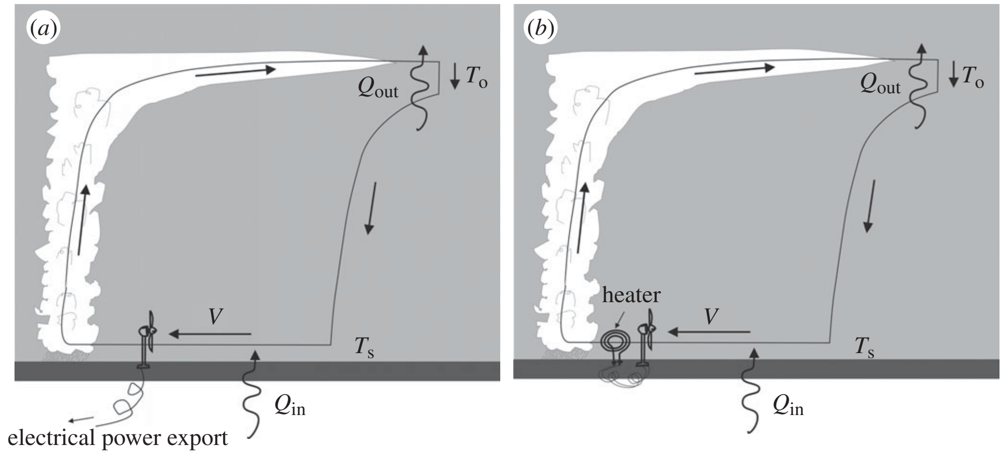

Figure 1. Two hypothetical hurricane systems. In both cases, heat is added from the ocean at a rate $Q_{\rm in}$ and at temperature $T_{\rm s}$ and removed from the top of the storm at a rate $Q_{\rm out}$ and temperature $T_{\rm o}$, the ocean is replaced by a stress-free surface and an array of wind turbines (one of which is illustrated) removes wind energy at a rate per unit surface area given by $\rho C_{\rm D} V^3$. (*a*) The electrical power generated by the turbines is exported, whereas in (*b*), it is returned to the system through a heater.

Now consider a second system identical to the first, except that the electrical current is directed back to a heating element placed in the atmospheric boundary layer (figure 1*b*). All the wind energy absorbed by the wind turbines is now used to heat the boundary layer and, in some sense, functions as an additional heat source, though, because it results from a strictly internal conversion, *it is not a net energy source and is therefore not added to* $Q_{\rm in}$. In this case, *no work is done on any external body*: $W=0$. This corresponds to MGLN's dissipative heat engine, and in agreement with their result, dissipation reduces the work performed by the engine: in this case, to zero. However, the conversion of mechanical energy to enthalpy is associated with an internal entropy source equal to the dissipation rate D divided by the temperature at which the dissipation takes place, in this case $T_{\rm s}$. One can then combine the energy and entropy equations for this cycle, as done by Bister & Emanuel (1998) or Pauluis & Held (2002) to obtain the dissipation rate:

$$D = \frac{T_{\rm s} - T_{\rm o}}{T_{\rm o}} Q_{\rm in}.$$

The wind speed is now that given by equation (1.1). No laws of thermodynamics have been violated. In the second case, the heat flowing out of the system $Q_{\rm out}$ has been increased by an amount W (given by equation (2.1) over the first case and is now equal to $Q_{\rm in}$. We emphasize that dissipation is not the same thing as work and hold that MGLN's characterization of dissipation as a form of mechanical work is misleading.

Note that, in reality, $Q_{\rm in}$ depends on the air temperature and wind speed, both of which are affected by dissipative heating. But we disagree with MGLN's statement (p. 6) that 'When work A dissipates to heat within the working body, the latter warms'. They use this premise then to argue that any heat added by dissipation necessarily subtracts from the external heat input. However, we find no support for their premise: the first law of thermodynamics states that heating can be used to increase internal energy or do work; it is silent as to the partition. By holding the expansion of the working fluid fixed during the isothermal expansion phase in their piston example, MGLN artificially restrict the physics of the system to arrive at their result that dissipative heating must subtract from external heating. We see no reason why the working fluid cannot further expand to accommodate the additional heat input from dissipation. In the numerical simulations reported by Bister & Emanuel (1998), dissipative heating led to a further reduction in the central pressure, corresponding to enhanced isothermal expansion and stronger wind speeds.

If the definition of a heat engine is one that uses heat energy to perform work on an external system, then the first system described earlier is an ideal Carnot heat engine, whereas the second is not an engine at all, even though, as stated by Emanuel (2003), the thermodynamic cycle is similar (not identical) to a Carnot cycle.

