Fluid Physics
12.330J/8.232J

Final Examination
(Closed Book)

1. In class, we discussed the development of extratropical disturbances as consequences of baroclinic instability and illustrated the concept through the Eday model using conservation of quasi-geostrophic potential vorticity. We found an instability that results in generally eastward-propagating disturbances of roughly 1000 km scale that take the form, at the suface, of high and low presure systems. We did not say anything, on the other hand, about rain and clouds.

Clouds and rain generally result from rising motion in the atmosphere. The rising air cools owing to adiabatic expansion, and the air thereby becomes saturated with respect to water, leading to clouds and precipitation. The ascent rates are of order 1 $cm\ s^{-1}$ and are therefore almost impossible to detect directly. One method of estimating the vertical motion is through the use of the *quasi-geostrophic vorticity equation*, which may be written

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_{\mathrm{g}} \cdot \nabla\right) \eta = f_0 \frac{\partial w}{\partial z}, \tag{1}$$

where

$$\eta = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y},$$

w is the vertical velocity, and the other symbols have their usual meanings. Remember the geostrophic relations between velocity and geopotential:

$$u_g = -\frac{1}{f_0}\frac{\partial \phi}{\partial y},$$

$$v_g = \frac{1}{f_0}\frac{\partial \phi}{\partial x}.$$

At the surface, the background geostrophic wind tends to be small, and (1) is dominated by a balance between the right side and the *first* term on the left side. Using this, describe the relationship you would expect between clouds and precipitation and the eastward-propagating surface low and high pressure systems. How would you use a barometer to detect changes in the weather?

2. Quasi-geostrophic Rossby waves can be excited by a variety of processes, including atmospheric flow over large-scale topography, such as the Rocky Mountains and the Himilayas. One interesting issue is whether and how these waves propogate upward into the stratosphere. We can estimate this using the conservation of quasi-geostrophic potential vorticity:

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_{\mathrm{g}} \cdot \nabla\right) q'_p = -\beta v_g, \tag{2}$$

where v_g is the northward component of the geostrophic flow, and q'_p is the perturbation potential vorticity, defined

$$q'_p = \frac{1}{f_0}\nabla^2\phi' + \frac{f_0}{N^2}\frac{\partial^2\phi'}{\partial z^2}. \tag{3}$$

If we linearize (2) about a mean west-to-east flow, $\overline{U}$, the only consequence is that $\mathbf{V}_{\mathrm{g}} \cdot \nabla$ in (2) is replace by $\overline{U}\partial/\partial x$. Look for modal solutions to (2) with (3) of the form

$$e^{ik(x-ct)+imz}.$$

Since the mountains are stationary, c will be zero. Find an expression for the vertical wavenumber, m.

Now it turns out that your solution will remain approximately valid even if $\overline{U}$ varies slowly with altitude. (In that case, m will vary with altitude too.) In the troposphere, the mean flow is from the west all year round (though it is stronger in the winter). In the stratosphere, however, the winds are from the west in the winter and switch to blowing from the east in summer.

During what season is one most likely to observe large amplitude, stationary Rossby waves in the stratosphere? Assuming for the moment that the mountains directly excite waves of all horizontal scales, what horizontal scales are we most likely to see in the stratosphere? What will the stratospheric flow look like in summer?