MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
DEPARTMENT OF EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Physics: 8.292J EAPS: 12.330J

Assignment #4

Reading

Faber, pp. 119–145, 240–262

Problems are due Tuesday, 4/2/96, before 4:00 PM, in 4–334

Problems

1. An ideal gas contained in a cylinder is kept under pressure by force applied to a piston. Near the piston, the gas pressure is P_0 and the temperature is T_0. The gas escapes through a small aperture at the opposite end of the cylinder from the piston. The gas pressure outside the cylinder is P_1, where $P_1 < P_0$. Ignoring gravitational effects and assuming that the flow is steady, adiabatic and inviscid, derive expressions for the gas velocity and mass flow rate per unit area normal to the flow at the exit aperture. As P_0 increases, show that there exists an upper bound for the mass flow rate per unit area, and write expressions for the maximum values of P_0/P_1 and the maximum mass flow rate per unit area. Also derive and compare expressions for the gas velocity and the *local* sound speed at the aperture when the mass flow rate is at its maximum value. What do you think happens when P_0 exceeds the value that maximizes the mass flow rate?

2. Consider an ideal gas at rest on a level surface under constant gravitational acceleration. The gas is saturated with water vapor and contains a uniform suspension of very small liquid water droplets, which may be considered to be in thermodynamic equilibrium with the water vapor. A small change in the thermodynamic state of the system results in evaporation or condensation of water, with an amount of heating per unit mass of air given by

$$\dot{Q} = -L_v \frac{dq}{dt},$$

where L_v is the latent heat of vaporization of water and q is the specific humidity (mass of water vapor per unit mass of air). The Clausius-Clapeyron equation relates the *saturation* specific humidity, q^*, to pressure and temperature:

$$q^* = \frac{a}{P} e^{-b/T},$$

where a and b are constants. (Note that in this system $q = q^*$ everywhere.) Now consider changes in the system for which the *only* heating is owing to condensation or evaporation, and for which the hydrostatic equation applies.

a. Derive an expression for the **moist adiabatic lapse rate**, Γ_m, defined

$$\Gamma_m = -\left(\frac{dT}{dz}\right)_s,$$

where s denotes constant entropy (no heating other than latent heating). Compare this to the (dry) adiabatic lapse rate (Γ_d) in the same gas but without water substance. Your expression should be in terms of one or more of a, b, L_v, R, C_p, T and q^*.

b. Calculate the ratio Γ_m/Γ_d for $P = 10^5 Kg\ m^{-1} s^{-2}$ and (1) T= 0 C (273 K), (2) T=30 C (303 K), given the following constants:

$$\begin{aligned} a &= 1.61x10^{11} Kg\ m^{-1} s^{-2} \\ b &= 5.42x10^3 K \\ L_v &= 2.5x10^6 J\ Kg^{-1} \\ R &= 287 J\ Kg^{-1}\ K^{-1} \\ C_p &= 10^3 J\ Kg^{-1}\ K^{-1} \end{aligned}$$