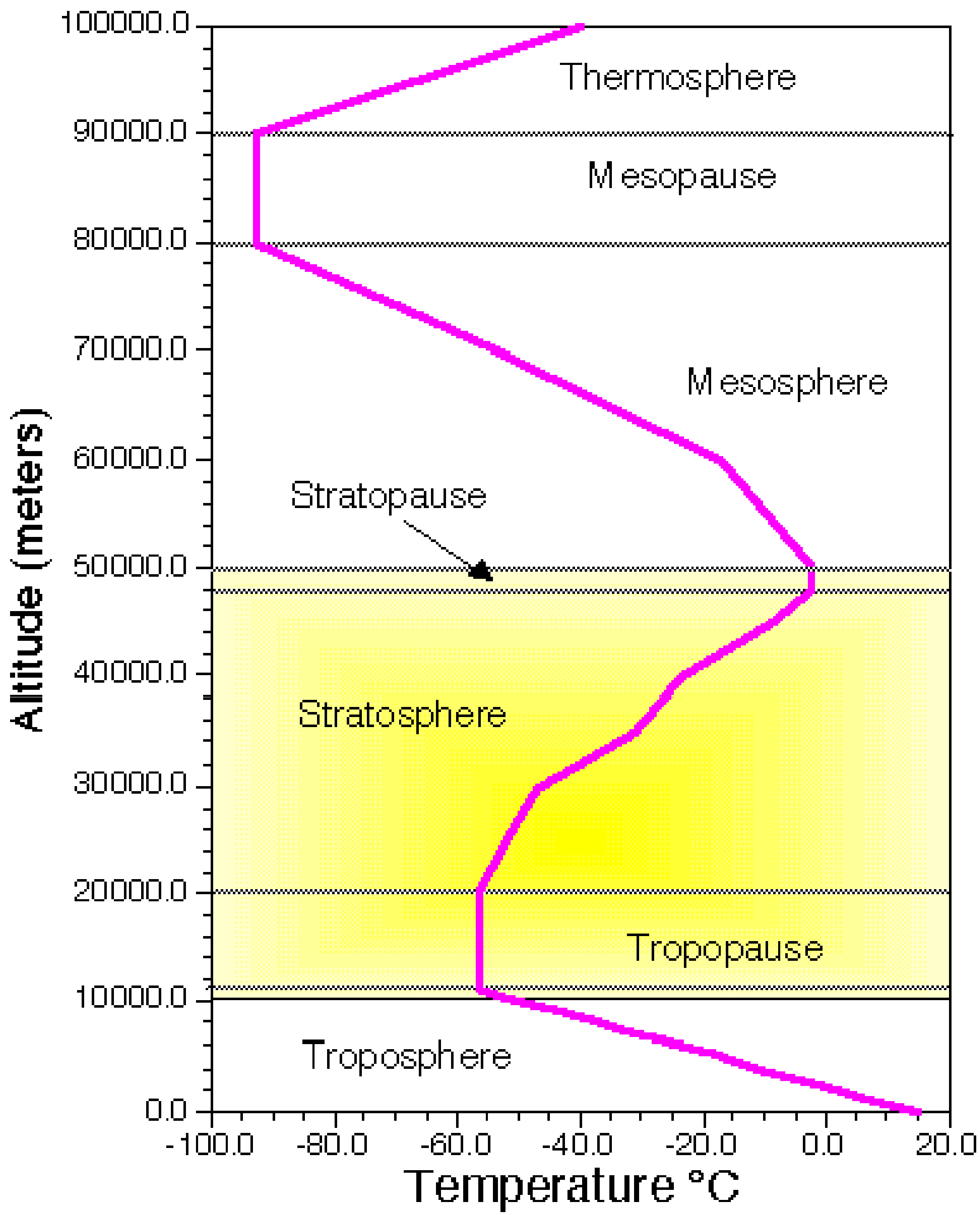
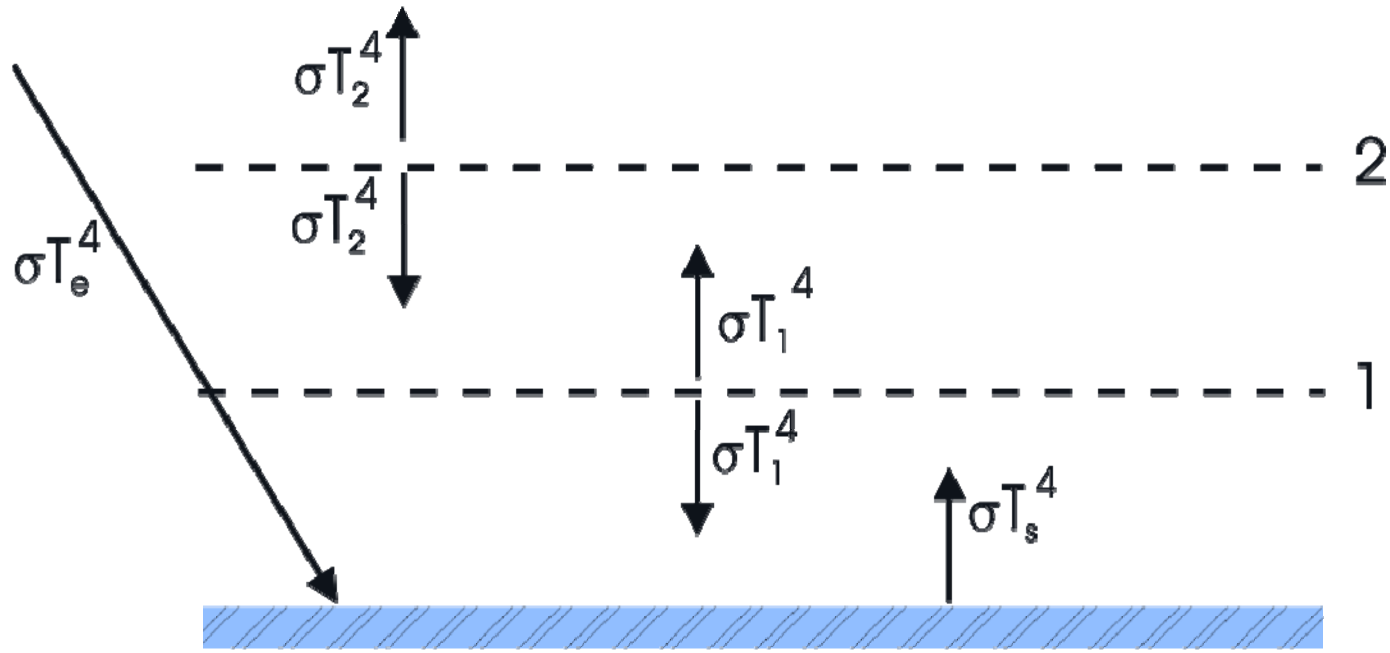
Radiative Equilibrium

- Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
- Radiative heating drives actual state toward state of radiative equilibrium



100000.0
90000.0
80000.0
70000.0
60000.0
50000.0
40000.0
30000.0
20000.0
10000.0
0.0
Altitude (meters)
Thermosphere
Mesopause
Mesosphere
Stratopause
Stratosphere
Tropopause
Troposphere
-100.0
-80.0
-60.0
-40.0
-20.0
0.0
20.0
Temperature °C

Extended Layer Models



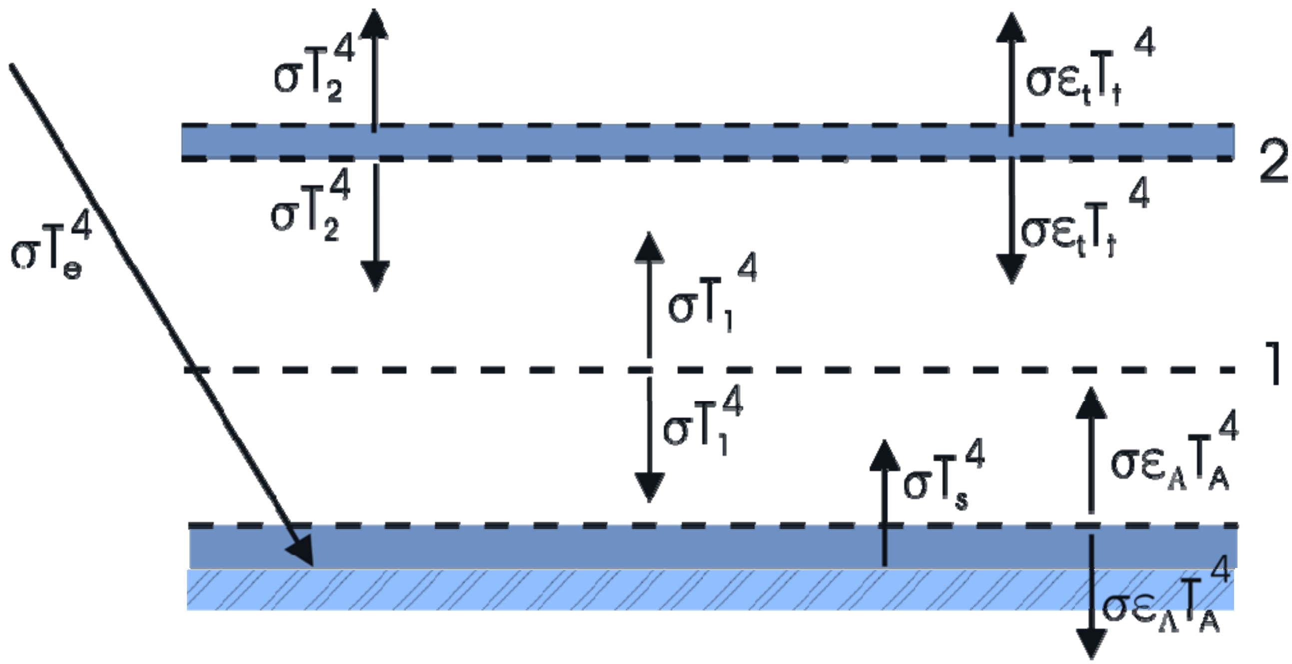
$$TOA: \quad \sigma T_2^{\,4} = \sigma T_e^{\,4} \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1^{\,4} = \sigma T_2^{\,4} + \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_s^{\,4}$$

$$Surface: \quad \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_1^{\,4}$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

Effects of emissivity<1



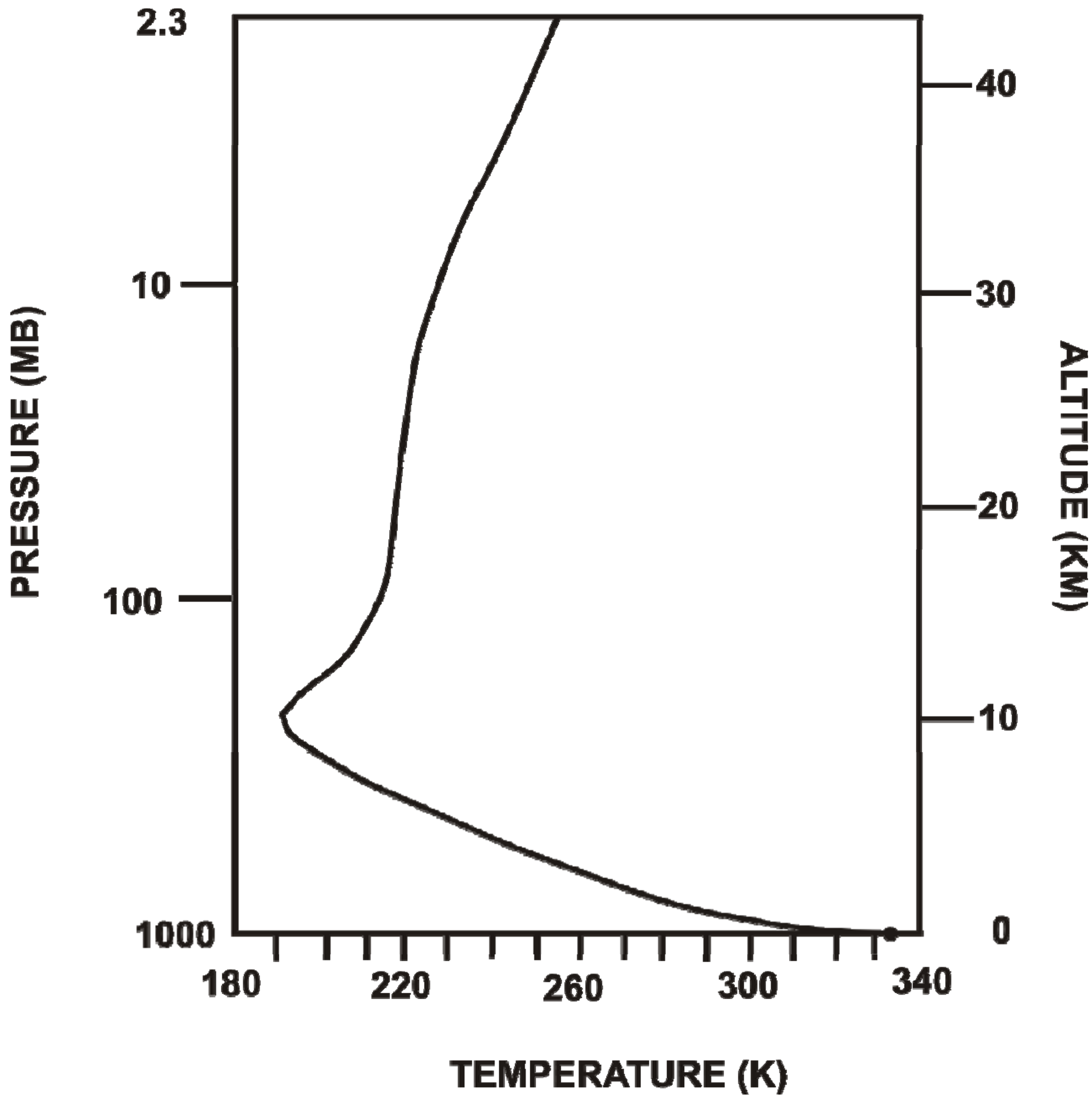
$$Surface: \quad 2\varepsilon_A \sigma T_A^{\,4} = \varepsilon_A \sigma T_1^{\,4} + \varepsilon_A \sigma T_s^{\,4}$$

