# A Similarity Hypothesis for Air–Sea Exchange at Extreme Wind Speeds

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ABSTRACT

Hurricane intensity is sensitive to fluxes of enthalpy and momentum between the ocean and atmosphere in the high wind core of the storm. It has come to be recognized that much of this exchange is likely mediated by sea spray. A number of representations of spray-mediated exchange have appeared in recent years, but when these are applied in numerical simulations of hurricanes, storm intensity proves sensitive to the details of these representations. Here it is proposed that in the limit of very high wind speed, the air–sea transition layer becomes self-similar, permitting deductions about air–sea exchange based on scaling laws. In particular, it is hypothesized that exchange coefficients based on the gradient wind speed should become independent of wind speed in the high wind limit. A mechanistic argument suggests that the enthalpy exchange coefficient should depend on temperature. These propositions are tested in a hurricane intensity prediction model and can, in principle, be tested in the field.

## 1. Introduction

Basic theory (e.g., Emanuel 1986) and numerical experiments (Ooyama 1969; Rosenthal 1971; Emanuel 1995a) show that the intensity of tropical cyclones depends strongly on the coefficients for the transfers of momentum ($C_D$) and enthalpy ($C_k$) between the ocean and the atmospheric boundary layer. The maximum wind speed, in particular, depends on $(C_k/C_D)^{1/2}$ in the high wind speed core of the storm (Emanuel 1986). Unfortunately, there are no simultaneous measurements of the effective values of these coefficients at wind speeds greater than about 25 m s$^{-1}$, and the theory of air–sea interaction at very high wind speeds is poorly developed. The agitated sea no doubt increases the effective roughness length and, thereby, $C_D$, and the dissipation rate of kinetic energy; while, for wind speeds up to about 20 m s$^{-1}$, there is little observational evidence to suggest a corresponding increase in $C_k$ (Geernaert et al. 1987). Emanuel (1995a) showed that if estimated values of the exchange coefficients at 20 m s$^{-1}$ are applied at higher wind speeds, maintaining a storm of much greater than marginal hurricane intensity would be impossible. Some mechanism must also serve to enhance air–sea enthalpy exchange at high wind speed.

One candidate for enhancing the sea–air enthalpy flux at high wind speeds is sea spray. Riehl (1954, p. 287) was perhaps the first to suggest that sea spray supplies a significant amount of heat for generating and maintaining tropical storms. Laboratory studies (e.g., Mestayer and Lefauconnier 1988), numerical spray droplet models (e.g., Rouault et al. 1991; Edson et al. 1996; van Eijk et al. 2001), and open-ocean observations (Korolev et al. 1990) all show that sea spray can redistribute enthalpy between the temperature and humidity fields in the marine boundary layer (MBL).

Emanuel (1995a) argued that sea spray could not affect enthalpy transfer, because droplets that completely evaporate absorb as much sensible heat as they give off in latent heat. Several parameterizations of sea spray (e.g., Fairall et al. 1994) also showed little net enthalpy transfer, and numerical simulations of hurricanes based on these (e.g., Kepert et al. 1999; Uang 1999; Wang et al. 1999) showed little effect of spray on storm intensity. Then Andreas and Emanuel (1999) considered the effect of re-entrant sea spray and concluded that a large sea–air enthalpy transfer could result from re-entrant spray. Andreas and Emanuel (2001) considered spray effects on momentum transfer and concluded that this effect could be large as well. They developed a new parameterization of air–sea fluxes at high wind speed and showed that numerical simulations of hurricanes are very sensitive to the details of such parameterizations. Several other numerical simulations (Bao et al. 2000; Wang et al. 2000, 2001) confirm that re-entrant sea spray can strongly affect hurricane intensity.

The conclusion of Andreas and Emanuel (2001) that the intensity of simulated hurricanes is very sensitive to the details of the spray formulations as well as to wave-induced drag is particularly disturbing. One in-

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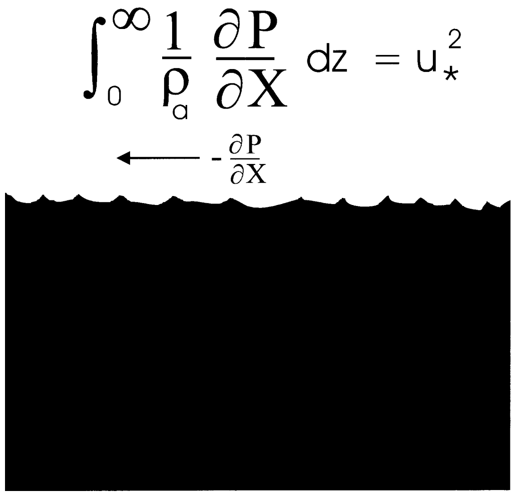


FIG. 1. Idealized experimental setup. A uniform, infinitesimal, horizontal pressure gradient is applied throughout a semi-infinite gas with no buoyancy stratification. This integrated momentum source is balanced by a momentum flux into the water. A slight slope of the water surface provides the net momentum sink needed to balance the atmospheric source.

ference that follows from this conclusion is that accurate hurricane intensity forecasting will not be possible without coupling hurricane models to detailed prognostic surface wave models. While this may indeed prove to be the case, it is at odds with the findings of Emanuel (1999), who showed that many hurricanes can be accurately hindcast using a simple coupled model with equal exchange coefficients for enthalpy and momentum.

