MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF PHYSICS

DEPARTMENT OF EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Physics: 8.292J EAPS: 12.330J

QUIZ #2

1. [35 points]
   A finite one-dimensional (i.e., planar) disturbance, which has not yet sharpened into a shock front, propagates through an ideal gas with ambient pressure and density P_0 and ρ_0, respectively. The ambient gas is at rest, and the ratio of specific heats is γ everywhere.

   (a) Determine the ratio, ζ, of pressure to density (P/ρ) for a point on the disturbance at which the propagation velocity is u.

   (b) Determine the propagation velocity, u', for a point on the disturbance at which the ratio of pressure to density is $\zeta/4$.

2. [30 points]
   A one-dimensional (i.e., planar) shock wave propagates with speed $v_{\rm sh}$ through an ideal gas of mean molecular weight μ everywhere. The unshocked fluid is at rest and has density and pressure P_0 and ρ_0, respectively. The gas velocity behind the shock is u. The adiabatic exponent is *not* necessarily the same on both sides of the shock. Determine the density, ρ_1, pressure, P_1, and temperature, T_1, of the gas behind the shock.

3. [35 points]
   An ideal gas of heat capacity c_p and gas constant R $(= R^*/\mu)$ is at rest, subject to a uniform gravitational acceleration g. The gas is isentropic...that is, it has the same specific entropy content everywhere. Derive expressions for the distributions of temperature and pressure in the gas, taking the surface temperature and pressure to be T_0 and P_0, respectively. Do you notice any peculiarities of these solutions? If so, describe how they arise.

SOME POTENTIALLY USEFUL FORMULAS

$$P = \frac{1}{3}\int_{p=0}^{\infty} pv(p)n(p)dp \ \ ; \ \ \alpha P = \frac{R^*}{\mu}T \ \ ; \ \ \epsilon = \left(\frac{1}{\gamma - 1}\right)\frac{R^*}{\mu}T \ \ ; \ \ P = K\rho^\gamma \ \ ; \ \ \alpha = 1/\rho$$

$$-\nabla P + \rho \vec{g} = 0$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \nabla)$$

$$\frac{d\rho}{dt} + \rho \nabla \cdot \vec{v} = 0$$

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla P + \vec{g}$$

$$c_{\mathrm{shallow}} = \sqrt{gh}$$

$$\frac{d}{dt}\left[\frac{1}{2}v^2 + \frac{1}{\rho}P + gh\right] = 0$$

$$\rho(x,t) = f\left(x - [c_s(\rho) + v(\rho)]\, t\right) \ \ ; \ \ c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \ \ ; \ \ v(\rho) = \int_{\rho_0}^{\rho} \frac{c_s(\rho')}{\rho'} d\rho'$$

$$\rho_0 u_0 = \rho_1 u_1 \ \ ; \ \ P_0 + \rho_0 u_0^2 = P_1 + \rho_1 u_1^2 \ \ ; \ \ \frac{1}{2}u_0^2 + \epsilon_0 + P_0/\rho_0 = \frac{1}{2}u_1^2 + \epsilon_1 + P_1/\rho_1$$

$$dQ = c_p dT - \alpha dP$$