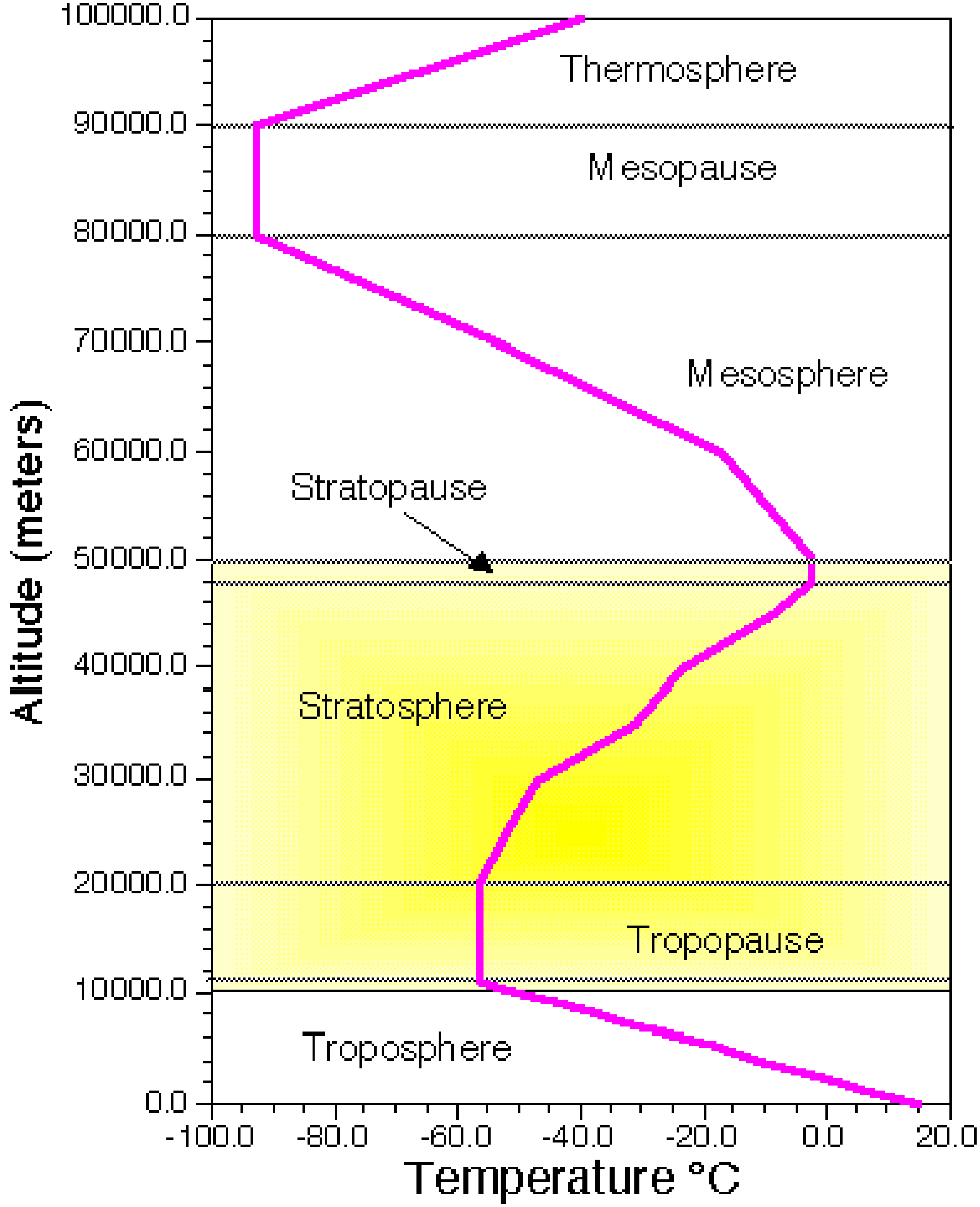
$$\rightarrow \quad T_A = \left(5/2\right)^{1/4} T_e \simeq 321K \quad < T_s$$

$$Stratosphere: \quad 2\varepsilon_t \sigma T_t^{\,4} = \varepsilon_t \sigma T_2^{\,4}$$

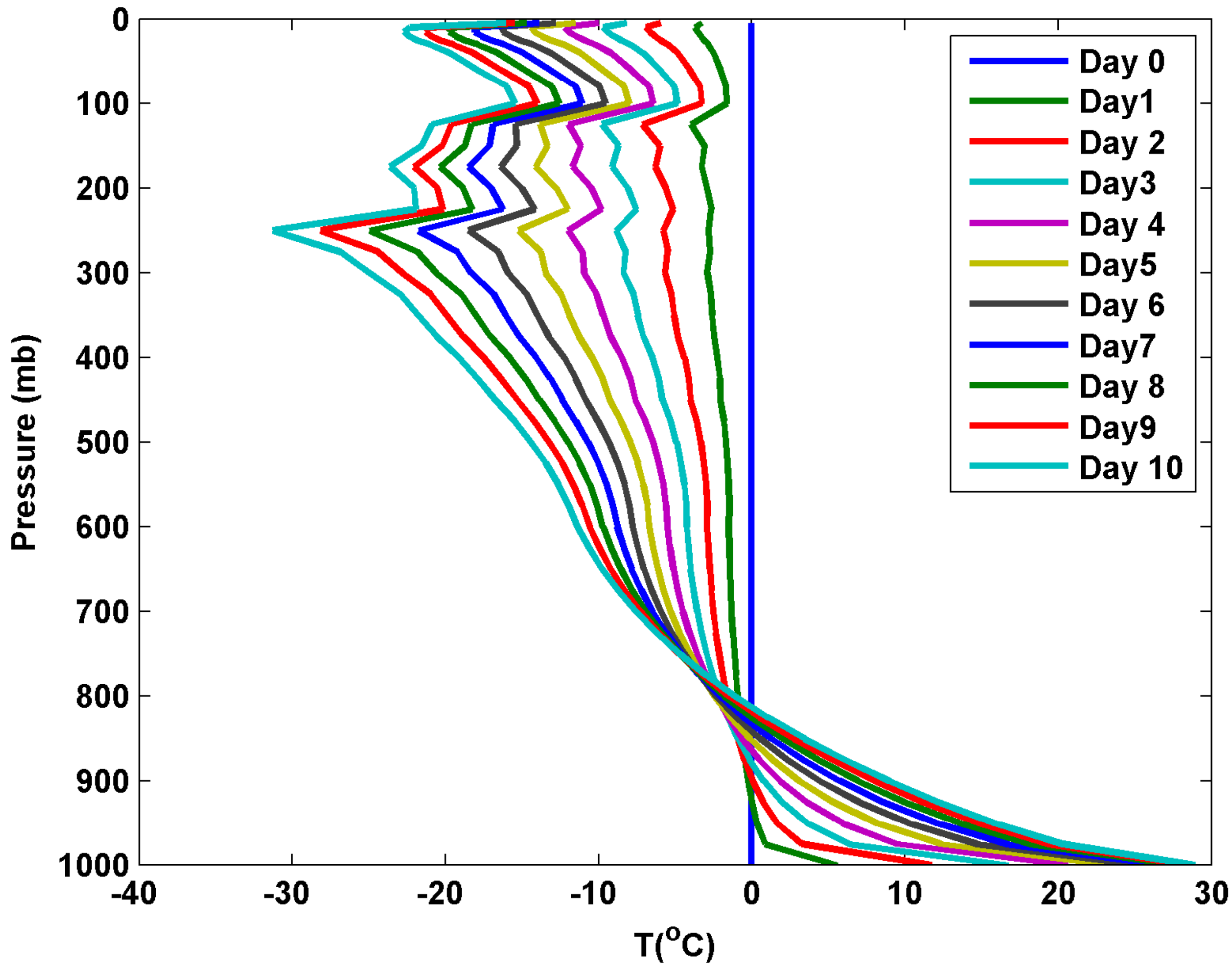
$$\rightarrow \quad T_t = \left(1/2\right)^{1/4} T_e \simeq 214K \quad < T_e$$

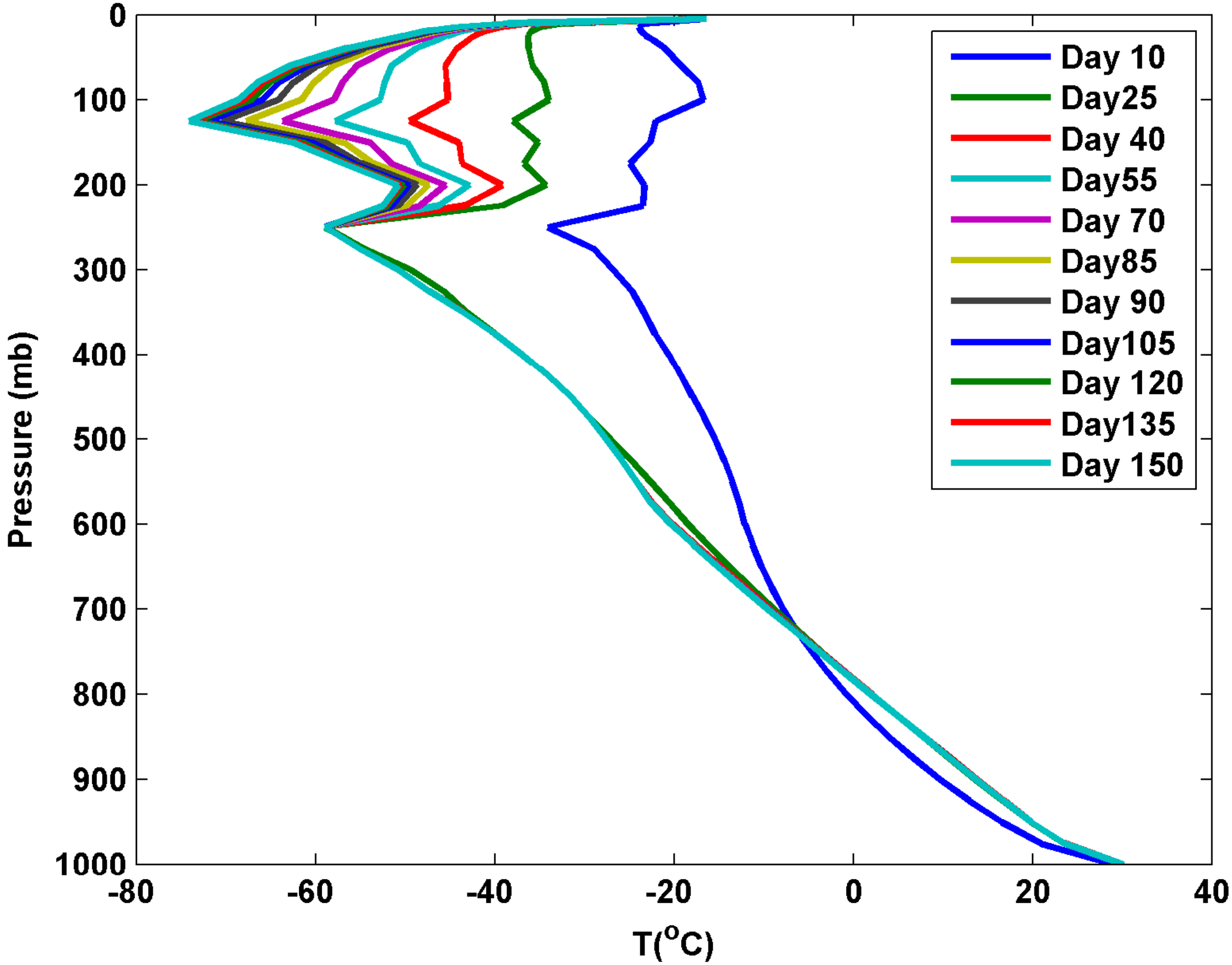
Full calculation of radiative equilibrium




100000.0
90000.0
80000.0
70000.0
60000.0
50000.0
40000.0
30000.0
20000.0
10000.0
0.0
Altitude (meters)
Thermosphere
Mesopause
Mesosphere
Stratopause
Stratosphere
Tropopause
Troposphere
-100.0
-80.0
-60.0
-40.0
-20.0
0.0
20.0
Temperature °C

