FLUID PHYSICS

12.330J/8.232J

Final Examination

(Closed Book)

Spring 2000

1. [30 points] Consider the flow of an inviscid, incompressible fluid on a rotating planet. In class, we showed that, to a good approximation, the equations of motion for such a fluid are

$$\frac{du}{dt} = -\alpha_0 \frac{\partial P}{\partial x} + fv,$$
$$\frac{dv}{dt} = -\alpha_0 \frac{\partial P}{\partial y} - fu,$$
$$\frac{dw}{dt} = -\alpha_0 \frac{\partial P}{\partial z} - g,$$

where x, y, and z are the eastward, northward, and upward directions, u, v, and w are the velocity components in those respective directions, and f is the Coriolis parameter, which for the purposes of this problem you may consider to be a constant. The specific volume, α_0, is strictly constant. Find the dispersion relation for small disturbances to the resting state of such a fluid. Do such disturbances correspond to any of the wave phenomena discussed in class? If time permits, discuss the physics of waves in this system.

2. [40 points] A shallow layer of incompressible, inviscid fluid of depth h_0 flows through an initially straight channel of width L at a uniform velocity U_0. The system is under a uniform gravitational acceleration g, and the atmospheric pressure at the surface of the fluid is constant. Somewhere down along the length of the channel, the channel takes a symmetric bend to the right, emerging as a straight channel at right angles to its former course. At the point of

maximum curvature, the radius of curvature at the mid-point of the channel is R. Assuming that the flow is steady and that the flow streamlines are always parallel to the channel walls, find the cross-channel profile of the flow at the place where the channel has maximum curvature. (Note: You may not be able to find a closed form expression for an integration constant; just write down the equation for it. If time permits, find a closed form expression in the limit that $L \ll R$ and $U_0^2 \ll gh_0$.)

3. [30 points] Quasi-geostrophic Rossby waves can be excited by a variety of processes, including atmospheric flow over large-scale topography, such as the Rocky Mountains and the Himalayas. One interesting issue is whether and how these waves propagate upward into the stratosphere. We can estimate this using the conservation of quasi-geostrophic potential vorticity:

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_g \cdot \nabla\right) q_p' = -\beta v_g, \tag{1}$$

where v_g is the northward component of the geostrophic flow, and q_p' is the perturbation potential vorticity, defined

$$q_p' = \frac{1}{f_0}\nabla^2\phi' + \frac{f_0}{N^2}\frac{\partial^2\phi'}{\partial z^2}. \tag{2}$$

If we linearize (1) about a mean west-to-east flow flow, $\overline{U}$, the only consequence is that $\mathbf{V}_g \cdot \nabla$ in (1) is replaced by $\overline{U}\partial/\partial x$. Look for modal solutions to (1) with (2) of the form

$$e^{ik(x-ct)+imz}.$$

Since the mountains are stationary, c will be zero. Find an expression for the vertical wavenumber, m.

Now it turns out that your solution will remain approximately valid even if $\overline{U}$ varies slowly with altitude. (In that case, m will vary with altitude too.) In the troposphere, the mean flow is from the west all year round (though it is stronger in the winter). In the stratosphere, however, the winds are from the west in the winter and switch to blowing from the east in summer.

During what season is one most likely to observed large amplitude, stationary Rossby waves in the stratosphere? Assuming for the moment that the mountains directly excite waves of all horizontal scales, what horizontal scales are we most likely to see in the stratosphere? What will the stratospheric flow look like in summer?