In this paper, I hypothesize that at high enough wind speeds, the sea–air transition is governed by a single length scale, implying that exchange coefficients based on a gradient wind become independent of wind speed. This idea is presented in section 2. In section 3, a mechanistic argument is advanced to estimate the ratio of the enthalpy and momentum exchange coefficients. Provisional formulations of the exchange coefficients are proposed in section 4 and tested using a simple hurricane intensity prediction model. Concluding remarks are presented in section 5.

## 2. Dimensional analysis

### *a. Nonrotating, saturated system*

Consider the idealized air–water system sketched in Fig. 1. A semi-infinite layer of water-saturated air overlays a semi-infinite body of fresh water. Neither fluid is buoyancy stratified, there is no temperature jump at the air–water interface, and both fluids experience a constant gravitational acceleration $g$. Initially, we take the air to be saturated to avoid evaporation effects. The air is subjected to a constant, infinitesimal horizontal pressure gradient acceleration, such that

$$\int_0^\infty \frac{\partial p}{\partial x}\,dz = \text{constant} = \rho_a u_*^2, \qquad (1)$$

where $\rho_a$ is the air density at the surface, $\partial p/\partial x$ is the horizontal pressure gradient, and $\rho_a u_*^2$ is the flux of momentum from the air to the water. The relation (1) simply states that, in equilibrium, the vertically integrated momentum source to the air must be balanced by a momentum flux into the water. The mean altitude of the surface of the water, which we take to be incompressible, has an infinitesimal slope given by

$$\int_{-\infty}^0 \rho_l g \frac{\partial h}{\partial x}\,dz = -\rho_a u_*^2,$$

where $\rho_l$ is the water density and $h$ is the mean altitude of the water surface. Such a water surface slope guarantees that the surface stress is balanced by a horizontal pressure gradient in the water. While the infinitesimal slope of the water surface enforces momentum balance, it does not otherwise play a role in the exchange processes we consider here.

The control parameters of this fluid system are considered to be $g$ and $u_*$, the densities of water and air $\rho_l$ and $\rho_a$, the kinematic surface tension $\sigma$, and the kinematic viscosities of water and air $\nu_l$ and $\nu_a$. In the governing equations, gravity only enters as a multiplier of $\Delta\rho/\rho_l$, where $\Delta\rho$ is the difference between the densities of the two fluids. Thus, we consider a slightly reduced gravity

$$g' \equiv g\frac{\Delta\rho}{\rho_l}$$

to be a single control parameter. Since the density of water is three orders of magnitude greater than that of air, this reduced gravity is very nearly equal to $g$.

There are three important length scales in this system: a Charnock-like length scale, a length scale governing the molecular diffusion of momentum, and an equilibrium drop size, assumed to pertain to sea spray, determined by gravity and surface tension. These are, respectively,

$$l_c = \frac{u_*^2}{g'}, \qquad l_\nu = \frac{\nu_a}{u_*}, \quad \text{and} \quad l_\sigma = \sqrt{\frac{\sigma}{g'}}. \qquad (2)$$

Here we note that the Charnock length increases rapidly with $u_*$, while the diffusive length scale decreases with $u_*$ and the equilibrium drop size remains constant. The Charnock length scale governs the length scales associated with surface gravity waves (Charnock 1955). We form three dimensionless combinations of the control parameters. We choose these to be, respectively, the square of the ratio of the Charnock length to a measure of the maximum stable size of a spray droplet in a gravitational field (e.g., see Pruppacher and Klett 1997, p. 411), a nondimensional measure of surface tension, and the ratio of the kinematic viscosities of the two fluids:

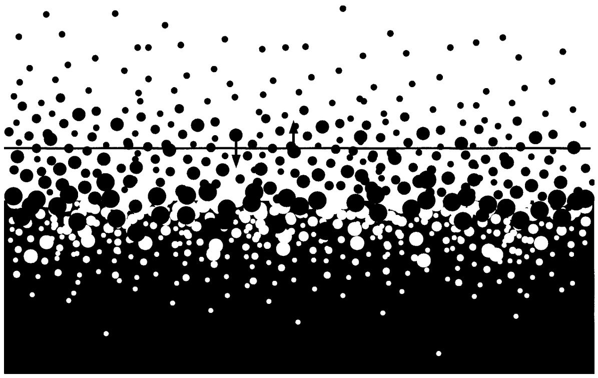

FIG. 2. Momentum and enthalpy transfer through an emulsion. Spray droplets are ejected upward and accelerate toward the free stream velocity, absorbing momentum from the atmosphere.

$$R_u \equiv \frac{u_*^4}{\sigma g'}, \qquad R_\sigma \equiv \frac{\sigma}{\nu_l^{4/3} g'^{1/3}}, \qquad R_\nu \equiv \frac{\nu_l}{\nu_a}. \qquad (3)$$

The last two of these depend only on gravity and the molecular properties of the fluids. Thus, for a given gravitational acceleration, and for the two given fluids (water and air, in this case), $R_\sigma$ and $R_\nu$ are constants.