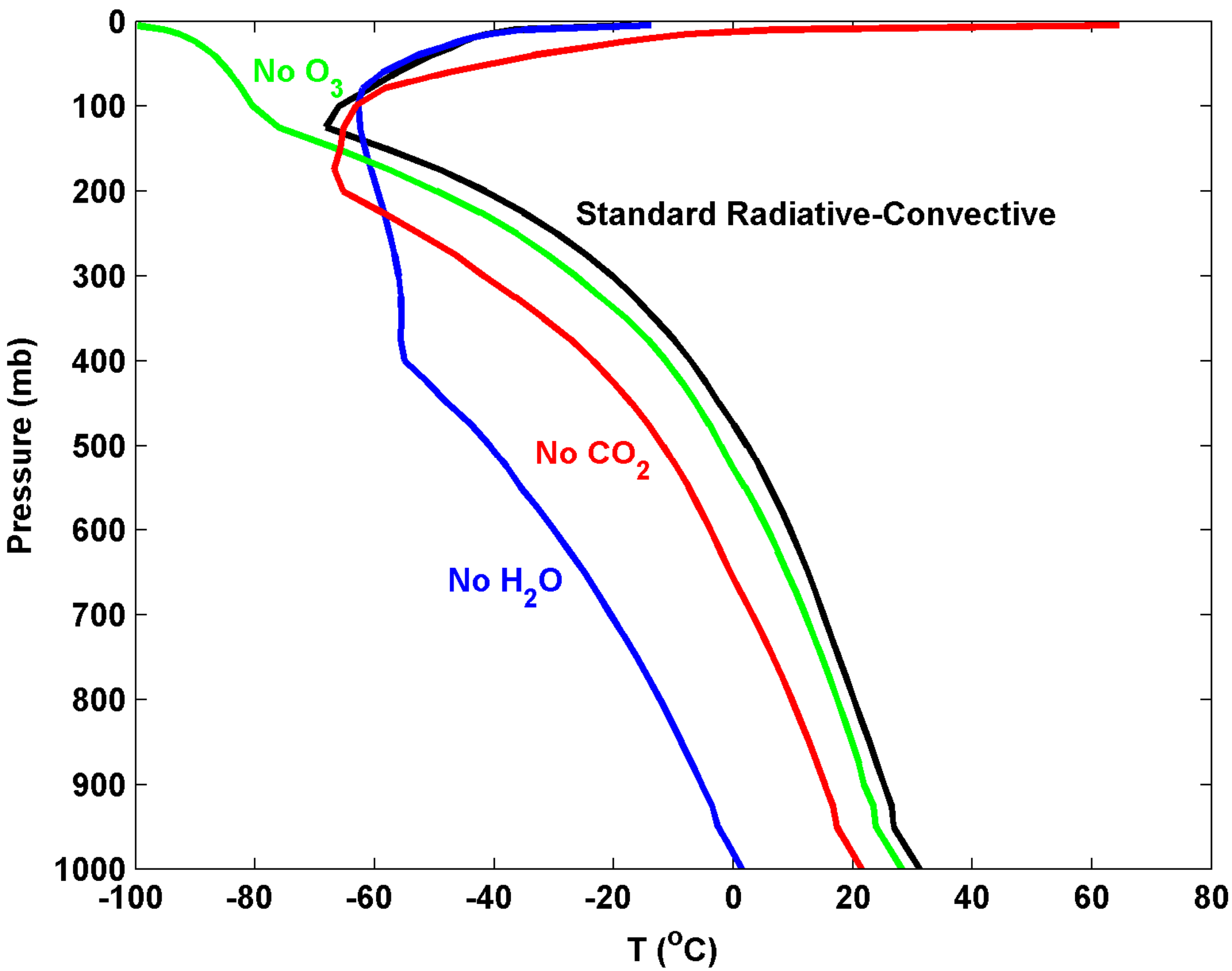
Time scale of approach to equilibrium



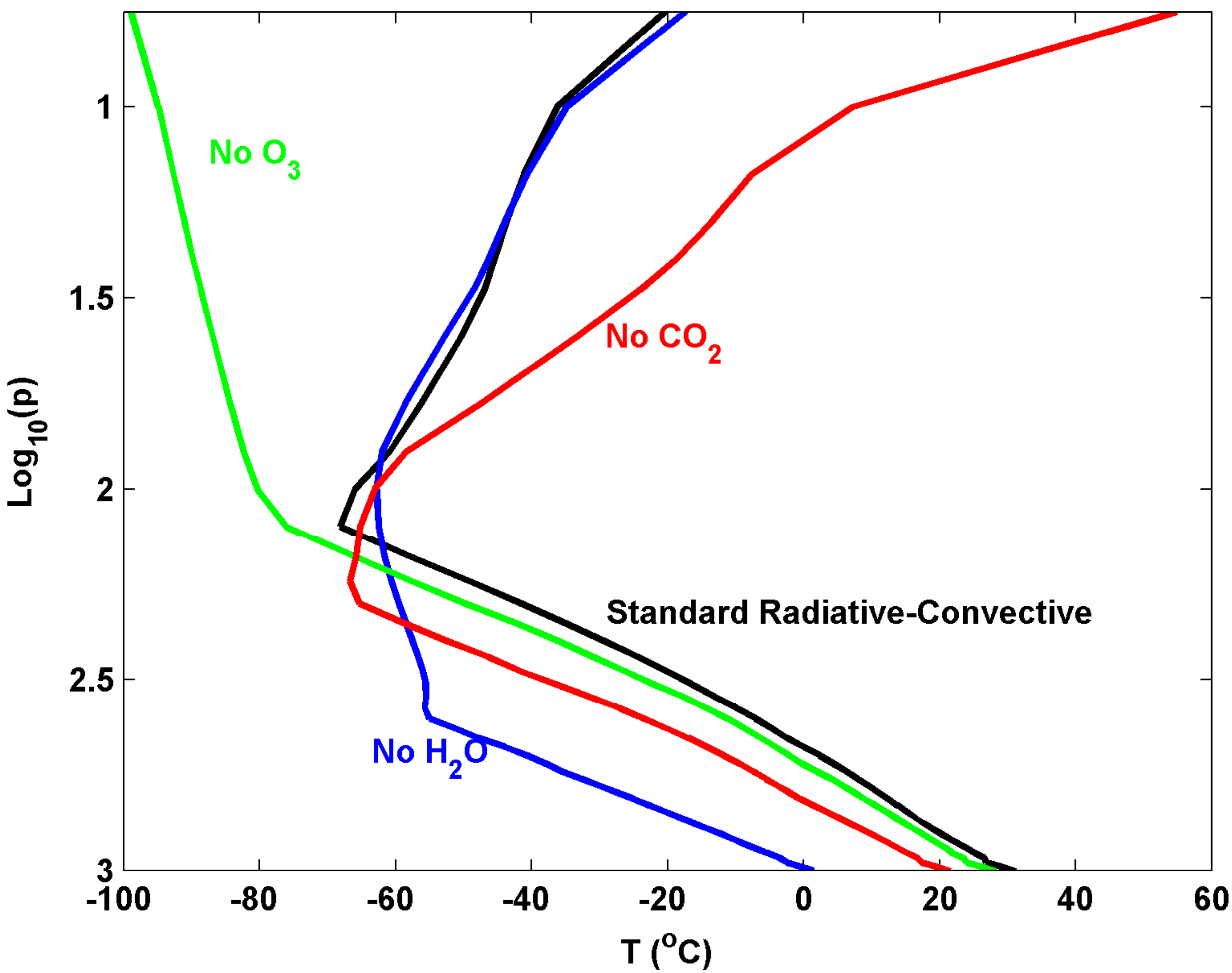


Pressure (mb)
T(^{o}C)
0
100
200
300
400
500
600
700
800
900
1000
-80
-60
-40
-20
0
20
40
Day 10
Day25
Day 40
Day55
Day 70
Day85
Day 90
Day105
Day 120
Day135
Day 150

Contributions of various absorbers



Note: All simulations have variable clouds interacting with radiation



No O3
No CO2
Standard Radiative-Convective
No H2O
Log10(p)
1
1.5
2
2.5
3
-100
-80
-60
-40
-20
0
20
40
60
T (oC)

Problems with radiative equilibrium solution

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

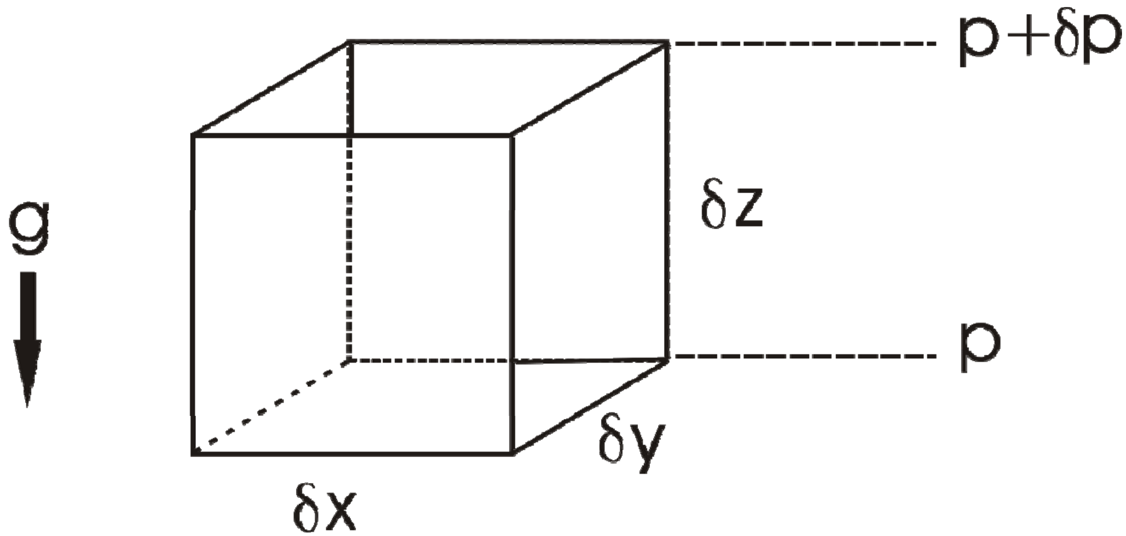
Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer

When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability

Hydrostatic equilibrium



$$Weight: \quad -g\rho\delta x\delta y\delta z$$

$$Pressure: \quad p\delta x\delta y-\left(p+\delta p\right)\delta x\delta y$$

$$F=MA: \quad \rho\delta x\delta y\delta z\frac{dw}{dt}=-g\rho\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha\frac{\partial p}{\partial z}, \qquad \alpha = {}^{1}\!/\!_{\rho} = specific\ volume$$

Pressure distribution in atmosphere at rest

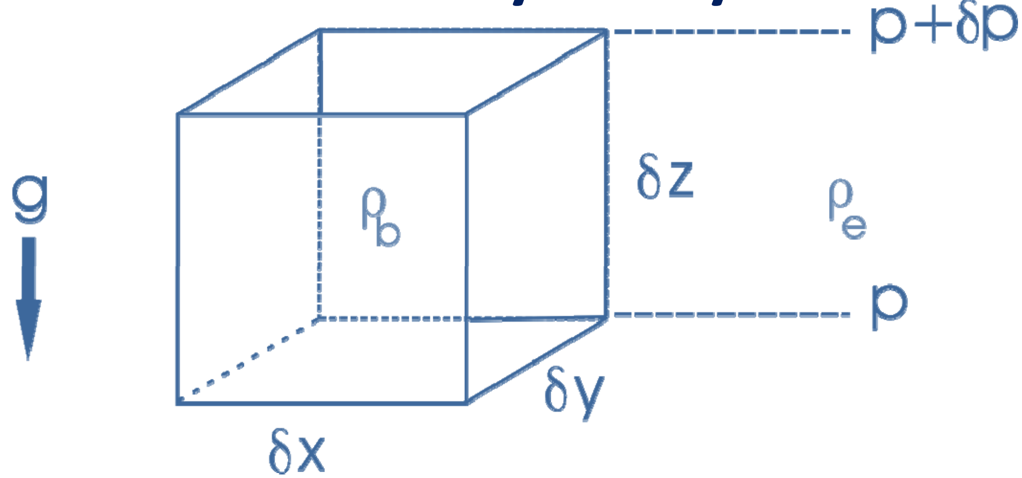
$$Ideal\ gas: \quad \alpha = \frac{RT}{p}, \qquad R \equiv \frac{R^*}{\bar{m}}$$

$$Hydrostatic: \quad \frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT}$$

$$Isothermal\ case: \quad p = p_0 e^{-z/H}, \qquad H \equiv \frac{RT}{g} = "scale\ height"$$

Earth: H~ 8 Km

Buoyancy



$$Weight: \quad -g\rho_b \delta x \delta y \delta z$$

$$Pressure: \quad p\delta x\delta y - (p+\delta p)\delta x \delta y$$

$$F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g\rho_b \delta x \delta y \delta z - \delta p \delta x \delta y$$

$$\frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \qquad but \quad \frac{\partial p}{\partial z} = -g/\alpha_e$$

$$\rightarrow \quad \frac{dw}{dt} = g\frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B$$

Buoyancy and Entropy

Specific Volume: $\alpha = {}^{1}\!/\!_{\rho}$

Specific Entropy: s

$$\alpha = \alpha(p, s)$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial \alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta\alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma \delta s$$

The adiabatic lapse rate

$$\textit{First Law of Thermodynamics}:$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d\left(\alpha p\right)}{dt} - \alpha\frac{dp}{dt}$$

$$= \left(c_v + R\right)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

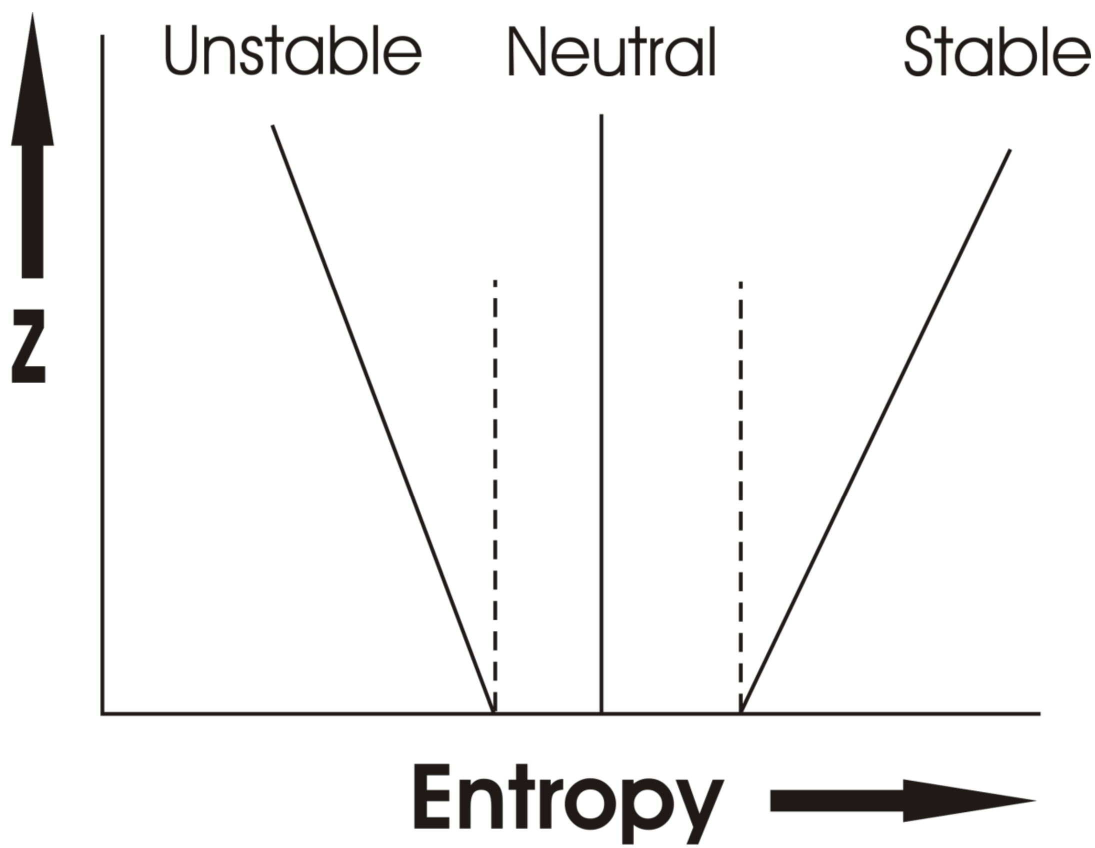
$$\textit{Adiabatic}: \quad c_p dT - \alpha dp = 0$$