We can generalize to the case in which a fraction β of the wind energy is exported, while $1-\beta$ is returned to the system through the heater. In that case, the entropy balance yields

$$\frac{Q_{\mathrm{in}}}{T_{\mathrm{s}}} - \frac{Q_{\mathrm{out}}}{T_{\mathrm{o}}} + (1-\beta)\frac{\rho C_{\mathrm{D}} V^3}{T_{\mathrm{s}}} = 0, \tag{2.2}$$

where the last term represents the entropy production caused by the heater. The total amount of work done (in this case, the exported electrical current) is then

$$W = \beta\rho C_{\mathrm{D}} V^3 = Q_{\mathrm{in}} - Q_{\mathrm{out}}, \tag{2.3}$$

while the internal dissipation rate is

$$D = (1-\beta)\rho C_{\mathrm{D}} V^3.$$

Note that here the heat input from the heater is *not* added to the right-hand side of equation (2.3), because it is not an external energy source. Using equation (2.2) for Q_{out}, equation (2.3) becomes

$$W = \beta\rho C_{\mathrm{D}} V^3 = Q_{\mathrm{in}}\frac{T_{\mathrm{s}} - T_{\mathrm{o}}}{T_{\mathrm{s}}} - \frac{T_{\mathrm{o}}}{T_{\mathrm{s}}}(1-\beta)\rho C_{\mathrm{D}} V^3. \tag{2.4}$$

Solving equation (2.4) for W gives

$$W = \left[\frac{\beta}{\beta + (T_{\mathrm{o}}/T_{\mathrm{s}})(1-\beta)}\right]\frac{T_{\mathrm{s}} - T_{\mathrm{o}}}{T_{\mathrm{s}}} Q_{\mathrm{in}}. \tag{2.5}$$

It is easily verified that the factor in brackets is always less than unity, so that diverting some of the power back into the hurricane system always results in a reduction in the efficiency (the ratio of work to Q_{in}) from the maximum value given by Carnot's theorem. Note that for the hurricane, $Q_{\mathrm{in}} = \rho C_{\mathrm{k}} V(k^* - k)$ and

$W = \beta\rho C_{\mathrm{D}} V^3$; substitution of these into equation (2.5) yields

$$V^2 = \left[\frac{1}{\beta + (T_{\mathrm{o}}/T_{\mathrm{s}})(1-\beta)}\right] \frac{T_{\mathrm{s}} - T_{\mathrm{o}}}{T_{\mathrm{s}}} \frac{C_{\mathrm{k}}}{C_{\mathrm{D}}} (k^* - k), \tag{2.6}$$

which reduces to equation (1.1) when $\beta = 0$. We reiterate that equations (1.1) and (2.6) are expressions for wind speed, not work. In a real hurricane, in which the wind does some work on the ocean (here considered an external system), β can be shown to scale as the ratio of the ocean surface current speed to the wind speed, a number of the order of 10^{-2}. Moreover, as shown by Emanuel (1986), among others, some work is done on the environment in the outflow; this scales as the square of the radial dimension of the storm and is a small fraction of the rate of dissipation of kinetic energy unless the storm is exceptionally large.

We would like to point out that the equation for wind velocity given by equation (1.1) was first derived by Bister & Emanuel (1998) from the basic conservation equations for mass and momentum and from the first law of thermodynamics, without explicitly invoking heat engine concepts. That this gives basically the same result as that derived from consideration of the thermodynamic cycle gives some added confidence to the results. We note that MGLN have not claimed that this derivation, given in §3.1 of Bister & Emanuel (1998), is wrong. Also, Bister & Emanuel (1998) carried out an integration of a numerical model that solves the fundamental equations of fluid dynamics and thermodynamics and showed that dissipative heating does indeed increase wind speeds, as predicted.

MGLN raise the separate and interesting issue of finite time thermodynamics, which we have not addressed in our previous work. This deserves a more extensive treatment than is practical in this comment, but we make two points about this here. First, the observed temperatures in the atmospheric boundary layer are usually only a degree or two less than those of the sea surface; as this is a small fraction of the temperature difference between the temperatures at which enthalpy flows in and out of the hurricane, we expect the finite time effects to be correspondingly small. Second, this issue is not unique to the dissipative heat engine.

MGLN claim that the dissipative hurricane violates fundamental thermodynamic principles in two ways. First, they point out that the limit $T_{\mathrm{o}} \to 0$ produces a singularity in equation (1.1), but in any event, the classical physics becomes invalid before that limit is reached. Indeed, it is impossible to cool a physical system that is already at absolute zero, as such a body is already at its lowest possible energy state. Hence, we agree with MGLN that a dissipative heat engine working with an energy sink at absolute zero is impossible. It does not follow, however, that because the dissipative heat engine is unphysical for $T_{\mathrm{o}} = 0$, it is also unphysical for $T_{\mathrm{o}} > 0$.

Second, MGLN argue that our treatment of frictional dissipation violates the second law of thermodynamics. In our analysis, frictional dissipation is an internal process in which mechanical energy in the atmosphere is irreversibly converted into thermal (internal) energy. As such, it is associated with an internal production of entropy given by D/T_{s}.

Finally, we note that MGLN misquote Rennó & Ingersoll (1996), who actually state that 'increases in the fraction of energy dissipated near the hot source, at the expense of decreases in the fraction of energy dissipated at the cold source, lead to increases in the *apparent* efficiency of the convective heat engine' (italics added here). Their definition of *apparent efficiency* (their eqn (29)) makes it clear that it has nothing to do with exported power; thus there is no violation of Carnot's theorem. Nowhere in that paper, or in Rennó (1997, 2001), Pauluis *et al.* (2000) or Pauluis & Held (2002), is it stated that internal dissipation increases work done on an external system; indeed, in all of those papers, it is assumed that all mechanical energy generated by the atmospheric flow is dissipated internally and that no work is done on any external system.

3. Summary

We fully concur with MGLN that no engine can do work on an external body with an efficiency that exceeds the Carnot efficiency; indeed, we never claimed otherwise. Confusion seems to have arisen by mistaking an equation for wind speed for one for external work, and we do not agree with MGLN's assertion that dissipative heating necessarily reduces external heat input.

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