MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
DEPARTMENT OF EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Physics: 8.292J EAPS: 12.330J

Assignment #6

Problems are due Wednesday, 5/9//96, before 4:00 PM, in 4–334

Problems

1. In class, we considered the stability of a class of two-dimensional flows in the x direction, whose velocity varies in the y direction. We developed a general equation for linear modal velocity perturbations, v, of the form

$$\frac{d^2\tilde{v}}{dy^2} - \tilde{v}\left[k^2 + \frac{d^2\overline{U}/dy^2}{\overline{U} - c}\right], \tag{1}$$

where $\overline{U}$ is the mean flow in the x direction, and the velocity perturbation in the y direction is given by

$$v = REAL\left[\tilde{v}e^{ik(x-ct)}\right],$$

with c a phase speed *which is in general complex*. In class, we solved equation (1) for the very special case of a piecewise continuous mean flow, $\overline{U}$, which varied linearly within each region.

Now consider a general class of mean flows $\overline{U}(y)$ bounded by rigid walls at $y = \pm L$, along which, by definition, $v = 0$. We will attempt to prove some general theorems about the stability of this general class of flows as follows:

   a. Multiply equation (1) through by $\tilde{v}^*$, the complex conjugate of $\tilde{v}$, and integrate the result over the domain bounded by $y = \pm L$. Using the boundary condition on v, integrate the first term by parts. Make use of the fact that the product of any complex number and its complex conjugate is equal to the absolute value of that number. Show that the first term is a negative definite real quantity.

b. Remembering that c is, in general, a complex quantity, find the imaginary part of the equation derived in (a) above. Use the result to formulate, in words, a general statement about the character of the mean flow, $\overline{U}$, that must be satisfied if the flow is to be unstable. Is this a necessary condition for instability? A sufficient condition? Both?

c. Now find the real part of the equation derived in (a) above. Use this together with the result of (b) to formulate a second statement about the stability of the mean flow.

d. Are the following flows provably stable?

1. $\overline{U} = sin\left(\frac{\pi y}{2L}\right)$.
2. $\overline{U} = ye^{y^2}$.