$$\textit{Hydrostatic}: \quad c_p dT + g dz = 0$$

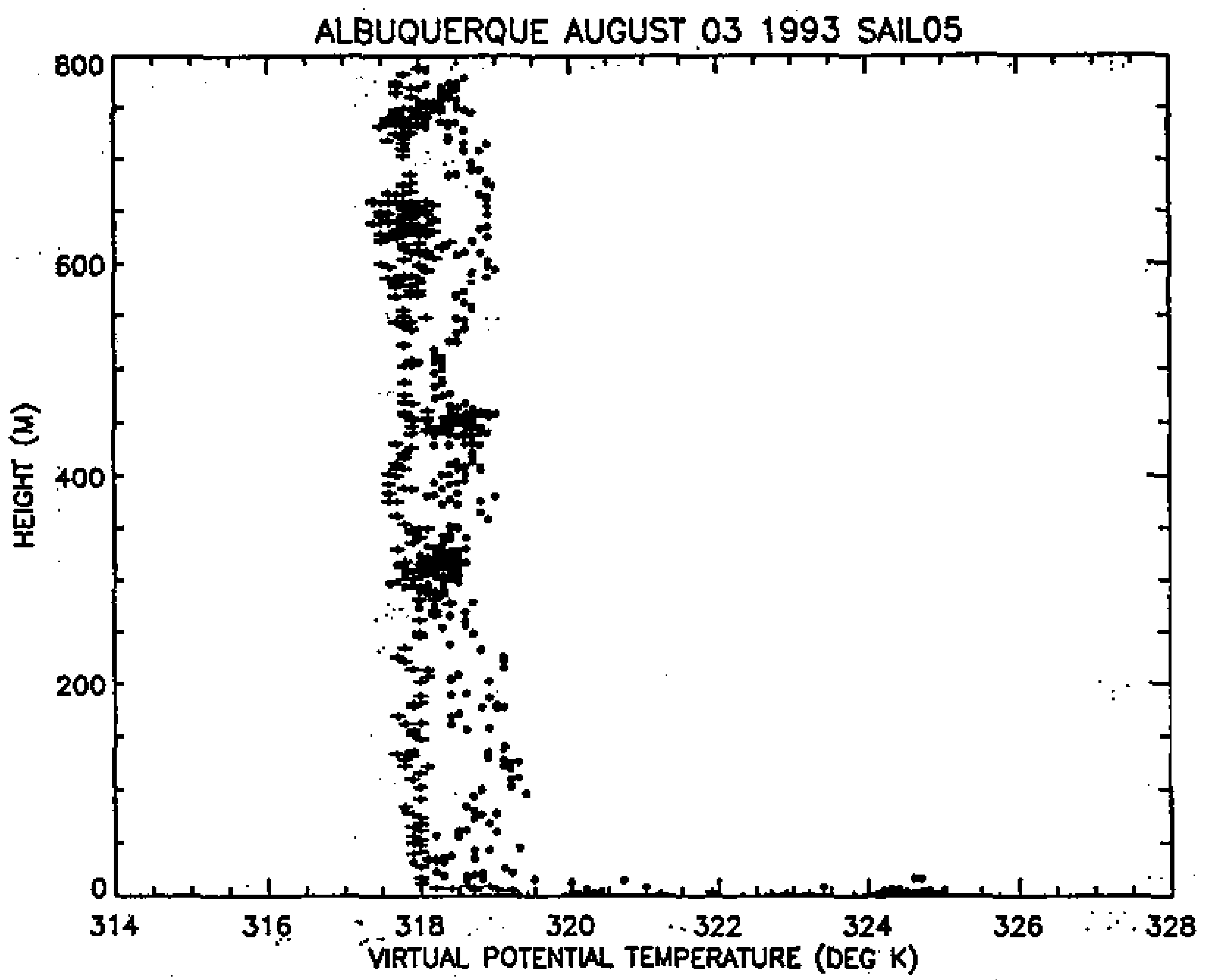
$$\rightarrow \quad \left({dT}/{dz}\right)_s = -{g}/{c_p} \equiv -\Gamma_d$$

$$\Gamma = {}^{g}\!/\!{}_{c_p}$$

Earth's atmosphere: $$\Gamma = {}^{1\,K}\!/\!{}_{100\,m}$$



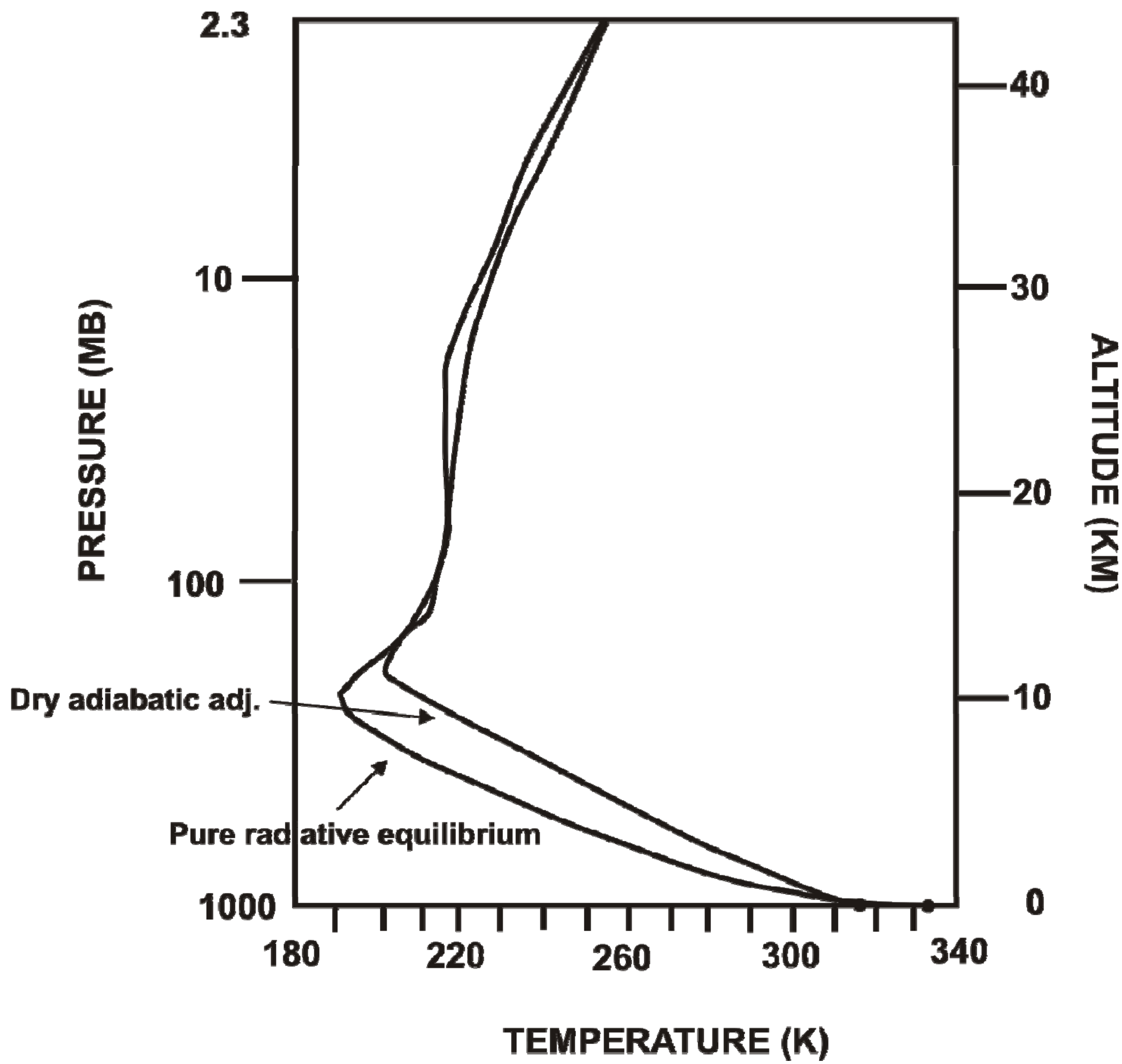
Model Aircraft Measurements (Renno and Williams, 1995)



Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium



Better, but still too hot at surface, too cold at tropopause

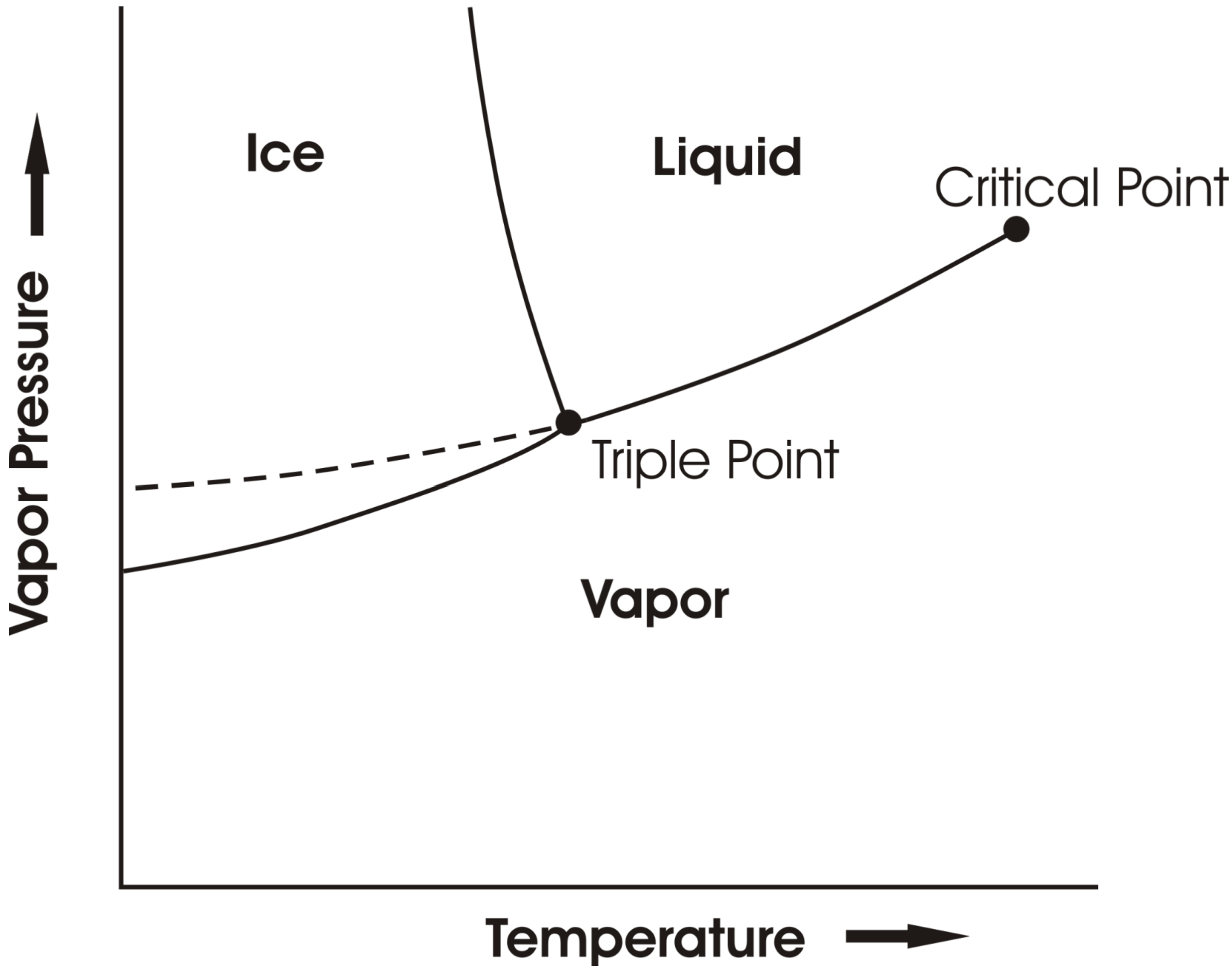
Above a thin boundary layer, most atmospheric convection involves phase change of water:

Moist Convection

Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

Phase Equilibria



When Saturation Occurs…

- Heterogeneous Nucleation
- Supersaturations very small in atmosphere
- Drop size distribution sensitive to size distribution of cloud condensation nuclei

Precipitation Formation

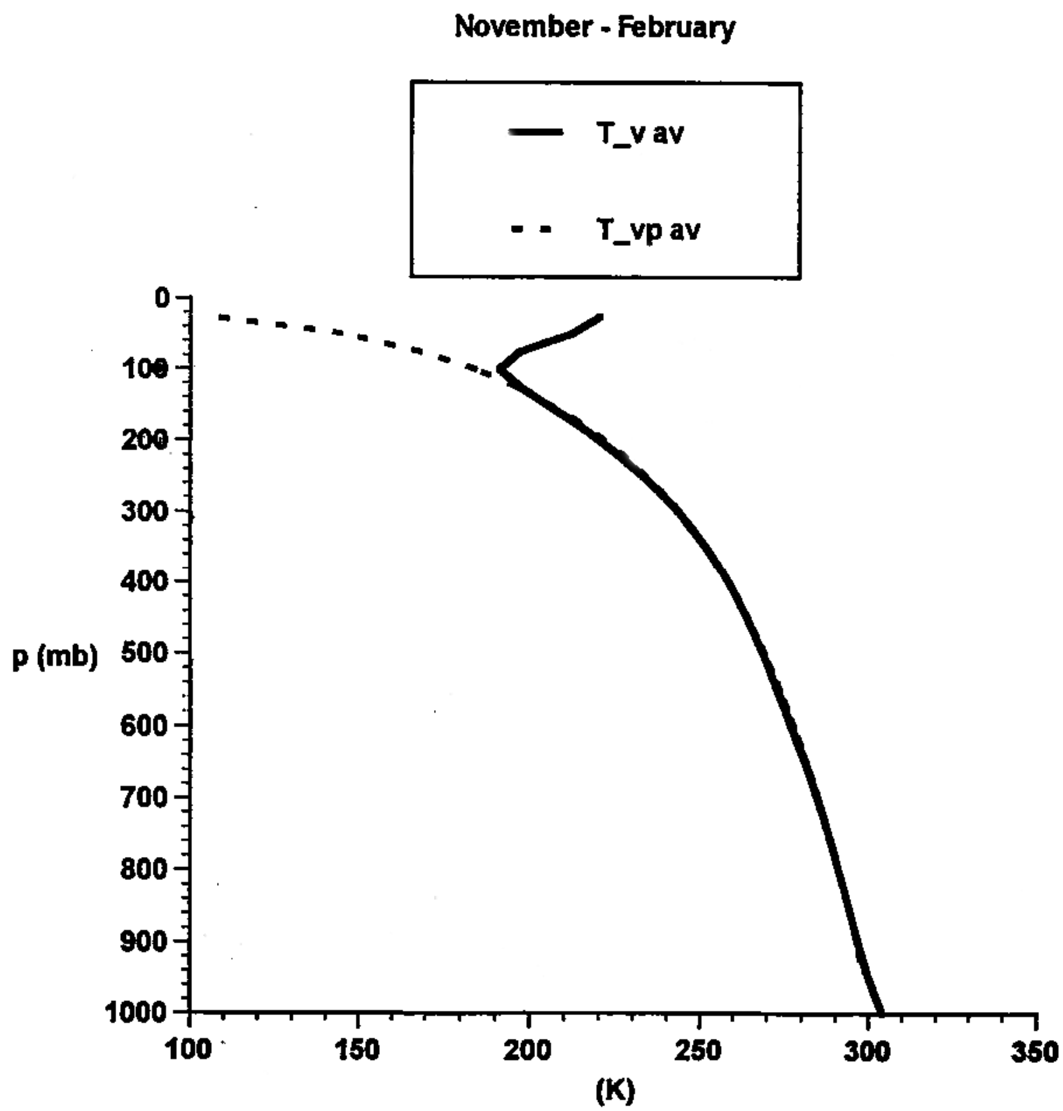
- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes

Stability

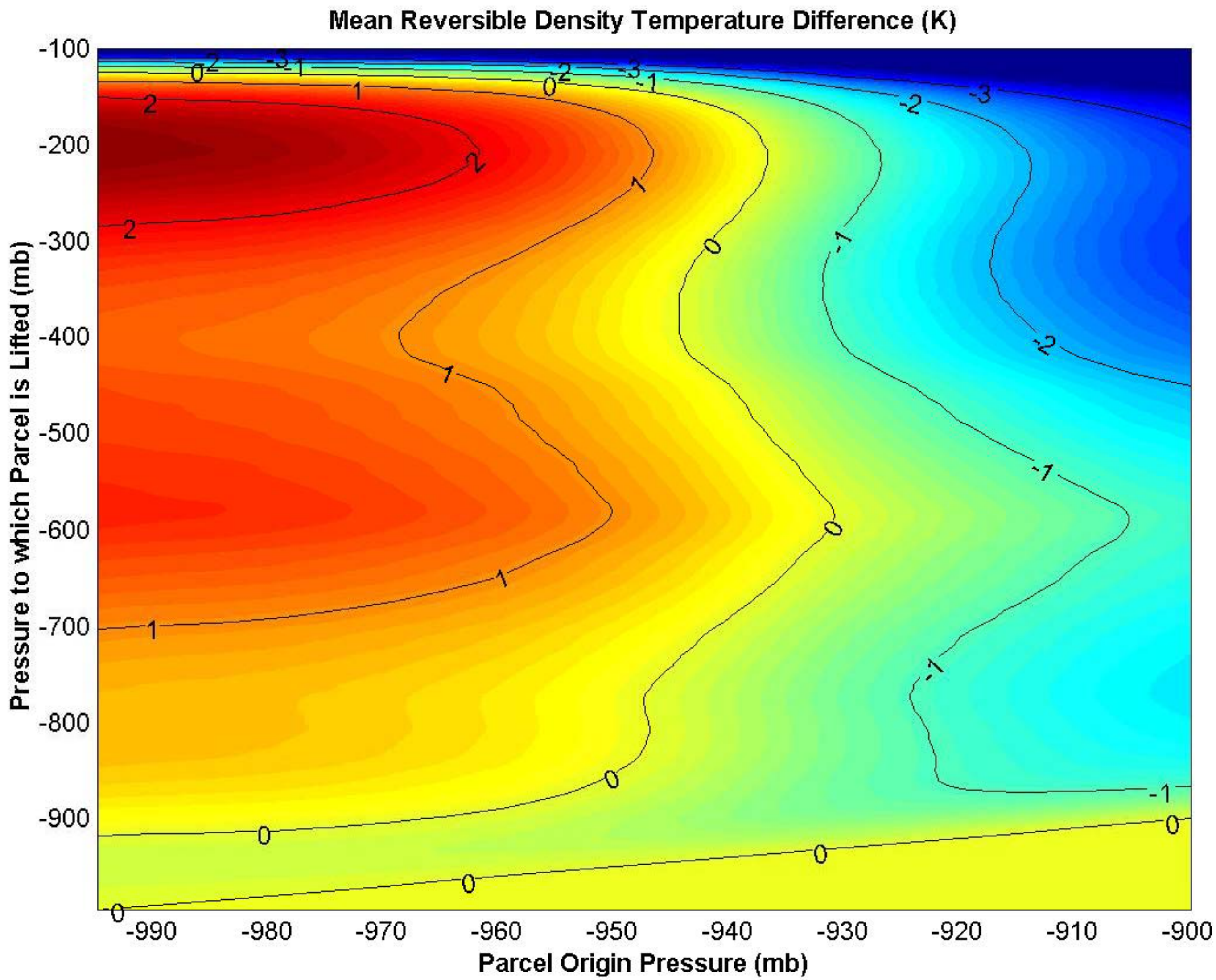
No simple criterion based on entropy. But air inside ascending cumulus turrets has roughly the same density as that of it environment. It can be shown that neutral stability corresponds to the constancy of the *saturation entropy* s*:

$$s^* = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right) + L_v \frac{q^*(T,p)}{T}$$

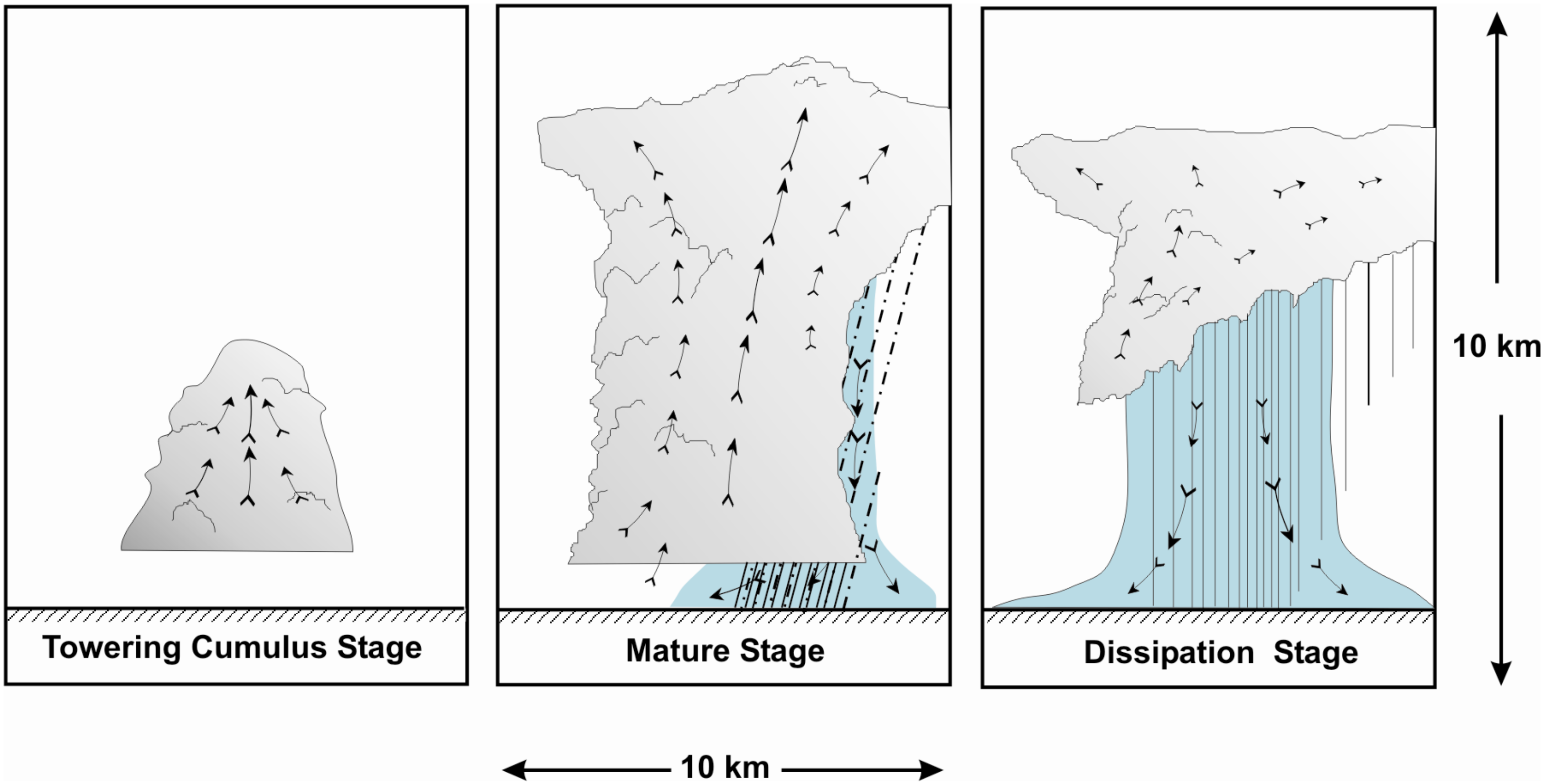
Tropical Soundings



Average density difference between reversibly lifted parcels and their environments, deep Tropics



“Air-Mass” Showers:



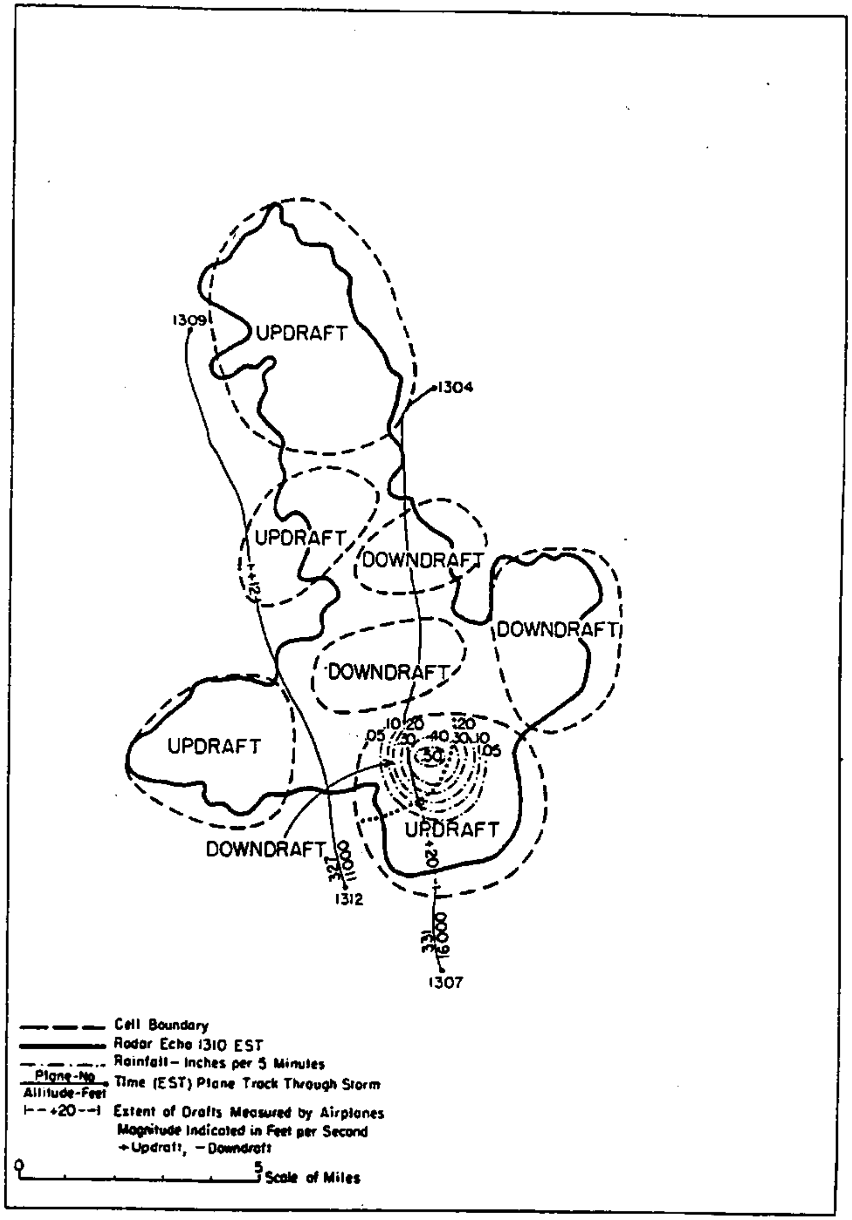


FIG. 15. Radar echo, plane paths, measured draft data, and cell outlines, 1310 EST 9 July 1946.

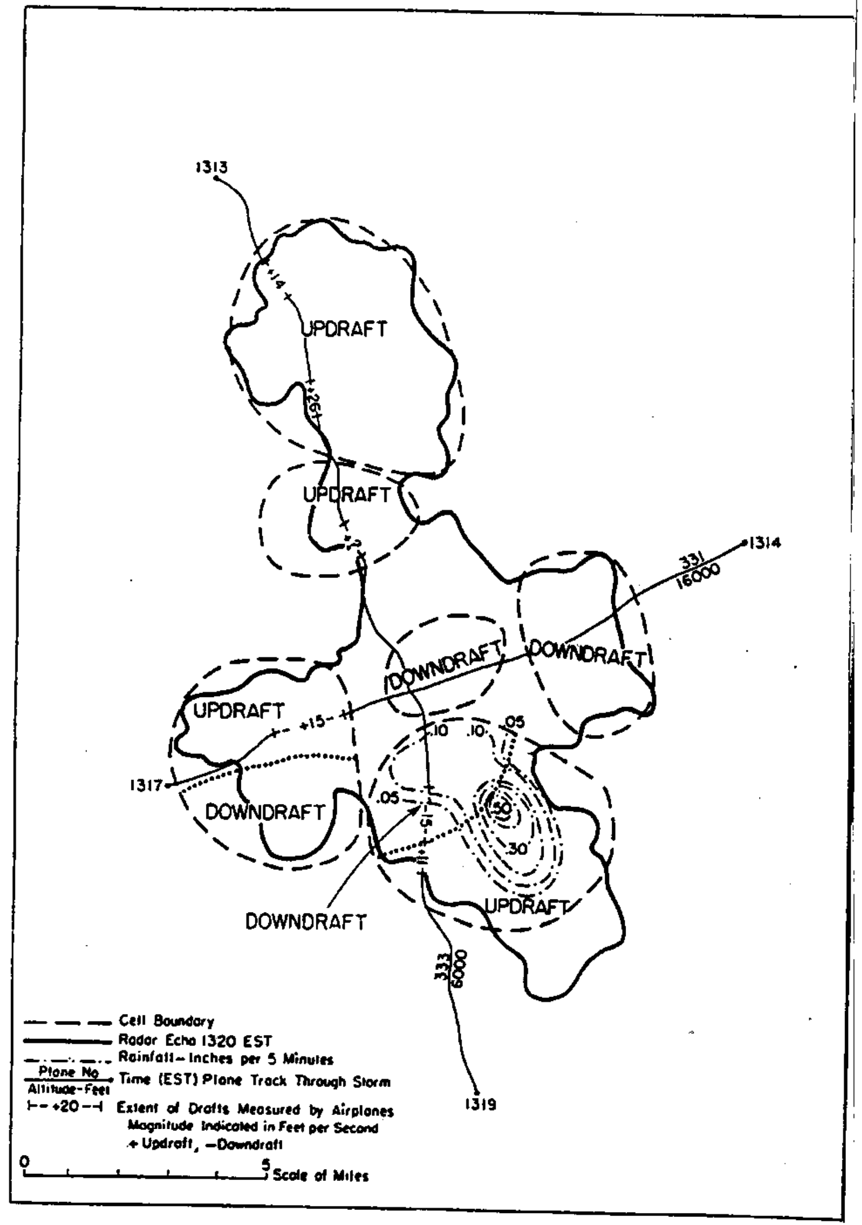
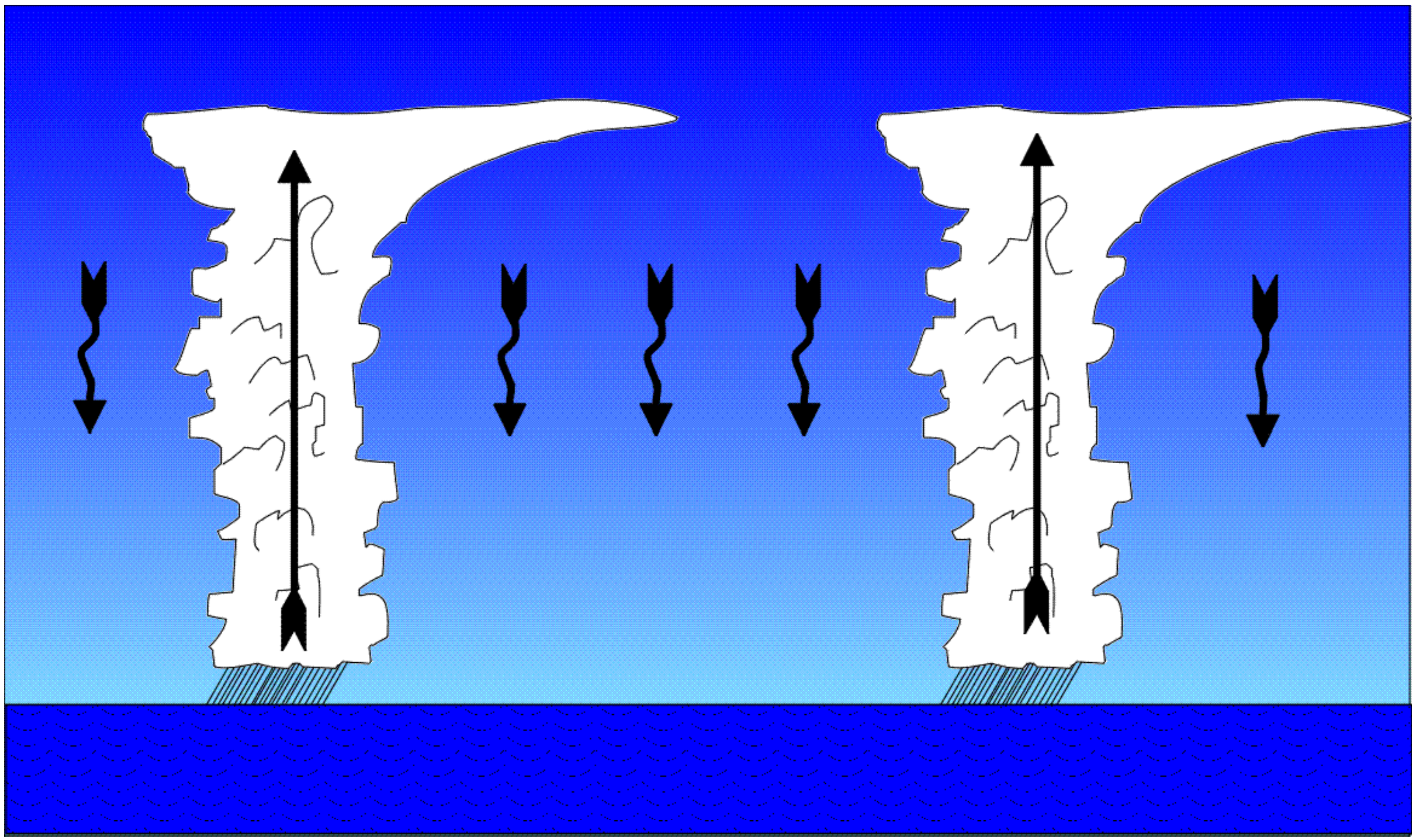


