MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
DEPARTMENT OF EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Physics: 8.292J EAPS: 12.330J

Assignment #2

Reading

Faber, pp. 78–105

<u>Problems are due Tuesday, 3/5/96, before 4:00 PM, in 4–334</u>

Problems

1. The principle gaseous components of the earth's atmosphere below 100 km, excluding water vapor, are molecular nitrogen (N_2, $\mu = 28.016$), molecular oxygen (O_2, $\mu = 32.00$), and argon (A, $\mu = 39.94$). The fraction of the total number of molecules in a sample of air for these three components are, approximately, 78%, 21%, and 1%, respectively.

   a. The effective gas constant for dry air, R_d, is defined as

   $$R_d \equiv R^*/\overline{\mu}_d,$$

   where $\overline{\mu}_d$ is a suitably defined average molecular weight of the constituents. Write an express for $\overline{\mu}_d$ and determine the numerical value of R_d.

   b. Water vapor can comprise up to 4% of the total number of molecules in a sample (with the percentage of other constituents reduced proportionately). The molecular weight of H_2O is 18.02. The variability of water vapor can be effectively accounted for by defining an effective gas constant $\overline{R}(q)$, where q is the *specific humidity,* which is the mass of water vapor in a unit mass of air. Determine the functional dependence of $\overline{R}$ on q.

c. Atmospheric scientists prefer to account for the effects of variable water content on density by using a quantity called *virtual temperature,* T_v, which is defined such that

$$R_d T_v \equiv \overline{R} T.$$

Write an expression for T_v, and write the ideal gas law in terms of R_d, T_v, ρ, and p.

d. A sample of moist air has the properties $p = 10^5 N\ \mathrm{m}^{-2}$, $T = 330$ K, $q = .02$. At the same pressure, what temperature would a sample have to have with $q = 0$ to yield the same density as the moist sample described above?

2. A cylindrical vessel contains an Euler fluid, which is rotating about the longitudinal axis of the cylinder. The velocity of rotation, V, is a function of radius r from the rotation axis.

   a. Write down Euler's equations cast in a cylindrical coordinate system r, z, θ, where r is radius, z is distance in the longitudinal direction, and θ is azimuth.

   b. Show that in the absence of variations in the θ direction the quantity M is conserved, where $M = rV$.

   c. Consider an infinitesimal ring of fluid located at r, z. Suppose this fluid ring is displaced radially outward a small distance δr. Assume that this displacement does not in any way affect the pressure distribution in the fluid; that is, $\partial p/\partial t = 0$ everywhere during the displacement. Examine the force balance on the displaced ring and write an expression for the radial acceleration of the displaced ring.

   d. Show that the expression derived in c above is a wave equation given a certain restriction on the distribution of M with r. State that condition and derive an expression for the oscillation frequency of the ring when that condition is met.

3. A completely rigid cylinder in a uniform gravitational field is completely filled with an Euler fluid of density ρ, which is at rest. Within the fluid is a small

balloon filled with an ideal gas. This balloon is attached to a string whose volume is negligible, by which means the balloon's vertical position can be changed. The tension of the balloon's material can be neglected, so the gas may be taken to be in pressure equilibrium with the adjacent Euler fluid. At the initial time the fluid pressure at the position of the balloon is p_0, at an altitude of z_0 above the bottom of the cylinder.

The balloon is then *slowly* raised or lowered to a new position z_1 at time t_1. Describe the pressure distribution in the fluid at time t_0 and at time t_1.