Note that as the applied wind stress increases, $R_u$ becomes larger. The central hypothesis of this paper is that as the wind stress becomes very large, $R_u$ becomes so large as to be irrelevant to the system dynamics. This is equivalent to supposing that the Charnock length becomes large compared to the typical scale of spray drops, and that air–sea transfer is dominated by spray physics. This is only a hypothesis and must be subject to experimental evaluation. If the hypothesis is correct, then at very large wind speeds, the character of the system must become independent of $R_u$ and thus of $u_*$. It follows that all variable length scales (such as the height of the spray layer) must scale as the Charnock length $u_*^2/g'$, and all variable timescales must scale as $u_*/g'$.

It is clear from simple visual observation that the sea surface is not self-similar at wind speeds much below hurricane force. The Beaufort scale, for example, makes use of the changes in the visual appearance (not just the scales) of the sea surface with increasing wind. (For example, the percentage of the sea surface covered with foam progressively increases with wind speed.) It would appear that the parameter $R_u$ is influential over the range of wind conditions in which the Beaufort scale was commonly used, that is, up to hurricane force. Visual observations at sea during severe tropical cyclones describe spray-filled air and suppressed wind waves, suggesting that a qualitatively different regime prevails at very high wind speeds. We hypothesize that the large $R_u$ limit is achieved at the upper end of the Beaufort scale, when and if the following conditions apply:

- it is no longer possible to define an interface corresponding to the sea surface, as illustrated in Fig. 2;
- the column-integrated surface area of spray droplets per unit horizontal area is a large number; and
- the depth of the spray layer is large compared to other length scales, such as the radius of spray droplets.

As sketched in Fig. 2, we envision that under these extreme conditions the concept of an air–sea interface becomes problematic, and there is a gradual transition from bubble-filled water to spray-filled air. We here present a physical argument for the dominant importance of the Charnock scale under these conditions.

Suppose, first, that the characteristic air velocities in the spray layer are much larger than the characteristic terminal velocities of the spray droplets, so that the latter are lofted by the turbulence. According to Pruppacher and Klett (1997), the terminal velocity of the largest drop that is stable in a gravitational field scales as

$$V_T \sim (g\sigma)^{1/4},$$

so that the ratio of the characteristic gust speed $u_*$ to this maximum terminal velocity is

$$\frac{u_*}{(g\sigma)^{1/4}} \approx R_u^{1/4}.$$

Thus the condition for significant lofting of spray by turbulence is that $R_u$ must be large. Conversely, in this limit, we may consider that all the spray is falling (with respect to the air) at or near its terminal velocity. Gravitational setting will cause the concentration of liquid water to increase downward (Lighthill 1999). If it does so too quickly, the resulting effective stability of the air column to vertical displacements will be such that the Richardson number based on it will be supercritical, and turbulence will cease. We hypothesize that the thickness of the transition from pure water to pure air (or, to within a numerical factor, the thickness of the spray layer) is such that the Richardson number based on it is near a critical value. The buoyancy frequency of air containing an upward decreasing suspension of liquid water droplets with mass concentration given by $l$ is $\sqrt{g(\partial l/\partial z)}$, so the Richardson number based on this is

$$\mathrm{Ri} = \frac{g(\partial l/\partial z)}{(\partial u/\partial z)^2}.$$

Taking this Richardson number to be equal to a critical value $\mathrm{Ri}_c$ in the spray layer, the characteristic depth over which the water concentration decreases from unity (its value in pure water) to zero, while the velocity increases from zero to $u_*$, is

$$\delta z_{\mathrm{sp}} = \mathrm{Ri}_c \frac{u_*^2}{g}.$$

This provides a physical hypothesis for why the depth of the spray layer scales as the Charnock length in the limit of large $R_u$.

If the similarity hypothesis holds, several interesting conclusions follow. The air–sea flux of any passive scalar $s$ with well-defined asymptotic concentrations for $z \to \pm\infty$ must follow

$$F_s = Cu_*(s_o - s_a), \tag{4}$$

where $s_o$ and $s_a$ are the values of the passive scalar deep in the water and high in the air, respectively, and $C$ is an exchange coefficient that is itself independent of $u$* (but which may depend on $R_\sigma$ and $R_\nu$ and thus on the properties of the water. For example, $C$ could be different for saltwater.).