FIG. 16. Radar echo, plane paths, measured draft data, and cell outlines, 1320 EST 9 July 1946.

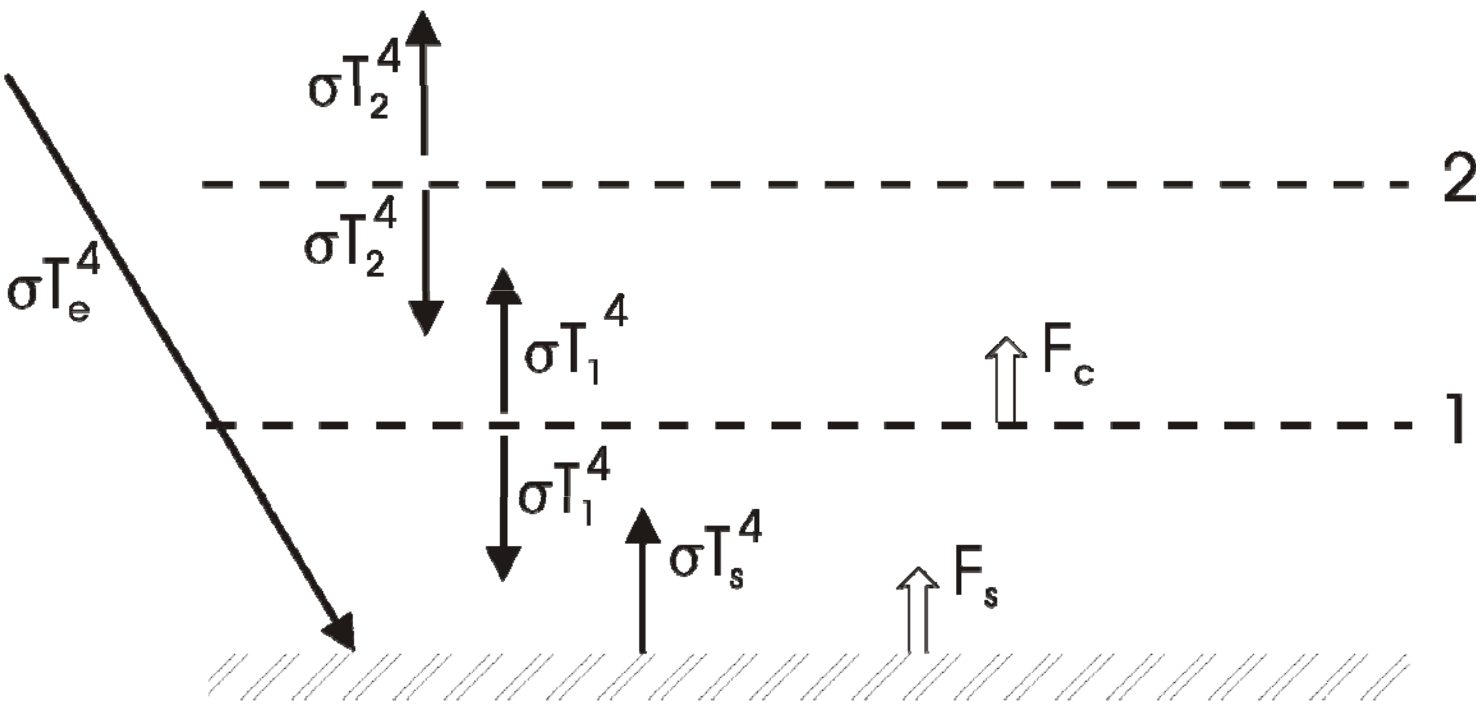
Precipitating Convection favors Widely Spaced Clouds (Bjerknes, 1938)



Properties of Moist Convection

- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
- $$M \frac{c_p T}{\theta} \frac{\partial \theta}{\partial z} = -\dot{Q}$$
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive

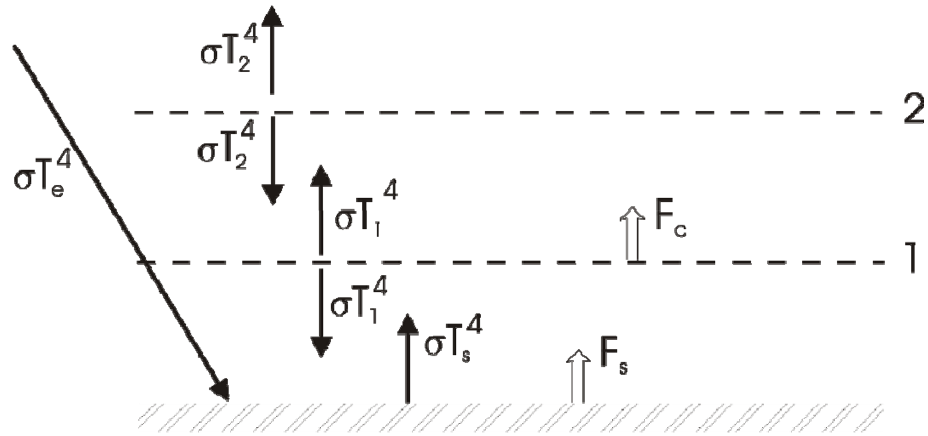
Simple Radiative-Convective Model



Enforce convective neutrality:

$$T_1 = T_2 + \Delta T,$$

$$T_s = T_2 + 2\Delta T$$



$$TOA: \quad T_2 = T_e \rightarrow T_1 = T_e + \Delta T, \quad T_s = T_e + 2\Delta T$$

$$Surface: \quad F_s + \sigma T_s{}^4 = \sigma T_e{}^4 + \sigma T_1{}^4$$

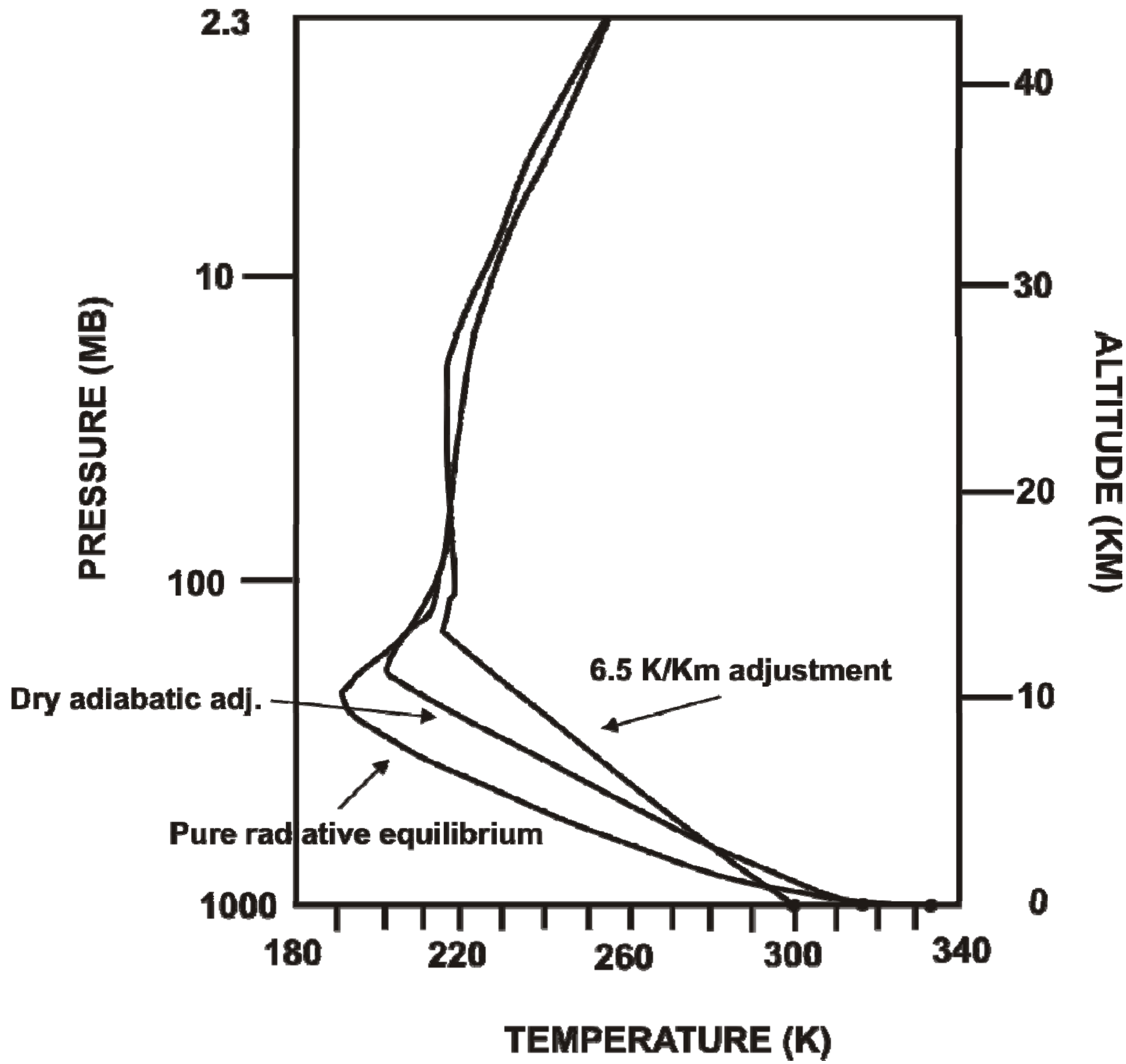
$$Layer\ 2: \quad 2\sigma T_e{}^4 = \sigma T_1{}^4 + F_c$$

$$Define \quad x \equiv \Delta T \big/ T_e ,$$

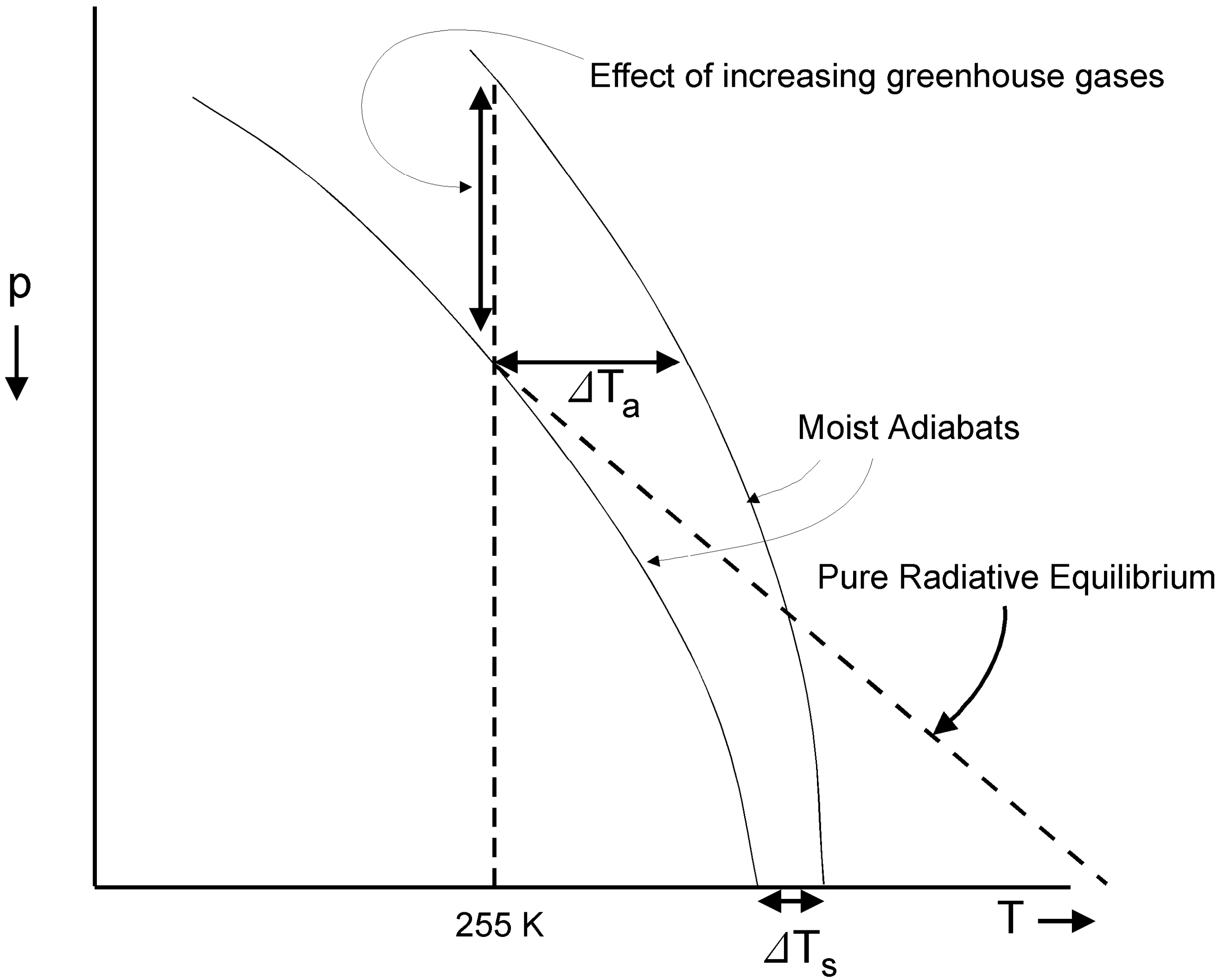
$$F_s = \sigma T_e{}^4 \left[1 + (1+x)^4 - (1+2x)^4 \right],$$

