Fluid Physics

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Problem Set 2

1. Consider the steady flow of an Euler fluid over a weir, as sketched below:

   The height of the fluid above $z = 0$ (the bottom of the reservoir) is $H(x)$ with an undisturbed value of H_0 upstream, well away from the weir, where we can assume that the fluid velocity is approximately zero. The total mass flow rate through the system is Q. The weir itself is elevated a distance $d(x)$ above the bottom of the reservoir. Using the Bernoulli equation and mass continuity, find the flow velocity $u(x)$ as a function of H_0, Q, and $d(x)$. At the highest point of the weir, d_{max}, find that value of u that minimizes H_0 given Q, and find the associated minimum value of H_0 and the local height of the fluid surface H at the point where $d = d_{\text{max}}$. Interpret these values.

2. A *density current* is a special class of fluid flow that can be easily approximated in the laboratory. Consider an apparatus consisting of a long tank with a central barrier separating two Euler fluids of differing density:

   To begin the experiment, the barrier is suddenly removed. The heavier fluid begins to flow under the lighter fluid, so that successive positions of the boundary between the two fluids look like:

   After awhile the noses of the advancing fluid become nearly steady in a coordinate system moving at the speed c of the nose. In that coordinate system, the flow looks like:

Approximate this as the steady flow of an Euler fluid.

a) Integrate the Bernoulli equation from point A, far to the right of the nose of the density current, to point C far downstream, where the flow velocity is u. Thereby find a relationship among p_0, p_2 (the pressure at C), c, u, and h.

b) Using the fact that the total integrated mass flow far to the right must equal that far to the left, find a relationship among u, c, H, and h.

c) The steady state x component of Euler's equation evaluated along the bottom plate is

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Integrate this fom A to S to find a relationship among C, p_0, and p. Assume that $u = 0$ at S.

d) Making use of the fact that $w = 0$ far to the left, integrate the hydrostatic equation between points B and C to find a relationship among p_2, p_1, ρ_2, and h.

e) Finally, eliminate the pressures and u from the above to find an expression for c as a function of H, h, ρ_2, and ρ_1. To what does this expression reduce in the special case that $H = 2h$? That $H \to \infty$?