We assume that altitude is scaled by the Charnock length rather than by the microscopic length scales in (2). For example, well into the spray layer, the air velocity should follow

$$u \sim u_* G\left(\frac{g'z}{u_*^2}\right), \tag{5}$$

where $G$ is some function. Experience suggests that for large $z$, $G$ should be a log function; indeed, it appears to be so in the case of sand lofted by wind (Janin and Cermak 1988). One consequence of (5) is that the drag coefficient based on wind at a fixed altitude (say, $z_a$) must be wind dependent:

$$u_*^2 = C_{Da}u_a^2 \sim C_{Da}u_*^2 G^2\left(\frac{g'z_a}{u_*^2}\right),$$

where $C_{Da}$ is the drag coefficient based on a fixed altitude wind. This implies that

$$C_{Da} = G^{-2}\left(\frac{g'z_a}{u_*^2}\right). \tag{6}$$

Therefore, if $G$ increases with its argument, then $C_{Da}$ should increase with $u_*$.

### *b. Extension to case with nonzero evaporation*

If we allow the air to be subsaturated, a nonzero upward water flux will ensue. Evaporation of spray droplets will cool the air, introducing a buoyancy flux in the system, and an additional length scale, which we take to be a Monin–Obukhov scale. For the similarity theory to hold, the Charnock length governing the depth of the spray layer must be small compared to this Monin–Obukhov length, so that we can be assured that buoyancy contributions to turbulence generation are negligible compared to shear generation.

An upper limit on the (downward) buoyancy flux can be estimated by taking the extreme limit in which all the spray evaporates. In that case, the heat of vaporization is supplied by some combination of a downward sensible heat flux from the air and an upward sensible heat flux from the sea; an upper bound is obtained by assuming that all the flux comes from the air. We suppose that the vapor flux satisfies the expression (4) for the flux of a passive scalar, subject to an a posteriori validation. Then, neglecting the direct effect of condensed or vapor phase water on density, the maximum downward buoyancy flux from (4) is

$$F_B = \frac{L_\nu g}{C_p T}Cu_*(q_w^* - q), \tag{7}$$

where $L_\nu$ is the latent heat of vaporization, $C_p$ is the heat capacity at constant pressure of air, $T$ is a representative air temperature, $q_w^*$ is the saturation specific humidity at undisturbed sea surface temperature and pressure, and $q$ is the specific humidity of the air at infinity. (Since some of the sea spray is re-entrant, the actual buoyancy flux will be less than this value. Note also that the buoyancy flux owing to the liquid water loading of the spray should not be included here as it is implicit in the problem with saturated air.) The Monin–Obukhov length associated with (7) is

$$L_{\mathrm{mo}} \equiv \frac{u_*^3}{kF_B} = \frac{u_*^2}{kg}\frac{C_pT}{L_\nu}\frac{1}{C(q_w^* - q)}, \tag{8}$$

where $k$ is von Kármán's constant. The depth of the spray layer must scale as Charnock's length if this theory holds:

$$L_{\mathrm{spray}} = C_s\frac{u_*^2}{g'}, \tag{9}$$

where $C_s$ is a dimensionless constant. Thus, neglecting the difference between $g'$ and $g$, the ratio of the depth of the spray layer to the Monin–Obukhov length is

$$R_{\mathrm{mo}} \equiv C_s k\frac{L_\nu}{C_pT}C(q_w^* - q). \tag{10}$$

If we take $C_s$ so as to give a (generous) spray layer depth of roughly 100 m at a 10-m wind speed of 50 m s$^{-1}$, $C = 0.05$, $k = 0.4$, $T = 300$ K, and a nominal surface relative humidity of 80%, then this ratio is about 0.07. Bearing in mind that this is an upper bound calculated using the extreme assumption that all the spray evaporates to calculate the maximum thermal buoyancy flux, it appears that we can neglect the buoyancy effects of evaporating spray. In that case, the enthalpy flux from the water to the air is given by (4), with $k$ denoting enthalpy.

### *c. Extension to a rotating system*

Consider another idealized system similar to that discussed in the previous section, but with a constant Coriolis parameter $f$. The horizontal pressure gradient acting on the air is now finite, such that the geostrophic wind $u_g$ is constant and independent of height. Now $u_g$, and not $u_*$, is the appropriate external parameter. Thus, instead of $R_u$, we have the dimensionless combination $R_{ug}$, defined as

$$R_{ug} \equiv \frac{u_g^4}{\sigma g'}. \tag{11}$$

In addition to the three length scales given by (2), we have a fourth length scale governing the height of the neutral boundary layer:

$$l_f \equiv \frac{u_g}{f}. \tag{12}$$

We also have a new dimensionless combination which we take here to be the ratio of the Charnock length to the neutral boundary layer depth, or the reciprocal of the surface Rossby number:

$$R_f \equiv \frac{u_g f}{g'}. \tag{13}$$

Observations make it clear that both the Charnock length and the neutral boundary layer depth are important scales in the system. We assume again that as $u_g$ becomes very large, $R_{ug}$ becomes large and that exchange coefficients based on the gradient wind become independent of $R_{ug}$. But here we are forced to make an additional hypothesis: in spite of large values of $u_g$, the Charnock length continues to be small compared to the neutral boundary layer depth; that is, $R_f$ is small. This is tantamount to assuming that the direct effect of rotation is small in the spray layer.