$$F_c = \sigma T_e{}^4 \left[2 - (1+x)^4 \right]$$

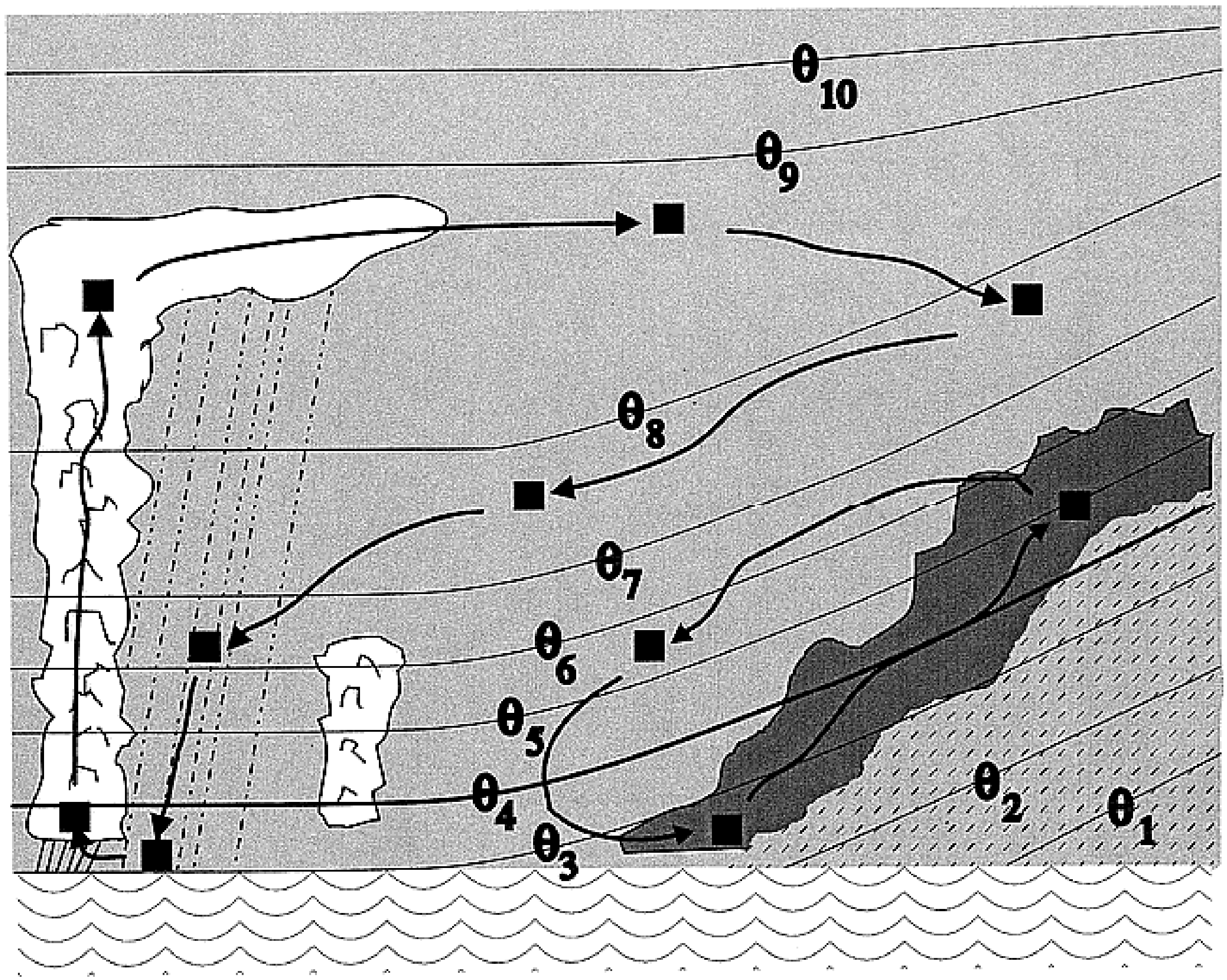
Manabe and Strickler 1964 calculation:

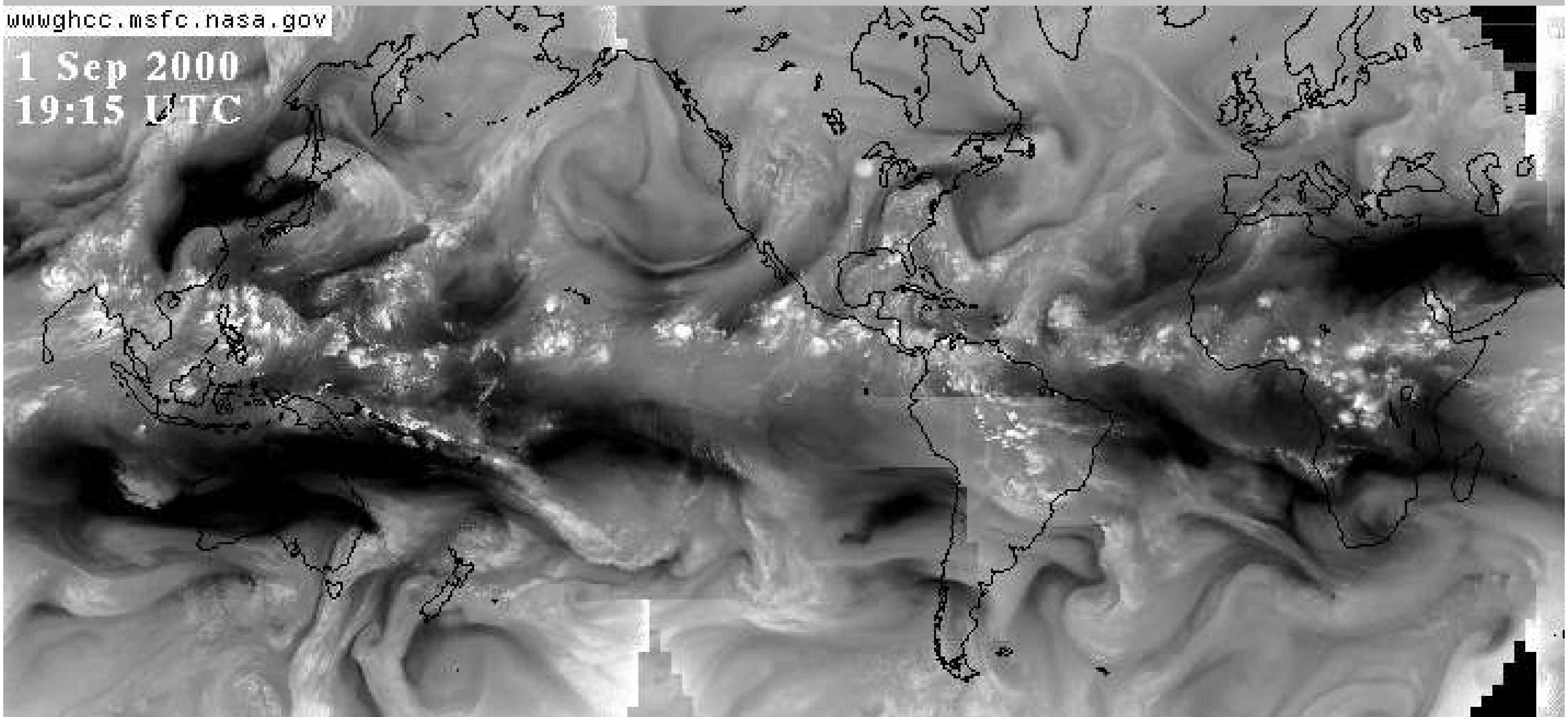


Effect of Moist Convective Adjustment on Climate Sensitivity

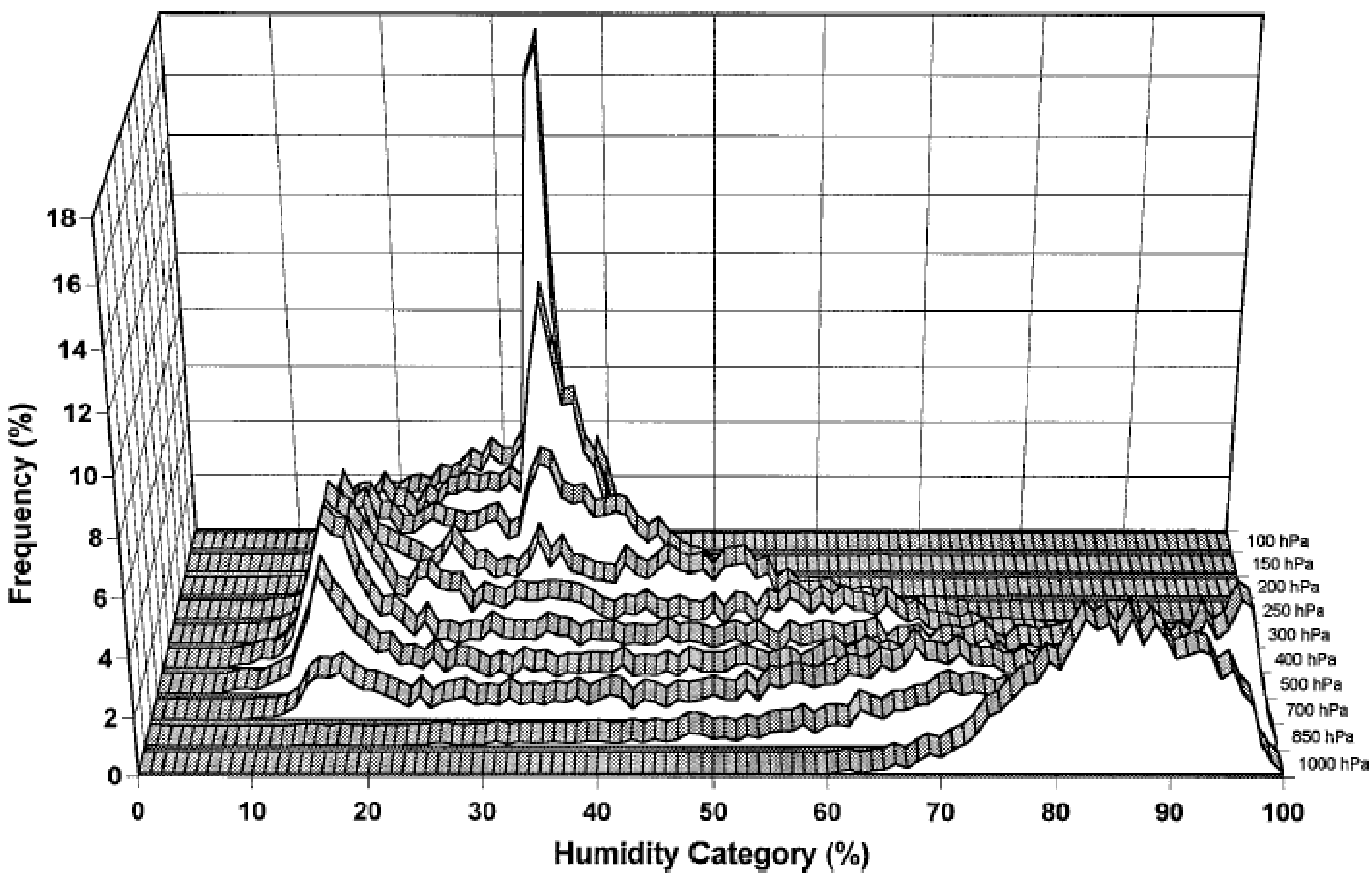


Flux of water by convection makes real problem complex




wwwghcc.msfc.nasa.gov
1 Sep 2000
19:15 UTC

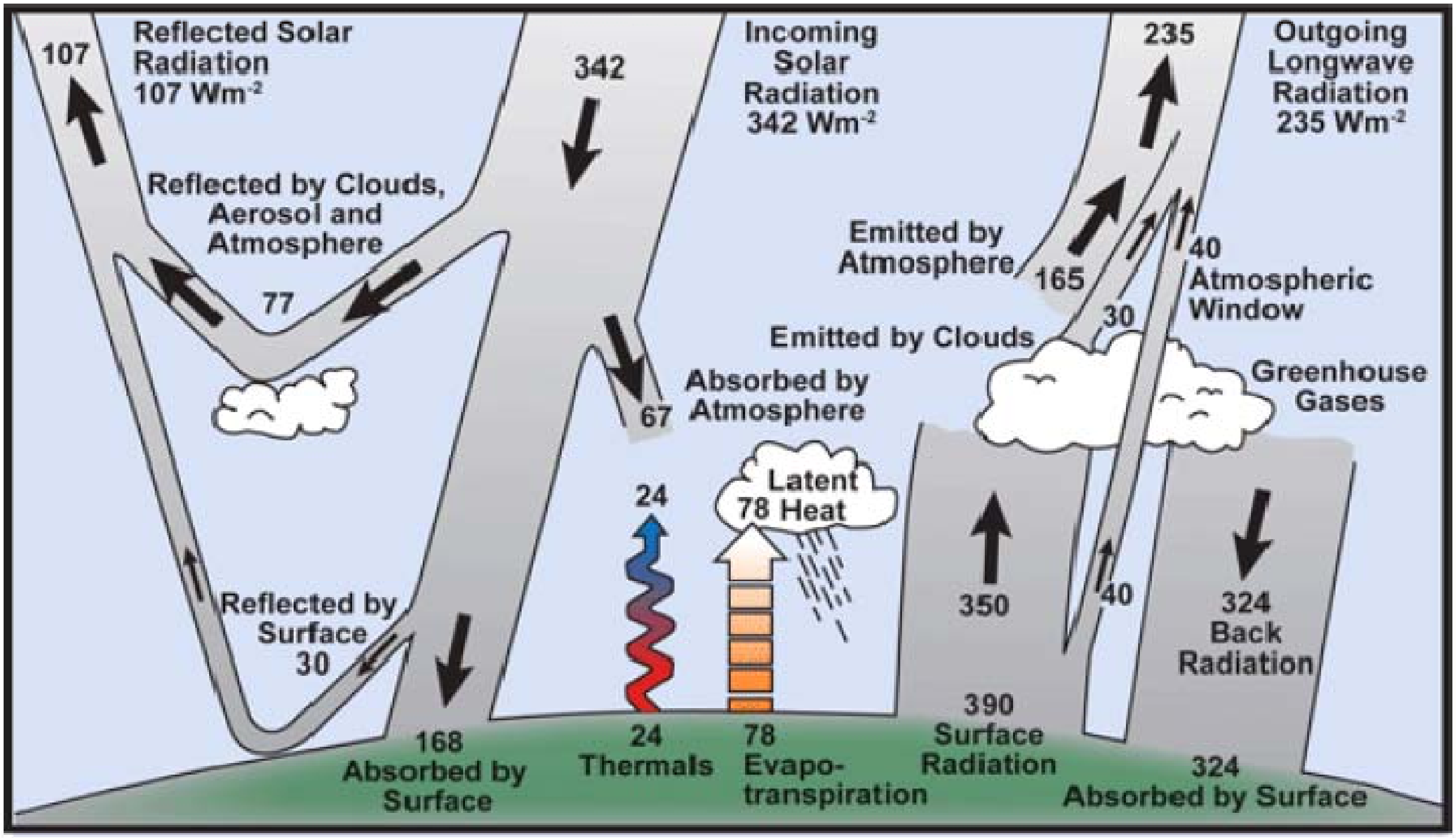
Frequency histogram of rawindsonde relative humidities from 1600 ascents at the tropical Pacific islands of Yap, Koror, Ponape and Majuro, January-May, 1994-95. Spencer and Braswell, *Bull. Amer. Meteor. Soc.,* 1997.

