MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
DEPARTMENT OF EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Physics: 8.292J EAPS: 12.330J

Quiz

1. A rectangular dam of length L holds back the water in a lake. The density of the water is ρ, and the depth of the water just behind the dam is H. The air pressure is P_a everywhere.

   (a) Determine the magnitude, F, of the *net* pressure force exerted on the dam.

   (b) Determine the magnitude, τ, of the net torque exerted on the dam about an axis passing along the base of the dam.

   (c) Use a sketch to show the directions of the net force and net torque exerted on the dam.

2. An infinitely long circular cylinder of radius A is composed of a solid with a uniform mass density ρ_c. The cylinder is surrounded by an atmosphere composed of an isothermal ideal gas of temperature T, mean molecular weight μ, and pressure P_0 at the surface of the cylinder. In parts (a), (b), and (c) below, neglect the self-gravity of the atmosphere.

   (a) Determine the magnitude, $g(h)$, of the gravitational field due to the cylinder as a function of the height, h, above the *surface* of the cylinder. [HINT: Rather than integrating directly, it is easier to use Gauss's Law. Simply replace the factor $1/4\pi\epsilon_0$ (as it appears in Coulomb's Law) with the universal gravitational constant, G.]

   (b) Determine the atmospheric density profile, $\rho(h)$ as a function of h, in terms of the given quantities and constants of nature.

   (c) Determine the height, $h_{1/2}$, at which the density of the atmosphere has fallen to one-half of its surface value.

(d) Determine the total mass of the atmosphere, and explain briefly whether neglecting the self-gravity of the atmosphere was, in fact, physically justified.

3. Consider two immiscible Euler fluids of density ρ_1 and ρ_2, respectively, at rest on an infinite plane and exposed to a uniform gravitational acceleration, g. The atmospheric pressure at the top of the system is P_a. The fluid of the smaller density is on top of the other fluid. The depth of the lower layer of fluid is h and the total depth of the fluid system is H. Find the dispersion relation for small amplitude disturbances whose wavelength is long compared to H. Write an approximate form of this relation valid when $|\rho_2 - \rho_1| \ll \rho_1$ and interpret the result.

SOME POTENTIALLY USEFUL FORMULAS

$$P = \frac{1}{3}\int_{p=0}^{\infty} pv(p)n(p)dp \ ; \ \alpha P = \frac{R^*}{\mu}T \ ; \ \alpha = 1/\rho$$

$$-\nabla P + \rho \vec{g} = 0$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \nabla)$$

$$\frac{d\rho}{dt} + \rho \nabla \cdot \vec{v} = 0$$

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla P + \vec{g}$$

$$c_{\text{shallow}} = \sqrt{gh}$$

$$\frac{d}{dt}\left[\frac{1}{2}v^2 + \frac{1}{\rho}P + gh\right] = 0$$