Therefore, within the spray layer, we assume that the similarity hypothesis pertaining to the nonrotating case continues to hold, and we further assume that $u_*$ scales with $u_g$. According to this hypothesis, the momentum flux from the air to the sea must be given by

$$u_*^2 = C_{Dg} u_g^2, \tag{14}$$

where the geostrophic drag coefficient is, in general, a function of $R_\sigma$ and $R_\nu$ (but not of $R_f$). Likewise, the flux of a passive scalar, in analogy to (4), is given by

$$F_s = C_g u_g (s_o - s_a), \tag{15}$$

where $C_g$ is a geostrophic exchange coefficient, likewise a function of $R_\sigma$ and $R_\nu$. Within the spray layer, the wind should vary with height according to

$$u = u_g G_g\left(\frac{g'z}{u_g^2}\right), \tag{16}$$

where $G_g$ is some function. The particular form or value of $G_g$ may depend on $R_\sigma$ and $R_\nu$.

This extension to the rotating case involves hypotheses beyond the single hypothesis that underlies similarity in the nonrotating case; these additional hypotheses are also subject to experimental evaluation.

Of course, in actual hurricanes, the Coriolis parameter itself plays almost no direct role in the intense core of the storm, where the predominant balance is more nearly cyclostrophic. In that case, the additional external parameter is the radius of curvature, not the Coriolis parameter, and to make the similarity hypothesis it is necessary to assume that length scales of interest in the spray layer are very small compared to the radius of curvature of the flow. Furthermore, $u_g$ should be regarded as the gradient, not the geostrophic wind.

## 3. Mechanistic argument

We here present a mechanistic argument for the ratio of the enthalpy flux coefficient to the drag coefficient in the limit of very high wind speeds. Consider the nature of the air–sea transition, as sketched in Fig. 2. There is a gradual transition from bubble-filled water to spray-filled air, and there is no longer a single identifiable interface between the two media. It is meaningless to speak in terms of interfacial fluxes.

It is convenient to define a certain level, $z = 0$, as the level of the undisturbed sea surface. We may define the volume fraction of water in terms of a probability distribution, whose mean is a function of altitude relative to this level. We suppose that in the fully developed state, the volume fraction of water at $z = 0$ is approximately 1/2. Since the density of water is nearly three orders of magnitude greater than that of air, it is likely that at this level the fluxes of all quantities of interest are predominantly carried by the water drops.

As illustrated in Fig. 2, the drops may be partitioned broadly into upward and downward moving sets. Collisions between drops will cause an exchange between these two sets, but we shall ignore that here. The net upward mass flux of droplets should be expected to exceed the net downward mass flux, owing to partial or complete evaporation of some of the drops or transport out of the boundary layer; however, the fractional amount by which it does so should be very small, as shown later in this section. Here we will assume that the exchanges of both enthalpy and momentum are dominated by relatively large, re-entrant drops.

To estimate the momentum flux, we assume that the bulk of the upward traveling component of the spray is accelerated to a horizontal velocity $u_{sp}$ characteristic of the spray layer. Taking $M_u$ to represent the upward mass flux of water, the downward flux of momentum into the spray layer necessary to accelerate the spray drops is

$$F \equiv \rho_a u_*^2 = M_u u_{sp}. \tag{17}$$

If we define a level $z_{sp}$ where the characteristic horizontal spray velocity is $u_{sp}$, then we can define a drag coefficient $C_D$ at that level, such that

$$F = \rho_a C_D u_{sp}^2, \tag{18}$$

hence we have, from (17),

$$M_u = \rho_a C_D u_{sp}. \tag{19}$$

The drag coefficient that appears in (18) and (19) is not identical to the conventional drag coefficient based on the wind at a fixed elevation. Assuming that the characteristic spray velocity scales with the wind velocity at some fractional depth of the spray layer, then $u_{sp}$ should scale as $u_g$, the gradient wind, in the rotating case.

Now consider the net enthalpy flux through $z = 0$. Again, we consider that the enthalpy flux is dominated by transport by the water drops at this level. According to the detailed microphysical calculations presented in Andreas (1995) and Andreas and Emanuel (2001), an evaporating spray droplet has a temperature very near the wet-bulb temperature of a saline, spherical droplet through most of its life. Here we assume that the upward traveling spray drops have a characteristic temperature equal to the undistributed water temperature $T_s$, while the downward traveling drops have a characteristic temperature equal to the wet-bulb temperature at the top of the spray layer $T_w$.[1] As shown presently, we may ignore the difference between the upward and downward spray mass transport. The net enthalpy flux across $z = 0$ is then