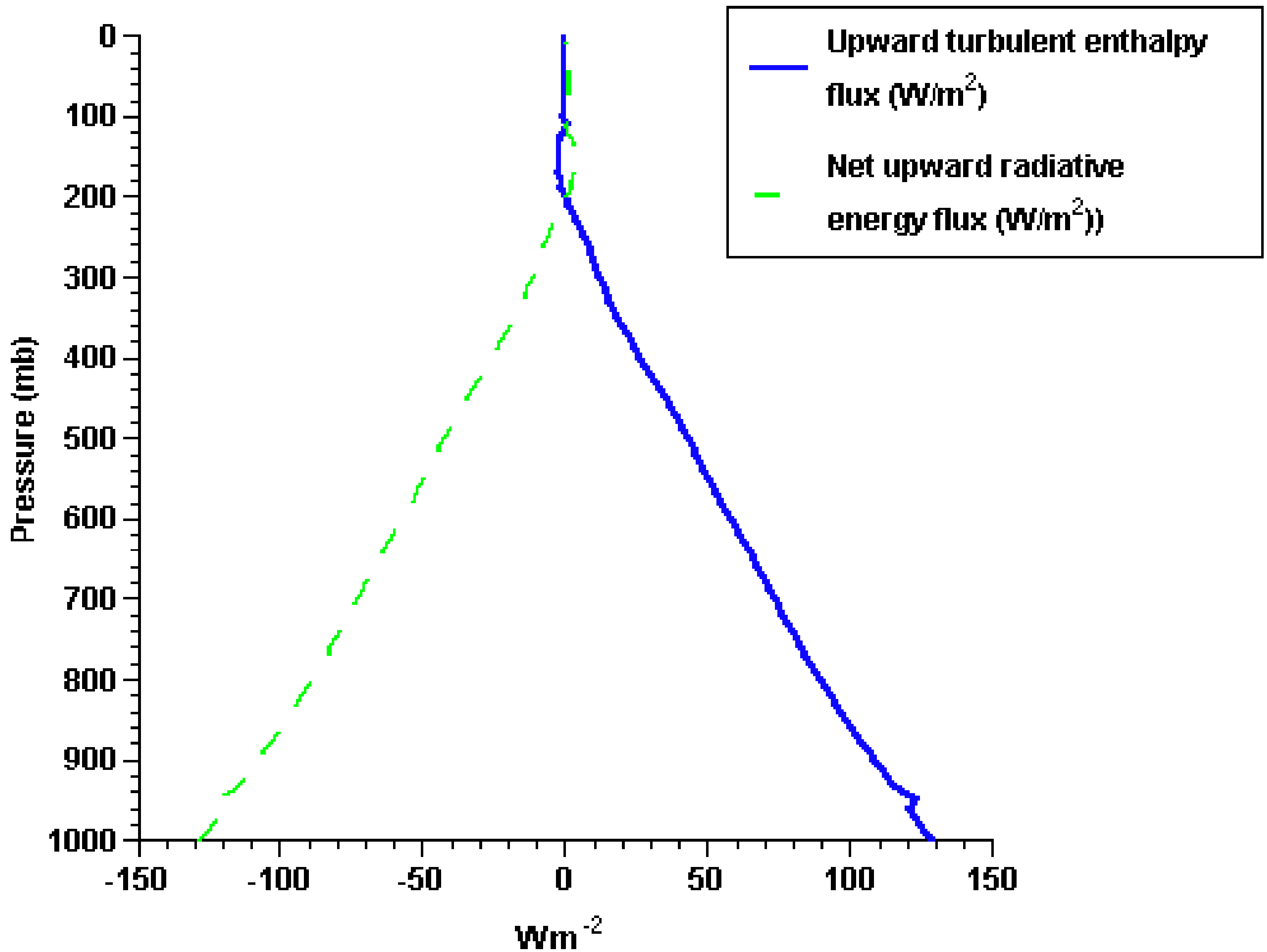


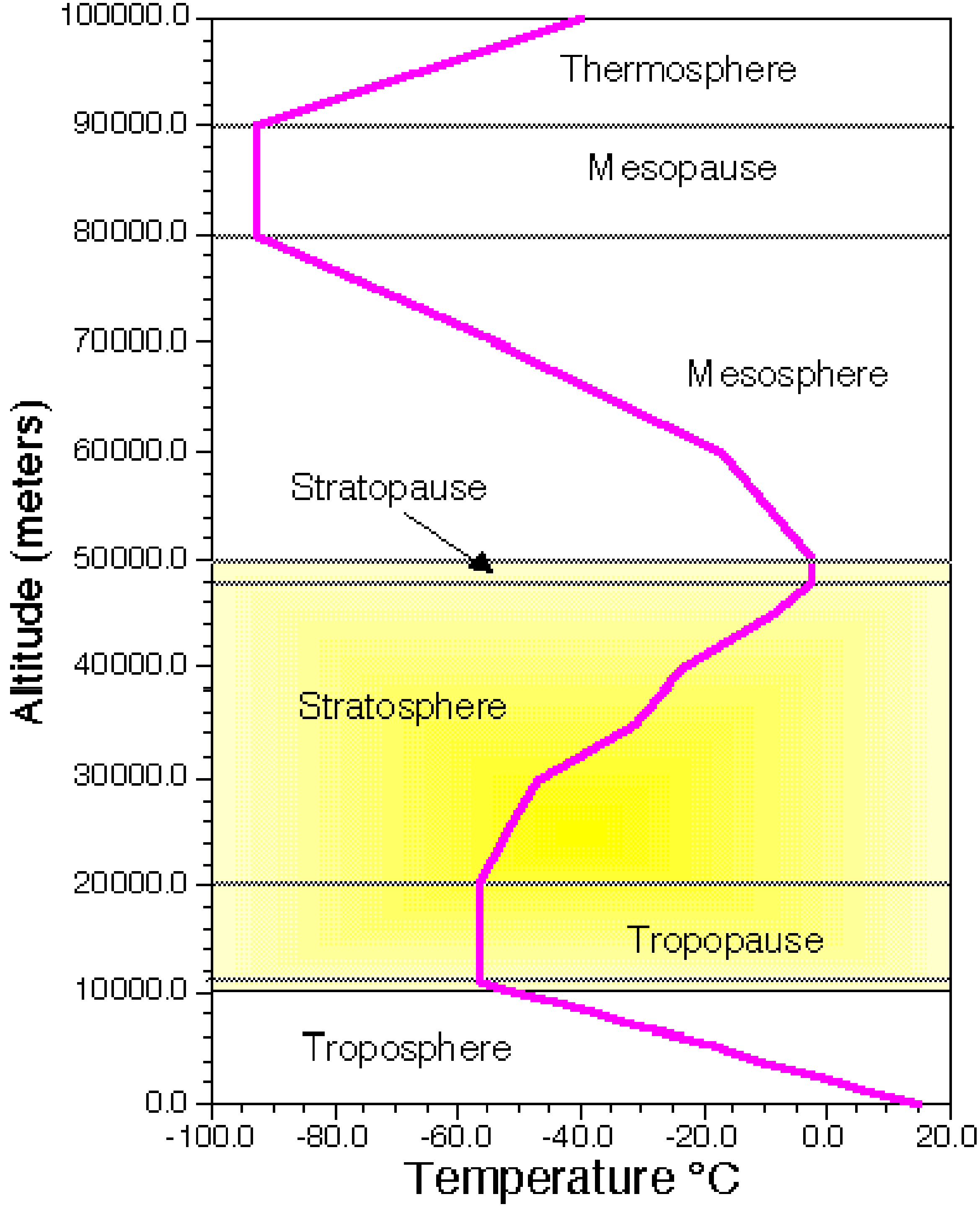
Effects of Clouds on Radiative Transfer

- Responsible for much of Earth's albedo
- Important greenhouse effect from longwave absorption and re-emission

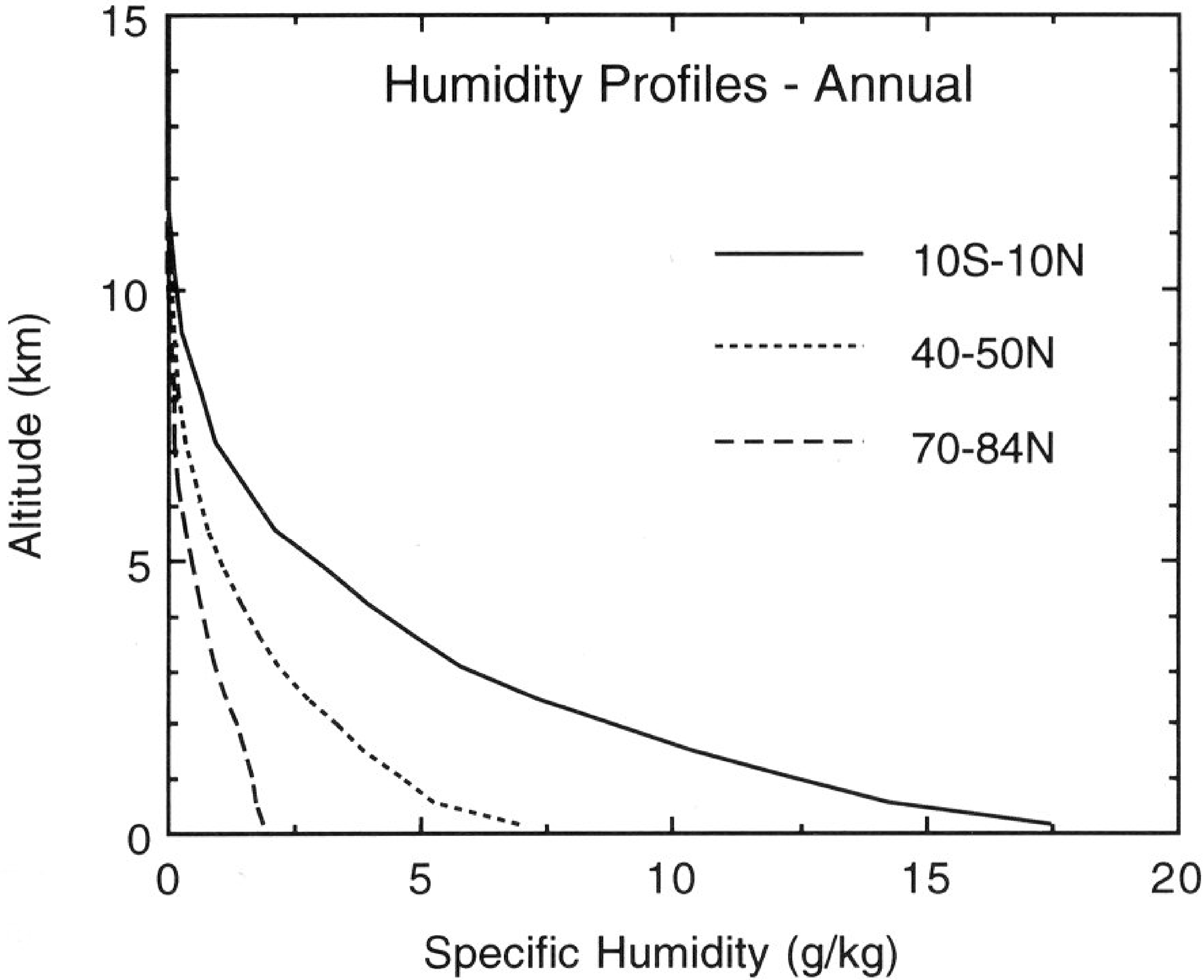
Globally Averaged Energy Balance




Upward turbulent enthalpy flux (W/m^2)
Net upward radiative energy flux (W/m^2))
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-150
-100
-50
0
50
100
150
Wm^{-2}



100000.0
90000.0
80000.0
70000.0
60000.0
50000.0
40000.0
30000.0
20000.0
10000.0
0.0
Altitude (meters)
Thermosphere
Mesopause
Mesosphere
Stratopause
Stratosphere
Tropopause
Troposphere
-100.0
-80.0
-60.0
-40.0
-20.0
0.0
20.0
Temperature °C


Humidity Profiles - Annual
10S-10N
40-50N
70-84N
Altitude (km)
Specific Humidity (g/kg)
0
5
10
15
20

Radiative-Convective Model

- Band-averaged radiation
- Radiation interacts with H_20, CO_2, O_3, clouds
- Two-stream approximation: radiation assumed to travel vertically
- Moist convection whenever instability exists
- Convection transports H_20, enthalpy
- Representation of layered clouds

Open MATLAB
Type “run_model”

- 1) Restart from last run? [n]
- 2) Interactive radiation? [y]
- 3) Interactive clouds? [y]
- 4) Interactive surface temp? [n]
- 5) Time-dependent radiation? [n]
- 6) Date-dependent radiation? [n]
- 7) Diurnal-average radiation? [n]
- 8) Annual-average radiation? [y]
- 9) Surface albedo [0.32]
- 10) Amount of CO2 [360 ppm]
- 11) End time of integration [100 days]
- 12) Graphics averaging time [1 days]
- 13) Frequency of radiation calls [3 hours]
- 14) Frequency of graphics output [2 hours]

- 0) Run model with current configuration.