$$F_k = M_u C_l (T_s - T_w) = \rho_a C_D u_{sp} C_l (T_s - T_w), \quad (20)$$

where $C_l$ is the heat capacity of liquid water and we have made use of (19). Now the bulk aerodynamic formula for the enthalpy flux at level $z_{sp}$, expressed in terms of $u_{sp}$, is

$$F_k = \rho_a C_k u_{sp} (k_s^* - k_a), \quad (21)$$

where $k$ is the specific enthalpy, $C_p T + L_v q$, (where $C_p$ is the heat capacity at constant pressure of air, $L_v$ is the latent heat of vaporization, and $q$ is the specific humidity), and $C_k$ is the transfer coefficient for enthalpy appropriate to the level at which the characteristic spray velocity is $u_{sp}$. The subscripts $s$ and $a$ in (21) refer to saturation at undisturbed water temperature and evaluation in the free atmosphere, respectively. Comparing (20) to (21) gives

$$\frac{C_k}{C_D} = \frac{C_l (T_s - T_w)}{k_s^* - k_a}. \quad (22)$$

We can simplify (22) by relating the difference in wet-bulb temperatures to the difference in enthalpy. From the definition of wet-bulb temperature, we have

$$C_p (T_a - T_w) = L_v (q_w^* - q_a), \quad (23)$$

where $q_w^*$ is the saturation specific humidity at the wet-bulb temperature. Assuming that the wet-bulb temperature is not too different from the air temperature, we can use the Clausius–Clapeyron equation to expand the saturation specific humidity to first order:

$$q_w^* \simeq q_a^* + \frac{L_v q_a^*}{R_v T_a^2}(T_w - T_a), \quad (24)$$

where $R_v$ is the gas constant for water vapor. Substituting (24) into (23) and collecting terms gives

$$T_a - T_w = \frac{L_v (q_a^* - q_a)}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}} = \frac{L_v (q_s^* - q_a) + L_v (q_a^* - q_s^*)}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}} = \frac{L_v (q_s^* - q_a) + \frac{L_v^2 q_a^*}{R_v T_a^2}(T_a - T_s)}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}}, \quad (25)$$

where we have again made use of a linearization of the Clausius–Clapeyron equation. Finally,

$$T_s - T_w = \frac{C_l}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}} \times [C_p (T_s - T_a) + L_v (q_s^* - q_a)] = \frac{C_l}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}}(k_s^* - k_a). \quad (26)$$

Substituting (26) into (22) gives

$$\frac{C_k}{C_D} = \frac{C_l}{C_p + \frac{L_v^2 q_a^*}{R_v T_a^2}}. \quad (27)$$

To the extent that the similarity theory is also approximately valid in rotating flows, (27) should apply as well to exchange coefficients based on the gradient wind if an allowance is made for the difference between the gradient wind and $u_{sp}$.

That it is safe to equate the upward and downward spray mass flux near $z = 0$ can be demonstrated by comparing the net evaporation rate to the upward mass flux. The former can be estimated by noting that above the spray layer, the enthalpy flux is carried by the air. An upper bound on the water flux above the spray layer is obtained by assuming that all the enthalpy flux is in the form of latent heat flux, whence the maximum water flux is just the enthalpy flux divided by the heat of vaporization. Using (20), we have

$$E \simeq M_u \frac{C_l (T_s - T_w)}{L_v}.$$

For typical values of ocean and wet-bulb temperatures, $E/M_u \leq 0.01$; thus the difference between upward and downward spray mass fluxes near $z = 0$ is of the order of 1% or less.

The ratio of exchange coefficients predicted by (27) is a decreasing function of temperature, as shown in Fig. 3. For air temperatures characteristic of the Tropics, this ratio is around unity. Experience with numerical simulations of hurricanes indicates that in order to sim-

[1] The wet-bulb temperature $T_w$ for a small, saline water drop is greater than the nominal wet-bulb temperature. On the other hand, $T_w$ may be expected to be less than the wet-bulb temperature of undisturbed air at nominal sea level, owing to the upward decrease of $T_w$ in a well-mixed layer. Thus the enthalpy flux will be larger for deeper spray layers, all other things being equal. This reflects an additional enthalpy source owing to spray drops moving through an ambient temperature gradient. Here we assume that the equilibrium temperature of the falling spray drops is equal to the nominal wet-bulb temperature at the top of the spray layer.

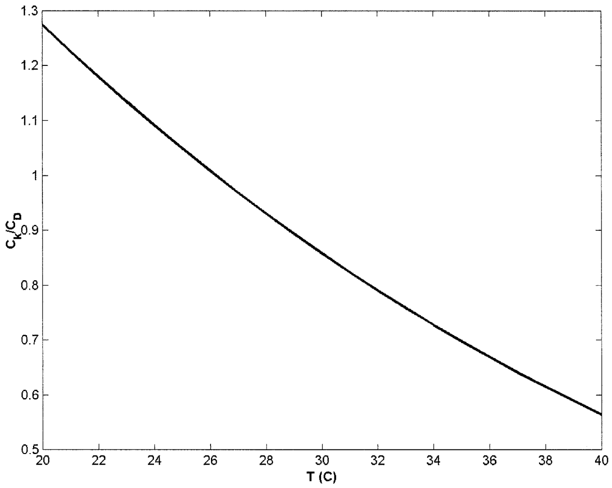


FIG. 3. The ratio of the enthalpy exchange coefficient to the drag coefficient, each based on the gradient wind, according to (27). The ratio is plotted as a function of the undisturbed air temperature.

ulate hurricane intensity with any accuracy, this ratio should not be too different from unity (Emanuel 1995a; Braun and Tao 2000).

Similarity predicts that at very high wind speeds, the exchange coefficients based on the gradient wind should be independent of wind speed, while the mechanistic argument presented in this section predicts that the enthalpy exchange coefficient should depend on temperature, as given by (27). These are testable hypotheses.

## 4. Numerical simulations

As a preliminary test of the proposition that the exchange coefficients become independent of wind speed at high winds, we test trial formulations of the exchange coefficients in a simple coupled ocean–atmosphere model of hurricane intensity, developed by the author (Emanuel 1999). This model consists of an axisymmetric, balanced hurricance model (Emanuel 1995b) coupled with a string of one-dimensional ocean columns along the track of the storm center. These ocean columns mix cold water to the surface when driven by surface wind stresses. As shown by Schade (1997), this simple ocean scheme produces feedbacks on the modeled hurricane intensity that are indistinguishable from those produced by a fully three-dimensional ocean model. Experience with this simple coupled model suggests that it is capable of accurate hindcasts of hurricane intensity in those events that are relatively unaffected by vertical wind shear (Emanuel 1999).

We test three formulations of the gradient wind-based surface exchange coefficients. The first is a close approximation to the formulation proposed by Large and Pond (1982) and used in many numerical models:

$$C_D = 8 \times 10^{-4} + 4 \times 10^{-5}V,$$
$$C_k = 8 \times 10^{-4}, \tag{28}$$

where $V$ is the gradient wind speed in meters per second. (In the Large and Pond formulation, the enthalpy exchange coefficient actually decreases slightly with wind speed, and the coefficients differ from the ones used here, but these authors expressed their results in terms of exchange coefficients operating on the 10-m wind speed.) The second formulation is similar to the first, but $C_k = C_D$; that is, both coefficients increase linearly with wind speed. The third formulation is similar to the second, but both exchange coefficients are capped at the values they attain when $V = 30$ m s$^{-1}$. The results are shown in Fig. 4 for hindcasts of Hurricanes Hugo, Andrew, Opal, and Camille. While these hindcasts are also somewhat sensitive to other model parameters, no set of parameters can overcome the severe underprediction of all events when the Large and Pond (1982) formulation is used. On the other hand, the results of capping the exchange coefficients at constant values, suggested by the similarity theory advanced here, are not significantly different from those using exchange coefficients that depend linearly on wind speed for all wind speeds.

Thus while the simulations presented here strongly suggest that the Large and Pond (1982) formulation is inadequate at wind speeds beyond those for which it was developed, they do not provide evidence for or against the similarity theory presented here.

## 5. Summary

As wind speeds increase over the ocean, the whole character of the sea surface changes, a fact made use of in the well-known Beaufort scale used by mariners for many years. But at the extreme wind speeds encountered in hurricanes, the sea surface becomes enveloped in spray and spume, to the extent that the transition between air and water begins to resemble an emulsion, with bubble-filled water gradually transitioning to spray-filled air. The dimensional analysis undertaken here assumes that at sufficiently high wind speeds, the transition becomes self-similar, with all relevant lengths scaling as the Charnock length. This in turn suggests that the rate of exchange of any passive scalar is proportional to the friction velocity $u_*$ or, in the case of a rotating system, to the gradient wind speed. Thus the exchange coefficients based on $u_*$ or the gradient wind should be constant. The mechanistic argument presented in section 3 predicts that the ratio of the enthalpy exchange coefficient to the drag coefficient should be a function of temperature, with values of order unity. This is consistent with the values of this ratio that are necessary to achieve realistic simulations of hurricane intensity (Emanuel 1995a). While tests of the predicted wind independence of the exchange coefficients in a numerical hurricane model are inconclusive on this point, they do show that the formulations often used at

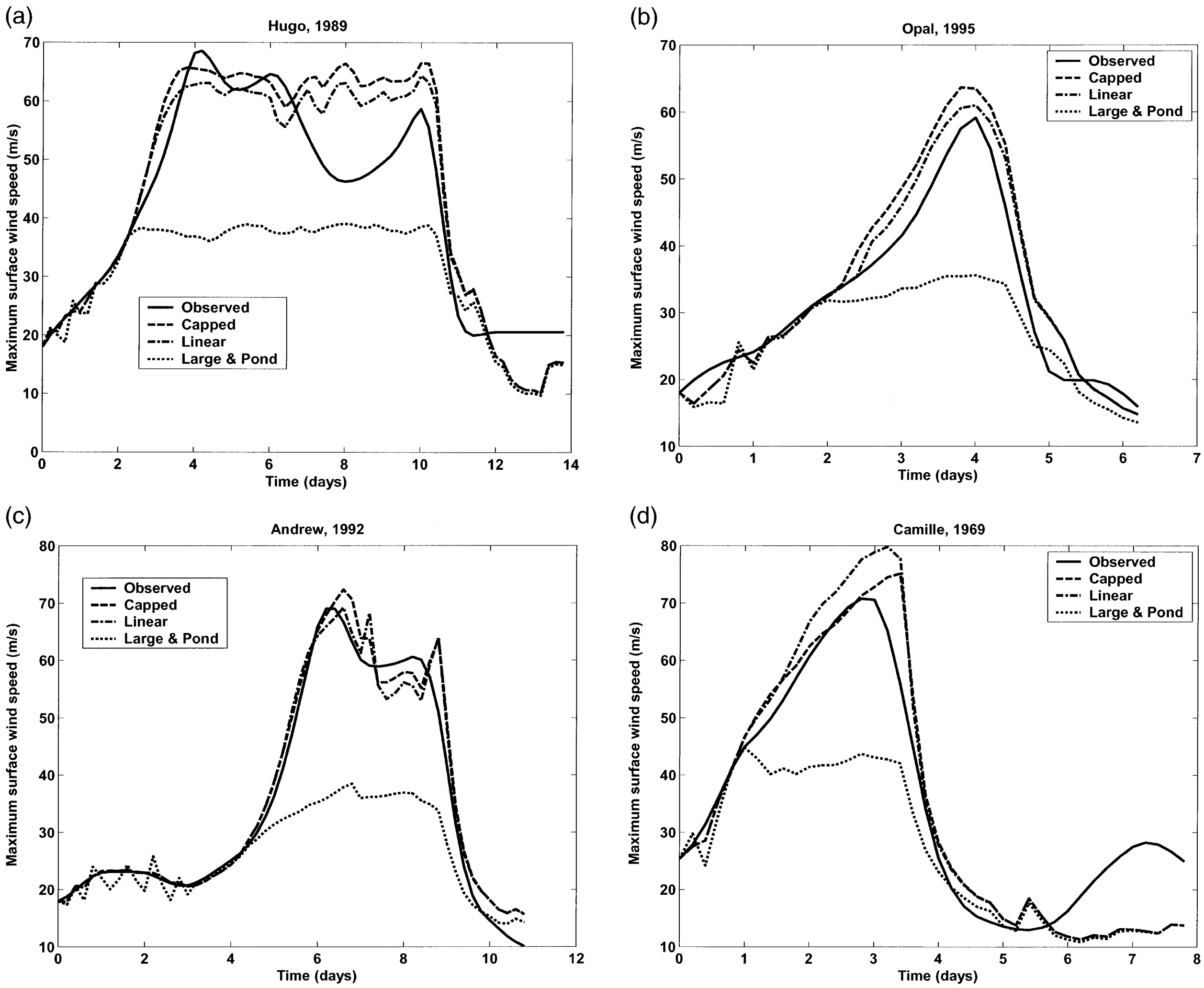


FIG. 4. Simulations of the evolution of the maximum wind speed with time in Atlantic Hurricanes (a) Hugo of 1989, (b) Opal of 1995, (c) Andrew of 1992, and (d) Camille of 1969, using the coupled model of Emanuel (1999). In each case, a simulated radial flux of low enthalpy into the core is adjusted continuously in time so that the modeled intensity matches as closely as possible the observed intensity for the first 2 days of the simulation, except in the case of Hurricane Opal where the matching is done for only 1 day, and Hurricane Andrew for which the matching is maintained for 4 days. In each plot, the solid curve shows the observed maximum wind speed, while those simulated with the model are shown by the dashed line, with exchange coefficients that vary linearly with wind speed but are capped at the values they have when $V = 30$ m s$^{-1}$; the dashed–dotted line, with uncapped linearly varying coefficients; and the dotted line, with coefficients given by (28).

low wind speed, based on the dominance of surface waves on drag, are greatly inadequate for simulating hurricanes.

An attractive aspect of the similarity theory is its prediction that surface fluxes are simple functions of gradient wind speeds and ambient thermodynamic conditions. The use of exchange formulations based on detailed consideration of spray drop microphysics and wave drag (e.g., Andreas and Emanuel 2001) lead to hurricane intensity predictions that are unrealistically sensitive to small details in the formulations. But, as shown here and in Emanuel (1999), very simple exchange formulations suffice for accurate hindcasts of the intensity evolution of many hurricanes.

Recent experimental deductions of the drag coefficient (Alamaro et al. 2002) suggest that its wind dependence is reduced at wind speeds characteristic of hurricanes. Although the deduction of air–sea exchange at very high wind speeds poses severe challenges for field and laboratory measurements, the importance of the issue for hurricane intensity forecasting makes such efforts worthwhile.

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